10722. *Proposed by Richard F. McCoart, Loyola College, Baltimore, MD.*
(**a**) In how many ways can $2n$ indistinguishable balls be placed into n distinguishable urns, if the first r urns may contain at most $2r$ balls for each $r \in \{1, 2, \dots, n\}$?
(**b**) Suppose that $0 \leq m \leq n$. In how many of the ways enumerated in part (**a**) are exactly m urns empty?

10723. *Proposed by Christopher J. Hillar, Yale University, New Haven, CT.* Let p be an odd prime. Prove that $\sum_{i=1}^{p-1} 2^i \cdot i^{p-2} \equiv \sum_{i=1}^{(p-1)/2} i^{p-2} \pmod{p}$.

10724. *Proposed by Serge Tabachnikov, University of Arkansas, Fayetteville, AR.*
(**a**) Let P be a convex plane polygon with vertices $A_1, \dots, A_n$, and let l be a continuous transverse field of directions along the boundary ∂P. (This means that through every point $X \in \partial P$ there passes a line $l(X)$ that intersects the interior of P and depends continuously on X.) Let α_i and β_i be the angles between the line $l(A_i)$ and the adjacent sides $A_i A_{i-1}$ and $A_i A_{i+1}$, respectively. Assume that $\prod_1^n \sin\alpha_i = \prod_1^n \sin\beta_i$. Prove that the lines $l(X)$ cover the interior of P twice, that is, every interior point of P belongs to at least two of these lines.
(**b**) Suppose $n \geq 3$, and let P be a convex polyhedron in n-dimensional space. As in (**a**), a continuous transverse line field l is given along the boundary ∂P. This field has the property that for every $(n-2)$-dimensional face E of P there exists a hyperplane $\pi(E)$ such that all the lines $l(X)$ with $X \in E$ belong to $\pi(E)$. Prove that the lines $l(X)$ cover the interior of P twice.

SOLUTIONS

Principal Ideals in Noetherian Rings

10534 [1996, 510]. *Proposed by Paul Arne Østvær, Oslo University, Oslo, Norway.* Suppose that R is a Noetherian ring in which all maximal ideals are principal. Show that all ideals in R are principal.

Solution by Robert Gilmer, Florida State University, Tallahassee, FL. If $M = (m)$ is a maximal ideal of R, then M/M^2 is a vector space over the field R/M of dimension at most 1. Hence there are no ideals of R properly between M and M^2. From this it follows (R. Gilmer, *Multiplicative Ideal Theory,* Queen's Papers Pure Appl. Math. **90** (1992), Theorem 39.2) that $R = D_1 \oplus \cdots \oplus D_n \oplus S_1 \oplus \cdots \oplus S_m$ is a finite direct sum of Dedekind domains D_i and special primary rings S_i. To show that each ideal of R is principal, it suffices to show that the D_i and S_i have this property. For S_i this is part of the definition of a special primary ring (Gilmer, p. 200). Moreover, D_i inherits from R the property that each of its maximal ideals is principal, and a Dedekind domain is a principal ideal domain whenever all of its maximal ideals are principal.

Editorial comment. D. D. Anderson mentions a stronger result that appears in R. Gilmer and W. Heinzer, Principal ideal rings and a condition of Kummer, *J. Algebra* **83** (1983) 285–292: If R has the ascending chain condition on *principal* ideals and each maximal ideal of R is principal, then every ideal of R is principal.

Solved also by Mahalal'el ben keinan (Israel), F. Calegari (Australia), J. E. Dawson (Australia), T. H. Foregger, O. Moubinool (France), S. Sertöz (Turkey), and M. Tabaâ (Morocco).

A Telescoping Constraint

10566 [1997, 68]. *Proposed by Gerry Myerson, Macquarie University, Australia.* Let S be a finite set of cardinality $n > 1$. Let f be a real-valued function on the power set of S, and suppose that $f(A \cap B) = \min\{f(A), f(B)\}$ for all subsets A and B of S. Prove that

$$\sum (-1)^{n-|A|} f(A) = f(S) - \max f(A),$$