PROBLEMS AND SOLUTIONS

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with the collaboration of Paul T. Bateman, Mario Benedicty, Paul Bracken, Duane M. Broline, Ezra A. Brown, Richard T. Bumby, Glenn G. Chappell, Randall Dougherty, Roger B. Eggleton, Ira M. Gessel, Bart Goddard, Jerrold R. Griggs, Douglas A. Hensley, Richard Holzsager, John R. Isbell, Robert Israel, Kiran S. Kedlaya, Murray S. Klamkin, Fred Kochman, Frederick W. Luttmann, Frank B. Miles, Richard Pfiefer, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Charles Vanden Eynden, and William E. Watkins.

Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before June 30, 1999; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

10704. *Proposed by Wiliam G. Spohn, Jr., Ellicott City, MD.* Show that there are infinitely many pairs $((a, b, c), (a', b', c'))$ of primitive Pythagorean triples such that $|a - a'|$, $|b - b'|$, and $|c - c'|$ are all equal to 3 or 4. Examples include $((12, 5, 13), (15, 8, 17))$ and $((77, 36, 85), (80, 39, 89))$.

10705. *Proposed by D. W. Brown, Marietta, GA.* A topological space has the *fixed point property* if every continuous function from the space to itself has a fixed point. Is there a countably infinite Hausdorff space with the fixed point property?

10706. *Proposed by James G. Propp, University of Wisconsin, Madison, WI.* Given a finite sequence $(a_1, \dots, a_n)$, define the *derived sequence* $(b_1, \dots, b_{n+1})$ by $b_i = s - a_{i-1} - a_i$, where $s = \min_{1\le i\le n+1}(a_{i-1} + a_i) + \max_{1\le i\le n+1}(a_{i-1} + a_i)$ and where we interpret both a_0 and a_{n+1} as 0. Let S_0 be the sequence (1) of length 1, and for $n \ge 1$ define S_k to be the derived sequence obtained from S_{k-1}. Thus $S_1 = (1, 1)$, $S_2 = (2, 1, 2)$, $S_3 = (3, 2, 2, 3)$, and $S_4 = (5, 3, 4, 3, 5)$. Show that the middle term of S_{2n} is a perfect square.

10707. *Proposed by John Isbell, State University of New York, Buffalo, NY.* Show that
(a) no vector space over an infinite field is a finite union of proper subspaces; and
(b) no vector space over an n-element field is a union of n or fewer proper subspaces.

10708. *Proposed by the Western Maryland College Problems Group, Westminster, MD.* Let

$$f(x) = \frac{1}{4}\int_0^{\pi} \frac{1}{t}\log\left(\frac{1-\cos(x+t)}{1-\cos(x-t)}\right) dt$$

for $x \in (0, \pi)$.
(a) Find the Fourier sine series for f.
(b) Find the L^2 norm of f.
(c) Find $\lim_{x\to 0} f(x)$.