

10708



Western Maryland College Problems Group

*The American Mathematical Monthly*, Vol. 106, No. 1. (Jan., 1999), p. 67.

Stable URL:

<http://links.jstor.org/sici?sici=0002-9890%28199901%29106%3A1%3C67%3A1%3E2.0.CO%3B2-9>

*The American Mathematical Monthly* is currently published by Mathematical Association of America.

---

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/maa.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

---

The JSTOR Archive is a trusted digital repository providing for long-term preservation and access to leading academic journals and scholarly literature from around the world. The Archive is supported by libraries, scholarly societies, publishers, and foundations. It is an initiative of JSTOR, a not-for-profit organization with a mission to help the scholarly community take advantage of advances in technology. For more information regarding JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

# PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Daniel H. Ullman, and Douglas B. West**

with the collaboration of Paul T. Bateman, Mario Benedicty, Paul Bracken, Duane M. Broline, Ezra A. Brown, Richard T. Bumby, Glenn G. Chappell, Randall Dougherty, Roger B. Eggleton, Ira M. Ges-  
sel, Bart Goddard, Jerrold R. Griggs, Douglas A. Hensley, Richard Holzsager, John R. Isbell, Robert  
Israel, Kiran S. Kedlaya, Murray S. Klamkin, Fred Kochman, Frederick W. Luttmann, Frank B. Miles,  
Richard Pfeifer, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Charles  
Vanden Eynden, and William E. Watkins.

*Proposed problems and solutions should be sent in duplicate to the MONTHLY problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before June 30, 1999; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk (\*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

---

## PROBLEMS

---

**10704.** *Proposed by Wiliam G. Spohn, Jr., Ellicott City, MD.* Show that there are infinitely many pairs  $((a, b, c), (a', b', c'))$  of primitive Pythagorean triples such that  $|a - a'|$ ,  $|b - b'|$ , and  $|c - c'|$  are all equal to 3 or 4. Examples include  $((12, 5, 13), (15, 8, 17))$  and  $((77, 36, 85), (80, 39, 89))$ .

**10705.** *Proposed by D. W. Brown, Marietta, GA.* A topological space has the *fixed point property* if every continuous function from the space to itself has a fixed point. Is there a countably infinite Hausdorff space with the fixed point property?

**10706.** *Proposed by James G. Propp, University of Wisconsin, Madison, WI.* Given a finite sequence  $(a_1, \dots, a_n)$ , define the *derived sequence*  $(b_1, \dots, b_{n+1})$  by  $b_i = s - a_{i-1} - a_i$ , where  $s = \min_{1 \leq i \leq n+1} (a_{i-1} + a_i) + \max_{1 \leq i \leq n+1} (a_{i-1} + a_i)$  and where we interpret both  $a_0$  and  $a_{n+1}$  as 0. Let  $S_0$  be the sequence (1) of length 1, and for  $n \geq 1$  define  $S_k$  to be the derived sequence obtained from  $S_{k-1}$ . Thus  $S_1 = (1, 1)$ ,  $S_2 = (2, 1, 2)$ ,  $S_3 = (3, 2, 2, 3)$ , and  $S_4 = (5, 3, 4, 3, 5)$ . Show that the middle term of  $S_{2n}$  is a perfect square.

**10707.** *Proposed by John Isbell, State University of New York, Buffalo, NY.* Show that  
(a) no vector space over an infinite field is a finite union of proper subspaces; and  
(b) no vector space over an  $n$ -element field is a union of  $n$  or fewer proper subspaces.

**10708.** *Proposed by the Western Maryland College Problems Group, Westminster, MD.* Let

$$f(x) = \frac{1}{4} \int_0^\pi \frac{1}{t} \log \left( \frac{1 - \cos(x+t)}{1 - \cos(x-t)} \right) dt$$

for  $x \in (0, \pi)$ .

- (a) Find the Fourier sine series for  $f$ .  
(b) Find the  $L^2$  norm of  $f$ .  
(c) Find  $\lim_{x \rightarrow 0} f(x)$ .