REVIEWS

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Life's Other Secret. By Ian Stewart. Wiley, New York, 1997, xiii + 285 pp., $24.95 hardcover.
The Magical Maze. By Ian Stewart. Wiley, New York, 1998, xii + 268 pp., $24.95 hardcover.

Reviewed by **Dan Schnabel**

In the city where I reside, smaller bookstores are disappearing due to market pressure from book superstores. The selection of mathematics titles available in these new superstores is astonishing both in its magnitude and in its eccentricity. Books on tricks to improve basic arithmetic skills stand next to highly specialized research texts. Scattered among these are the mathematics popularization books. It is not obvious from the increased shelf space whether mathematics is actually becoming more popular, but these two recent books by Ian Stewart certainly further that goal.

Any good high school mathematics teacher recognizes that to learn mathematics, one must appreciate it; and real-world relevance helps students to appreciate mathematics. Beyond high school, the people who toil or play at mathematics are most likely to be those whose appreciation of the subject is well established. Relevance becomes a lesser concern if it remains a concern at all. Good efforts to popularize mathematics must return to the matter of relevance, and the best works do so in a manner that intrigues even those for whom relevance no longer matters. Ian Stewart's books are among the best; they change the way we look at the world.

Or even the way we look at ourselves.

Standing in front of a mirror waving your left hand, you see an image of yourself waving its right hand. This leads to the question: "Why does a mirror reverse left and right, but not top and bottom?" *The Magical Maze* provides an explanation. For a bilaterally symmetrical object such as the human body, the image created by reflection in a mirror can also be achieved by a 180° rotation in space. Our visual processing system is conditioned to assume that the image results from a rotation, because in the real world it is possible for us to rotate objects manually, but it is not possible for us to reflect them. A left shoe will always be a left shoe, no matter how we manipulate it before our eyes. Seeing symmetry broken by the waving of one hand, we still assume that the image is the result of a rotation—the rotation of a person waving the other hand.

Stewart points out that top and bottom are not switched if you turn the mirror on its side, but he does not consider the more intriguing case in which you lie on your side. Lie on your right side in front of a mirror so that your left leg is on top and your right leg is on the bottom. The image in the mirror has the left leg on the bottom and the right leg on top. Ignoring anything else that might appear in the mirror, we can no longer distinguish whether the mirror has switched left and right or switched top and bottom.

Stewart's ideas and presentation frequently inspired me, even required me, to think beyond his writing, as when I considered the following thought experiment. Imagine an intelligent creature having both left-right symmetry and top-bottom symmetry. (How many eyes would such a creature have?) Would this creature think that mirrors switch left and right, or top and bottom? I suspect that, even though its image can be achieved through two different rotations (as well as through reflection), the creature would still see the mirror as swapping left and right, because of the uniqueness of the vertical orientation it learns from the pull of gravity. What, then, is the role of gravitation in the way our own visual conditioning interprets mirror images? I found myself wondering what Stewart would have to say about this.

Symmetry is a recurring motif in both these books, and the books themselves are somewhat symmetrical. *The Magical Maze* is a mathematics book in which one encounters some biology, while *Life's Other Secret*, subtitled *The New Mathematics of the Living World*, is essentially a biology text in which one encounters some mathematics.

How much mathematics *Life's Other Secret* can be said to contain depends on what one regards as mathematics. Stewart would like readers to acknowledge a broad meaning for mathematics and to recognize a large role for mathematics in the biological sciences.

The title *Life's Other Secret* refers to the idea that genetics and DNA do not provide the complete picture for life on earth. Stewart suggests a different role for genes:

> The cell carries out its genetic instructions; the laws of physics and chemistry produce certain results, and when you put the two together, you get an organism.

Consequently, an understanding of the laws of physics and chemistry and of the underlying mathematics is equal in importance to genetics in the understanding of life.

Given the symmetrical relationship between the two books, it is no surprise that Stewart's best examples of the intersection of biology and mathematics are addressed in both books.

Each of the books contains a discussion of why the number of petals on most flowers is a Fibonacci number. The question is quickly reduced to the arrangement of tissue called primordia at the tip of a plant shoot. As a plant grows, these primordia appear in places that are determined by the need to be closely packed around a circle. The best packings occur when the primordia are separated by an irrational multiple of 360°, because rational multiples of 360° would generate "spokes" of the primordia. The theory of continued fractions can be used to show that the golden ratio $\phi = (1 + \sqrt{5})/2$ is the "most irrational" number. Measurements confirm that primordia are usually positioned around a "generative spiral" separated by an angle that, measured externally, is approximately $360/\phi$ degrees; measured internally this is approximately 137.5°.

The most intriguing aspect of this discussion is that nature is consistent with number theorists on the matter of determining "how irrational" an irrational number is, at least in the case of the golden ratio. It is also interesting that nature appears unwilling to settle for anything other than the most irrational number. Reading Stewart left me wondering what sort of packings would result from irrational numbers that are not the most irrational.

Stewart focuses on the sunflower plant, the large head of which clearly demonstrates the packing problem. He suggests that the position of primordia provides a nearly complete explanation of the pattern of spirals on the head of a sunflower plant. Missing are the mathematical details explaining why the number of clockwise spirals and the number of counterclockwise spirals that we perceive are two consecutive Fibonacci numbers. We read only that it is because of the close relationship between Fibonacci numbers and the golden ratio, but I would like to have been shown more of the mathematics explaining how the number of spirals we see results from the separation on a "generative spiral". Moreover, Stewart never clarifies the connection between the actual number of petals and the position of primordia—the connection between "how many to arrange" and "how to arrange them".

Both of these books will frequently frustrate the more mathematically minded reader, as they often stop short of the interesting, nitty-gritty mathematical details. This is not necessarily a bad thing: the books are quite effective at whetting one's appetite for more mathematics. But they are not always good at indicating where to turn for more details. *The Magical Maze*, which is the more mathematical of the two books, does provide more details in the endnotes, but they are referred to in the text in a manner that, while intended to be consistent with the maze metaphor of the book, is awkward. The endnotes are numbered, but the references to them are not, so unless you always read all the endnotes, it is unclear which one is being cited.

On the matter of mathematical details, *Life's Other Secret* is the weaker, more frustrating, of the books, as it is essentially a book of biology. It talks about mathematics without actually including a great deal of traditional mathematics. When Stewart simplifies explanations with expressions such as "The mathematical machinery reveals...," I cannot help wanting to see the machinery in action, not just the results. While there are complicated details of how a tobacco virus develops, nowhere is there mathematics of a comparable level of difficulty.

How much mathematics should be included in a popular mathematics book? This is one of the fundamental questions facing writers in the genre. Certainly *The Magical Maze* is a book in which mathematicians will feel at home. It contains a great deal of mathematics, and it attempts explanations that are unusually complex for a book of its type. Its level of sophistication led me to speculate on the possibility that it succeeded in being published primarily because it was written to accompany the televised 1997 Christmas Lectures of the Royal Institution of Great Britain. Not that I mean by this to disparage the book itself; rather, I wonder about the system that makes books of this calibre a rare occurrence.

No doubt readers with only basic high school mathematics training will find *The Magical Maze* difficult going, made more so by an uncharacteristically high number of mistakes in the explanations and diagrams. While some mistakes are typographical and others appear to be printing errors, there are genuine calculation errors as well. Such challenges to comprehensibility may prevent this book from achieving its popularization aims, but do not significantly diminish its quality.

Meritorious as these two books are, publishers and the general public only seem to readily embrace more superficial books, such as Stewart's earlier work *Nature's Numbers*. The dust jacket for *The Magical Maze* includes *Nature* magazine's praise of *Nature's Numbers*:

> Stewart achieves what other popular mathematics writers merely strive for: an accurate, informative portrayal of contemporary mathematics without a single equation in sight.

As much as I liked *Nature's Numbers*, I hope that other popular mathematics writers are not striving to eliminate equations from their books. The ability to express abstract ideas in the form of equations is a cornerstone of mathematics. The near-taboo status of equations in popular books is irksome. Although doing mathematics and writing about mathematics are two different things, books that avoid equations altogether are not popularizing mathematics as it truly is, but are merely making mathematics marketable.

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An Introductory Course in Commutative Algebra. By A. W. Chatters and C. R. Hajarnavis. Oxford University Press, 1998, viii + 144 pp., $35 softcover, $75 hardcover.
Introduction to Algebra. By Peter J. Cameron. Oxford University Press, 1998, x + 295 pp., $32 softcover, $65 hardcover.

Reviewed by **Cynthia Woodburn**

Driving home last night, I was unhappy with the particular selection playing on my favorite classical radio station, so I switched to my second favorite classical radio station. Imagine my surprise to hear a voice discussing mathematics, and even more surprisingly, discussing the new pop fascination with mathematics. Evidently, there is a trend in pop culture towards the notion that "math is cool". There is even a cologne for men available now named "Pi". The two books under review may not make it onto any popular best-seller lists, be made into movies, or have colognes named after them, but both could be useful in helping students to appreciate that "abstract algebra is cool".

An Introductory Course in Commutative Algebra is a "lean and lively" introduction to commutative algebra with a definite number-theory perspective. Many examples are number theoretic in nature, and number theory is used frequently to motivate new concepts. For example, the chapter on ruler and compass constructions includes a discussion of the connection between Fermat primes and the constructibility of regular n-gons. Written for use by undergraduates, the book is appropriate for a second-semester course in abstract algebra. Its prerequisites include knowledge of equivalence relations, some elementary group theory such as Lagrange's Theorem, and some basic linear algebra. With caution being used at the places where some elementary group theory is assumed (for instance, Chapter 10 on finite cyclic groups and finite fields), the book could be used as a text for a first-semester abstract algebra course; it would be especially good for a class composed of secondary mathematics education majors.

The book begins with an introductory chapter on rings. Since the focus is on commutative algebra, a ring is defined as a commutative ring with identity. For those more familiar with a ring not necessarily being commutative or having a multiplicative identity, some minor adjustments in thinking must be made. For example, if one defines a ring in this fashion, then ideals are not typically subrings, and the even integers do not form a subring of the integers. Chapters 2 and 3 cover Euclidean rings and the highest common factor. I was disappointed to find that the