

10724



Serge Tabachnikov

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**10722.** Proposed by Richard F. McCoart, Loyola College, Baltimore, MD.

(a) In how many ways can  $2n$  indistinguishable balls be placed into  $n$  distinguishable urns, if the first  $r$  urns may contain at most  $2r$  balls for each  $r \in \{1, 2, \dots, n\}$ ?

(b) Suppose that  $0 \leq m \leq n$ . In how many of the ways enumerated in part (a) are exactly  $m$  urns empty?

**10723.** Proposed by Christopher J. Hillar, Yale University, New Haven, CT. Let  $p$  be an odd prime. Prove that  $\sum_{i=1}^{p-1} 2^i \cdot i^{p-2} \equiv \sum_{i=1}^{(p-1)/2} i^{p-2} \pmod{p}$ .

**10724.** Proposed by Serge Tabachnikov, University of Arkansas, Fayetteville, AR.

(a) Let  $P$  be a convex plane polygon with vertices  $A_1, \dots, A_n$ , and let  $l$  be a continuous transverse field of directions along the boundary  $\partial P$ . (This means that through every point  $X \in \partial P$  there passes a line  $l(X)$  that intersects the interior of  $P$  and depends continuously on  $X$ .) Let  $\alpha_i$  and  $\beta_i$  be the angles between the line  $l(A_i)$  and the adjacent sides  $A_i A_{i-1}$  and  $A_i A_{i+1}$ , respectively. Assume that  $\prod_1^n \sin \alpha_i = \prod_1^n \sin \beta_i$ . Prove that the lines  $l(X)$  cover the interior of  $P$  twice, that is, every interior point of  $P$  belongs to at least two of these lines.

(b) Suppose  $n \geq 3$ , and let  $P$  be a convex polyhedron in  $n$ -dimensional space. As in (a), a continuous transverse line field  $l$  is given along the boundary  $\partial P$ . This field has the property that for every  $(n-2)$ -dimensional face  $E$  of  $P$  there exists a hyperplane  $\pi(E)$  such that all the lines  $l(X)$  with  $X \in E$  belong to  $\pi(E)$ . Prove that the lines  $l(X)$  cover the interior of  $P$  twice.

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## SOLUTIONS

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### Principal Ideals in Noetherian Rings

**10534** [1996, 510]. Proposed by Paul Arne Østvær, Oslo University, Oslo, Norway. Suppose that  $R$  is a Noetherian ring in which all maximal ideals are principal. Show that all ideals in  $R$  are principal.

*Solution by Robert Gilmer, Florida State University, Tallahassee, FL.* If  $M = (m)$  is a maximal ideal of  $R$ , then  $M/M^2$  is a vector space over the field  $R/M$  of dimension at most 1. Hence there are no ideals of  $R$  properly between  $M$  and  $M^2$ . From this it follows (R. Gilmer, *Multiplicative Ideal Theory*, Queen's Papers Pure Appl. Math. **90** (1992), Theorem 39.2) that  $R = D_1 \oplus \dots \oplus D_n \oplus S_1 \oplus \dots \oplus S_m$  is a finite direct sum of Dedekind domains  $D_i$  and special primary rings  $S_i$ . To show that each ideal of  $R$  is principal, it suffices to show that the  $D_i$  and  $S_i$  have this property. For  $S_i$  this is part of the definition of a special primary ring (Gilmer, p. 200). Moreover,  $D_i$  inherits from  $R$  the property that each of its maximal ideals is principal, and a Dedekind domain is a principal ideal domain whenever all of its maximal ideals are principal.

*Editorial comment.* D. D. Anderson mentions a stronger result that appears in R. Gilmer and W. Heinzer, Principal ideal rings and a condition of Kummer, *J. Algebra* **83** (1983) 285–292: If  $R$  has the ascending chain condition on principal ideals and each maximal ideal of  $R$  is principal, then every ideal of  $R$  is principal.

Solved also by Mahalal'el ben keinan (Israel), F. Calegari (Australia), J. E. Dawson (Australia), T. H. Foregger, O. Moubinool (France), S. Sertöz (Turkey), and M. Tabaâ (Morocco).

### A Telescoping Constraint

**10566** [1997, 68]. Proposed by Gerry Myerson, Macquarie University, Australia. Let  $S$  be a finite set of cardinality  $n > 1$ . Let  $f$  be a real-valued function on the power set of  $S$ , and suppose that  $f(A \cap B) = \min\{f(A), f(B)\}$  for all subsets  $A$  and  $B$  of  $S$ . Prove that

$$\sum (-1)^{n-|A|} f(A) = f(S) - \max f(A),$$