**10709.** *Proposed by Zoltán Sasvári, Technical University of Dresden, Dresden, Germany.* Let $X$ be a standard normal random variable, and choose $y > 0$. Show that

$$e^{-ay} < \frac{Pr(a \leq X \leq a + y)}{Pr(a - y \leq X \leq a)} < e^{-ay+(1/2)ay^3}$$

when $a > 0$. Show that the reversed inequalities hold when $a < 0$.

**10710.** *Proposed by Bogdan Suceava, Michigan State University, East Lansing, MI.* Let $ABC$ be an acute triangle with incenter $I$, and let $D$, $E$, and $F$ be the points where the circle inscribed in $ABC$ touches $BC$, $CA$, and $AB$, respectively. Let $M$ be the intersection of the line through $A$ parallel to $BC$ and $DE$, and let $N$ be the intersection of the line through $A$ parallel to $BC$ and $DF$. Let $P$ and $Q$ be the midpoints of $DM$ and $DN$, respectively. Prove that $A$, $E$, $F$, $I$, $P$, and $Q$ are on the same circle.

# SOLUTIONS

### When O-H-I Is Isosceles

**10547** [1996, 695]. *Proposed by Dan Sachelarie, ICCE Bucharest, and Vlad Sachelarie, University of Bucharest, Bucharest, Romania.* In the triangle $ABC$, let $O$ be the circumcenter, $H$ the orthocenter, and $I$ the incenter. Prove that the triangle $OHI$ is isosceles if and only if

$$\frac{a^3 + b^3 + c^3}{3abc} = \frac{R}{2r}.$$

*Solution by Walther Janous, Ursulinengymnasium, Innsbruck, Austria.* We denote by MPV the reference D. S. Mitrinović, J. E. Pečarić, and V. Volenec, *Recent Advances in Geometric Inequalities,* Kluwer, 1989. Neither $IO$ nor $HI$ is ever as large as $HO$ [MPV, p. 288], so the only way triangle $IHO$ can be isosceles is if $IO = HI$. Also $IO^2 = R^2 - 2Rr$ [MPV, p. 279] and $HI^2 = 4R^2 + 4Rr + 3r^2 - s^2$ [MPV, p. 280], where $s$ is the semiperimeter. Hence $HI = IO$ if and only if $R^2 - 2Rr = 4R^2 + 4Rr + 3r^2 - s^2$. This rearranges to $2s(s^2 - 3r^2 - 6Rr)/12Rrs = R/2r$, or, using $abc = 4Rrs$ [MPV, p. 52] and $a^3 + b^3 + c^3 = 2s(s^2 - 3r^2 - 4Rr)$ [MPV, p. 52], to $(a^3 + b^3 + c^3)/3abc = R/2r$.

*Editorial comment.* Another condition equivalent to $HI = IO$, given in problem E2282 [1971, 196; 1972, 397] from this MONTHLY, is that $ABC$ has one angle equal to 60°.

Solved also by J. Anglesio (France), R. Barbara (Lebanon), F. Bellot Rosado (Spain), C. W. Dodge, J. S. Frame, Z. Franco, M. S. Klamkin (Canada), W. W. Meyer, V. Mihai (Canada), C. R. Pranesachar (India), B. Prielipp, V. Schindler (Germany), I. Sofair, M. Tabaâ (Morocco), T. V. Trif (Romania), M. Vowe (Switzerland), GCHQ Problems Group (U. K.), and the proposers.

### The Divisible Differences Property

**10553** [1996, 809]. *Proposed by Bjorn Poonen, Mathematical Sciences Research Institute, Berkeley, CA, Jim Propp, Massachusetts Institute of Technology, Cambridge, MA, and Richard Stong, Rice University, Houston, TX.* Say that a sequence $\langle q \rangle = q_1, q_1, q_2, \ldots$ of integers has the *divisible differences property* if $(n - m)|(q_n - q_m)$ for all $n$ and $m$.
**(a)** Show that if $\langle q \rangle$ has the divisible differences property and $\limsup |q_n|^{1/n} < e - 1$, then there is a polynomial $Q$ such that $q_n = Q(n)$.
**(b)** Show that there is a sequence $\langle q \rangle$ that has the divisible differences property and satisfies $\limsup |q_n|^{1/n} \leq e$, for which $q_n$ is not given by a polynomial in $n$.
**(c)*** Is it true that $\limsup |q_n|^{1/n} \geq e$ for all non-polynomial $\langle q \rangle$ with the divisible differences property?