10705

D. W. Brown

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PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Daniel H. Ullman,** and **Douglas B. West**

> *Proposed problems and solutions should be sent in duplicate to the* MONTHLY *problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before June 30, 1999; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

10704. *Proposed by Wiliam G. Spohn, Jr., Ellicott City, MD.* Show that there are infinitely many pairs $((a, b, c), (a', b', c'))$ of primitive Pythagorean triples such that $|a - a'|$, $|b - b'|$, and $|c - c'|$ are all equal to 3 or 4. Examples include $((12, 5, 13), (15, 8, 17))$ and $((77, 36, 85), (80, 39, 89))$.

10705. *Proposed by D. W. Brown, Marietta, GA.* A topological space has the *fixed point property* if every continuous function from the space to itself has a fixed point. Is there a countably infinite Hausdorff space with the fixed point property?

10706. *Proposed by James G. Propp, University of Wisconsin, Madison, WI.* Given a finite sequence $(a_1, \ldots, a_n)$, define the *derived sequence* $(b_1, \ldots, b_{n+1})$ by $b_i = s - a_{i-1} - a_i$, where $s = \min_{1 \le i \le n+1}(a_{i-1} + a_i) + \max_{1 \le i \le n+1}(a_{i-1} + a_i)$ and where we interpret both a_0 and a_{n+1} as 0. Let S_0 be the sequence (1) of length 1, and for $n \ge 1$ define S_k to be the derived sequence obtained from S_{k-1}. Thus $S_1 = (1, 1)$, $S_2 = (2, 1, 2)$, $S_3 = (3, 2, 2, 3)$, and $S_4 = (5, 3, 4, 3, 5)$. Show that the middle term of S_{2n} is a perfect square.

10707. *Proposed by John Isbell, State University of New York, Buffalo, NY.* Show that
(a) no vector space over an infinite field is a finite union of proper subspaces; and
(b) no vector space over an n-element field is a union of n or fewer proper subspaces.

10708. *Proposed by the Western Maryland College Problems Group, Westminster, MD.* Let

$$f(x) = \frac{1}{4} \int_0^\pi \frac{1}{t} \log \left(\frac{1 - \cos(x + t)}{1 - \cos(x - t)} \right) dt$$

for $x \in (0, \pi)$.
(a) Find the Fourier sine series for f.
(b) Find the L^2 norm of f.
(c) Find $\lim_{x \to 0} f(x)$.