express abstract ideas in the form of equations is a cornerstone of mathematics. The near-taboo status of equations in popular books is irksome. Although doing mathematics and writing about mathematics are two different things, books that avoid equations altogether are not popularizing mathematics as it truly is, but are merely making mathematics marketable.

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An Introductory Course in Commutative Algebra. By A. W. Chatters and C. R. Hajarnavis. Oxford University Press, 1998, viii + 144 pp., $35 softcover, $75 hardcover.
Introduction to Algebra. By Peter J. Cameron. Oxford University Press, 1998, x + 295 pp., $32 softcover, $65 hardcover.

Reviewed by **Cynthia Woodburn**

Driving home last night, I was unhappy with the particular selection playing on my favorite classical radio station, so I switched to my second favorite classical radio station. Imagine my surprise to hear a voice discussing mathematics, and even more surprisingly, discussing the new pop fascination with mathematics. Evidently, there is a trend in pop culture towards the notion that "math is cool". There is even a cologne for men available now named "Pi". The two books under review may not make it onto any popular best-seller lists, be made into movies, or have colognes named after them, but both could be useful in helping students to appreciate that "abstract algebra is cool".

An Introductory Course in Commutative Algebra is a "lean and lively" introduction to commutative algebra with a definite number-theory perspective. Many examples are number theoretic in nature, and number theory is used frequently to motivate new concepts. For example, the chapter on ruler and compass constructions includes a discussion of the connection between Fermat primes and the constructibility of regular n-gons. Written for use by undergraduates, the book is appropriate for a second-semester course in abstract algebra. Its prerequisites include knowledge of equivalence relations, some elementary group theory such as Lagrange's Theorem, and some basic linear algebra. With caution being used at the places where some elementary group theory is assumed (for instance, Chapter 10 on finite cyclic groups and finite fields), the book could be used as a text for a first-semester abstract algebra course; it would be especially good for a class composed of secondary mathematics education majors.

The book begins with an introductory chapter on rings. Since the focus is on commutative algebra, a ring is defined as a commutative ring with identity. For those more familiar with a ring not necessarily being commutative or having a multiplicative identity, some minor adjustments in thinking must be made. For example, if one defines a ring in this fashion, then ideals are not typically subrings, and the even integers do not form a subring of the integers. Chapters 2 and 3 cover Euclidean rings and the highest common factor. I was disappointed to find that the

Euclidean algorithm is not included (although it is mentioned on p. 39 in Chapter 6). The optional Chapter 4 uses "the ring of Gaussian integers to prove one of the most famous theorems in number theory: every positive integer is the sum of four squares." Next are chapters on the traditional topics of fields and polynomials, unique factorization domains, the field of quotients of an integral domain, factorization of polynomials, fields and field extensions, finite cyclic groups and finite fields, and algebraic numbers. Chapter 12 on ruler and compass constructions contains instructions for classical constructions, such as bisecting angles and line segments and dropping perpendiculars, along with the algebra of constructible numbers. The three impossible constructions from antiquity are discussed, as well as the proof by Gauss that the regular 17-gon is constructible (the longest proof of the text). The final three chapters cover homomorphisms, ideals and quotient rings (some familiarity with quotient groups is assumed), principal ideal domains and a method for constructing fields, and finite fields.

The text is extremely readable with a very concrete approach. It is evident that the authors strove to make the text understandable by undergraduate students. Comments are included to explain the significance of results. Definitions are often repeated when terminology is reused, and difficult concepts are explained in everyday language. The book abounds with examples, many of which include actual numbers, something students who are struggling with proofs and abstraction will appreciate. Most of the exercises are straightforward. Some are very easy, such as proving that every field is an integral domain (which appears as Exercise 5.1 and Exercise 9.2). More challenging exercises have hints or contain sketches of a solution with the details to be filled in by students. Answers to selected exercises can be found in the back of the book, although there is no notation within the exercise sets to indicate which problems have solutions provided.

The second text under review, *Introduction to Algebra*, is "lively" but not "lean". While the text by Chatters and Hajarnavis contains fifteen short chapters (the shortest—on constructing the field of quotients of an integral domain—is $2\frac{1}{2}$ pages, and the longest—on ruler and compass constructions—is 15 pages), the text by Cameron, twice the length, is organized into eight large chapters with sections and subsections. This book contains more information than can be covered in a year-long algebra course, which allows for flexibility in its use. After an introductory chapter containing preliminary concepts and motivating material about algebra, the book begins with a study of ring theory guided by the familiar example of the integers. Group theory follows. As pointed out by the author in the preface, these two chapters could form the basis for an introductory one-semester course in abstract algebra. Next are chapters on linear algebra and module theory. Chapter 6 is a change of pace: it contains a formal construction of the natural numbers, integers, rational numbers, real numbers (via Cauchy sequences), and complex numbers; also included are algebraic and transcendental numbers. Chapter 7 contains further topics from group theory, ring theory, and field theory, along with other advanced topics not typically found in a book at this level: namely, universal algebra, lattices, and category theory. The final chapter, entitled "Applications", discusses Galois theory (a classical application), and error-correcting codes (a modern application).

Cameron's writing style is very enjoyable and reader-friendly. He uses entertaining verbs such as "whittle" and "blur" and gives many examples throughout the book. Modern and up-to-date analogies help students relate to concepts: for example, rings with special properties are compared to personal computers with extra features. Connections between concepts are emphasized. The use of certain terminology and notation is explained, and differences in notation are pointed out. For instance, maps and functions are written on the right in the text, and the reader is urged to "remember that not everybody uses this convention!" Solutions to selected exercises are provided in the back of the book. There is no notation within the exercise sets to denote those problems whose solutions are given, but more difficult exercises are marked with an asterisk. Some of the exercises are even marked with two asterisks.

Even though the text is reader-friendly, a high level of rigor is maintained. Kernels are first defined as equivalence relations, polynomials are defined as infinite sequences, and three different proofs of the existence of transcendental numbers are given.

Both texts include some historical background of terminology and results. Cameron's book also includes some interesting mathematical folklore, which piques the interest of many students. For example, he relates that "legend has it that the [irrationality of the square root of 2] was discovered by Hippasos, a member of the Pythagorean Brotherhood; he was expelled from the Brotherhood (or, in some versions, drowned in a shipwreck) to prevent him from revealing the shameful truth that nature contains irrationality." Each text also includes references or sources for further reading. About half of the references in *An Introductory Course in Commutative Algebra* are from the area of number theory. *Introduction to Algebra* has a more extensive and comprehensive list of sources for further reading, with several introductory paragraphs expounding upon the sources.

Missing from both texts are ideas for cooperative learning activities or writing assignments. Cameron's text does have a very nice web page at `http://www.maths.qmw.ac.uk/~pjc/algebra/`. It contains solutions to all of the exercises from the first three chapters of the text in both LaTeX and PostScript formats, further material and problems, links to other sites of interest to algebraists, and corrections.

There are many abstract algebra books on the market. A subject search through a popular online book distributor yielded a list of 148 abstract algebra books and a list of 52 books classified under commutative algebra. The books by Chatters and Hajarnavis and by Cameron are fine additions to the collection of abstract algebra books available for use at the undergraduate level, and each in its own way does a great job of advancing the notion that "abstract algebra is cool".

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