10709. *Proposed by Zoltán Sasvári, Technical University of Dresden, Dresden, Germany.* Let X be a standard normal random variable, and choose $y > 0$. Show that

$$e^{-ay} < \frac{Pr(a \le X \le a+y)}{Pr(a-y \le X \le a)} < e^{-ay+(1/2)ay^3}$$

when $a > 0$. Show that the reversed inequalities hold when $a < 0$.

10710. *Proposed by Bogdan Suceava, Michigan State University, East Lansing, MI.* Let ABC be an acute triangle with incenter I, and let D, E, and F be the points where the circle inscribed in ABC touches BC, CA, and AB, respectively. Let M be the intersection of the line through A parallel to BC and DE, and let N be the intersection of the line through A parallel to BC and DF. Let P and Q be the midpoints of DM and DN, respectively. Prove that A, E, F, I, P, and Q are on the same circle.

SOLUTIONS

When O-H-I Is Isosceles

10547 [1996, 695]. *Proposed by Dan Sachelarie, ICCE Bucharest, and Vlad Sachelarie, University of Bucharest, Bucharest, Romania.* In the triangle ABC, let O be the circumcenter, H the orthocenter, and I the incenter. Prove that the triangle OHI is isosceles if and only if

$$\frac{a^3+b^3+c^3}{3abc} = \frac{R}{2r}.$$

Solution by Walther Janous, Ursulinengymnasium, Innsbruck, Austria. We denote by MPV the reference D. S. Mitrinović, J. E. Pečarić, and V. Volenec, *Recent Advances in Geometric Inequalities,* Kluwer, 1989. Neither IO nor HI is ever as large as HO [MPV, p. 288], so the only way triangle IHO can be isosceles is if $IO = HI$. Also $IO^2 = R^2 - 2Rr$ [MPV, p. 279] and $HI^2 = 4R^2 + 4Rr + 3r^2 - s^2$ [MPV, p. 280], where s is the semiperimeter. Hence $HI = IO$ if and only if $R^2 - 2Rr = 4R^2 + 4Rr + 3r^2 - s^2$. This rearranges to $2s(s^2 - 3r^2 - 6Rr)/12Rrs = R/2r$, or, using $abc = 4Rrs$ [MPV, p. 52] and $a^3+b^3+c^3 = 2s(s^2 - 3r^2 - 4Rr)$ [MPV, p. 52], to $(a^3+b^3+c^3)/3abc = R/2r$.

Editorial comment. Another condition equivalent to $HI = IO$, given in problem E2282 [1971, 196; 1972, 397] from this MONTHLY, is that ABC has one angle equal to 60°.

Solved also by J. Anglesio (France), R. Barbara (Lebanon), F. Bellot Rosado (Spain), C. W. Dodge, J. S. Frame, Z. Franco, M. S. Klamkin (Canada), W. W. Meyer, V. Mihai (Canada), C. R. Pranesachar (India), B. Prielipp, V. Schindler (Germany), I. Sofair, M. Tabaâ (Morocco), T. V. Trif (Romania), M. Vowe (Switzerland), GCHQ Problems Group (U. K.), and the proposers.

The Divisible Differences Property

10553 [1996, 809]. *Proposed by Bjorn Poonen, Mathematical Sciences Research Institute, Berkeley, CA, Jim Propp, Massachusetts Institute of Technology, Cambridge, MA, and Richard Stong, Rice University, Houston, TX.* Say that a sequence $\langle q \rangle = q_1, q_1, q_2, \ldots$ of integers has the *divisible differences property* if $(n-m)|(q_n - q_m)$ for all n and m.
(a) Show that if $\langle q \rangle$ has the divisible differences property and $\limsup |q_n|^{1/n} < e - 1$, then there is a polynomial Q such that $q_n = Q(n)$.
(b) Show that there is a sequence $\langle q \rangle$ that has the divisible differences property and satisfies $\limsup |q_n|^{1/n} \le e$, for which q_n is not given by a polynomial in n.
(c)* Is it true that $\limsup |q_n|^{1/n} \ge e$ for all non-polynomial $\langle q \rangle$ with the divisible differences property?