

10726



Donald E. Knuth

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# PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Daniel H. Ullman, and Douglas B. West**

with the collaboration of Paul T. Bateman, Mario Benedicty, Paul Bracken, Duane M. Broline, Ezra A. Brown, Richard T. Bumby, Glenn G. Chappell, Randall Dougherty, Roger B. Eggleton, Ira M. Gessel, Bart Goddard, Jerrold R. Griggs, Douglas A. Hensley, Richard Holzinger, John R. Isbell, Robert Israel, Kiran S. Kedlaya, Murray S. Klamkin, Fred Kochman, Frederick W. Luttman, Vania Mascioni, Frank B. Miles, Richard Pfeifer, Cecil C. Rousseau, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Charles Vanden Eynden, and William E. Watkins.

*Proposed problems and solutions should be sent in duplicate to the MONTHLY problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before September 30, 1999; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk (\*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

## PROBLEMS

**10725.** *Proposed by Vasile Mihai, Toronto, ON, Canada.* Fix a positive integer  $n$ . Given a permutation  $\alpha$  of  $\{1, 2, \dots, n\}$ , let  $f(\alpha) = \sum_{i=1}^n (\alpha(i) - \alpha(i+1))^2$ , where  $\alpha(n+1) = \alpha(1)$ . Find the extreme values of  $f(\alpha)$  as  $\alpha$  ranges over all permutations of  $\{1, 2, \dots, n\}$ .

**10726.** *Proposed by Donald E. Knuth, Stanford University, Stanford, CA.* Start in state 0. For every nonnegative integer  $k$ , stay in state  $k$  for  $X_k$  units of time, then go to state  $k+1$ . What is the probability of being in state  $s$  after  $t$  units of time, assuming that  $X_k$  is distributed exponentially (a) with mean  $1/(k+1)$ ? (b) with mean  $1/2^k$ ?

**10727.** *Proposed by Jean Anglesio, Garches, France.* Let  $m$  be a fixed positive integer. For a positive integer  $n$ , let  $s_m(n)$  be the sum of the  $m$ th powers of the decimal digits of  $n$ . For example,  $s_3(172) = 1^3 + 7^3 + 2^3 = 352$ . Starting with any positive integer  $n_0$ , construct a sequence of positive integers by setting  $n_k = s_m(n_{k-1})$  for every  $k \geq 1$ .

(a) Show that  $n_0, n_1, n_2, \dots$  is eventually periodic.

(b) Show that only finitely many periods are possible as  $n_0$  varies.

**10728.** *Proposed by Titu Andreescu, American Mathematics Competitions, Lincoln, NE.* Determine all functions  $f: \mathbb{Z} \rightarrow \mathbb{Z}$  satisfying

$$f(x^3 + y^3 + z^3) = (f(x))^3 + (f(y))^3 + (f(z))^3$$

for all integers  $x, y$ , and  $z$ .

**10729.** *Proposed by David P. Bellamy and Felix Lazebnik, University of Delaware, Newark, DE.* Let  $I \subset \mathbb{R}$  be an open interval, and let  $n$  be a positive integer. Characterize the functions  $f: I \rightarrow \mathbb{R}$  that have a continuous  $n$ th derivative and satisfy

$$f^{(n)} + p_1 f^{(n-1)} + \dots + p_{n-1} f' + p_n f = 0$$

for some continuous functions  $p_1, p_2, \dots, p_n$  on  $I$ .