by (4). Therefore $\lim_{n\to\infty} U_{n+1}/U_n = \lim_{n\to\infty} S_{n+1}/S_n = 1/4$. Putting $\lim_{n\to\infty} A_n = \lim_{n\to\infty} B_n = \lim_{n\to\infty} C_n = \pi/3$ into (5) yields $d_{n+1}^2/d_n^2 \to 1/16$, or $d_{n+1}/d_n \to 1/4$.

Solved also by J. Anglesio (France), G. L. Body (U. K.), R. J. Chapman (U. K.), J. E. Dawson (Australia), N. Lakshmanan, J. H. Lindsey II, P. G. Poonacha (India), V. Schindler (Germany), A. Tissier (France), and the proposer.

An Appearance of the Beta Function

10632 [1997, 975]. *Proposed by William F. Trench, Trinity University, San Antonio, TX.* For given nonnegative integers m and n, evaluate

$$\sum_{k=0}^{m} \frac{(-1)^k}{n+k+1}\binom{m}{k}(1-y)^{n+k+1} + \sum_{k=0}^{n} \frac{(-1)^k}{m+k+1}\binom{n}{k} y^{m+k+1}.$$

Solution by Ronald A. Kopas, Clarion University of Pennsylvania, Clarion, PA. The sum is $m!\,n!/(m+n+1)!$. To see this, note that

$$\int_0^y t^m(1-t)^n\,dt = \int_0^y t^m \sum_{k=0}^{n}\binom{n}{k}(-1)^k t^k\,dt$$
$$= \sum_{k=0}^{n}\binom{n}{k}(-1)^k \int_0^y t^{m+k}\,dt = \sum_{k=0}^{n}\binom{n}{k}(-1)^k \frac{y^{m+k+1}}{m+k+1}.$$

Substituting $1-t$ for t and then computing in the same way yields

$$\int_y^1 t^m(1-t)^n\,dt = \int_0^{1-y} t^n(1-t)^m\,dt = \sum_{k=0}^{m}\binom{m}{k}(-1)^k \frac{(1-y)^{n+k+1}}{n+k+1}.$$

Hence the desired sum equals $\int_0^1 t^m(1-t)^n\,dt$, which repeated integration by parts reduces to $m!\,n!/(m+n+1)!$.

Editorial comment. Most solvers first differentiated the given expression to show that it was independent of y. They then evaluated the expression at $y = 0$ or $y = 1$ and got the final result either by induction or by reducing it to the beta integral that appears in the published solution.

Solved also by U. Abel (Germany), K. F. Andersen (Canada), P. J. Anderson (Canada), J. Anglesio (France), G. W. Arnold, G. Bach (Germany), D. Beckwith, J. C. Binz (Switzerland), G. L. Body (U. K.), P. Bracken (Canada), D. Callan, R. J. Chapman (U. K.), Q. H. Darwish (Oman), M. N. Deshpande (India), P. Devaraj & R. S. Deodhar (India), S. B. Ekhad, Z. Franco & M. Wood, R. García-Pelayo (Spain), C. Georghiou (Greece), T. Hermann, V. Hernández & J. Martín (Spain), D. Huang, G. Kesselman, M. S. Klamkin (Canada), R. A. Leslie, N. F. Lindquist, J. H. Lindsey II, S. McDonald & K. Adzievski, J. G. Merickel, C. A. Minh, D. A. Morales (Venezuela), R. G. Mosier, A. Nijenhuis, M. Omarjee (France), G. Peng, H. Qin, V. Schindler (Germany), H.-J. Seiffert (Germany), P. Simeonov, N. C. Singer, J. H. Steelman, R. F. Swarttouw (The Netherlands), A. Tissier (France), E. I. Verriest, M. Vowe (Switzerland), H. Widmer (Switzerland), M. Woltermann, Y. Yang, Q. Yao, Anchorage Math Solutions Group, BARC Problems Group (India), GCHQ Problems Group (U. K.), NSA Problems Group, WMC Problems Group, and the proposer.

Apéry's Constant

10635 [1998, 68]. *Proposed by Nicholas R. Farnum, California State University, Fullerton, CA.* Show that the value of $\zeta(s) = \sum_{k=1}^{\infty} k^{-s}$ at $s = 3$, also called Apéry's constant, can be expressed as $\zeta(3) = \sum_{n=1}^{\infty} r_n/n$, where $r_n = (\pi^2/6) - \sum_{k=1}^{n} k^{-2}$ is the nth remainder of the series expansion of $\zeta(2)$.

Solution by Alain Tissier, Montfermeil, France. We prove a generalization: For each positive integer k,

$$k!\,\zeta(k+2) = \sum_{n_1=1}^{\infty}\sum_{n_2=1}^{\infty}\cdots\sum_{n_k=1}^{\infty} \frac{1}{n_1 n_2 \ldots n_k} \sum_{p=1+n_1+n_2+\cdots+n_k}^{\infty} \frac{1}{p^2}. \qquad (*)$$