



Mathematics for Hikers: 10557

Nick MacKinnon

The American Mathematical Monthly, Vol. 106, No. 1. (Jan., 1999), p. 70.

Stable URL:

<http://links.jstor.org/sici?sici=0002-9890%28199901%29106%3A1%3C70%3AMFH1%3E2.0.CO%3B2-H>

The American Mathematical Monthly is currently published by Mathematical Association of America.

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

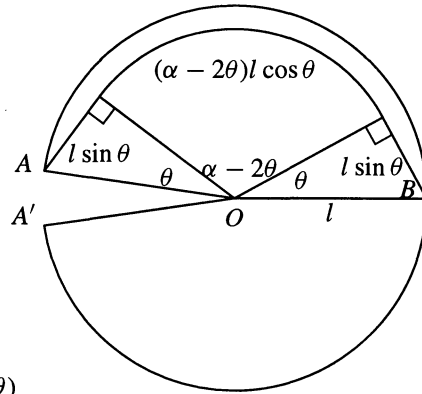
Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/maa.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

The JSTOR Archive is a trusted digital repository providing for long-term preservation and access to leading academic journals and scholarly literature from around the world. The Archive is supported by libraries, scholarly societies, publishers, and foundations. It is an initiative of JSTOR, a not-for-profit organization with a mission to help the scholarly community take advantage of advances in technology. For more information regarding JSTOR, please contact support@jstor.org.

10557 [1996, 902]. *Proposed by Nick MacKinnon, Winchester College, Winchester, U. K.* Naismith's rule allows walkers to compute the time for their journeys. The time is given by allowing a walking speed of 4 km/hr, but adding an extra minute for each 10m of ascent. A conical mountain has base radius 1650m and vertical height 520m. Points A and B are diametrically opposite at the base of the mountain. How should a path be constructed between A and B on the surface of the mountain that minimizes the time taken to walk from A to B ?

Solution by the proposer. The surface of the mountain can be unrolled isometrically onto a plane, forming a sector (of angle $2\alpha = 165/173 \cdot 2\pi$ radians) of a circle (of radius $l = 1730$ m, the slant height of the mountain), as shown in the figure at right.



Call the time-minimising path the *Naismith geodesic* for the cone. This geodesic must reach some maximum height h . The figure shows a potential Naismith geodesic with maximum height $h = 520(1 - \cos \theta)$ meters. It is composed of a circular arc following the contour with height h , together with tangents joining the arc to A and B . No alternative path taking less time attains the maximum height h of the given path since such a path must at least touch the circular arc, must not cross the circular arc, and must leave the circular arc without subsequent reascent. The Naismith geodesic must therefore be a path of the given shape. The length of such a path is $2l \sin \theta + (\alpha - 2\theta)l \cos \theta$, and Naismith's rule gives a time of

$$t(\theta) = 0.015(2l \sin \theta + (\alpha - 2\theta)l \cos \theta) + 520(1 - \cos \theta)/10$$

minutes for the journey. The lone critical value of $t(\theta)$ for $0 < \theta < \alpha/2$ occurs when $t'(\theta) = (52 - 0.015(\alpha - 2\theta)l) \sin \theta = 0$ at $\theta^* = (\alpha - 52/(0.015l))/2 \doteq .49623$ radians. This gives the optimal time $t(\theta^*) = 76.71037$ minutes. A path around the bottom of the cone takes $t(0) = 77.75442$ minutes, while the path of shortest distance takes $t(\alpha/2) = 99.98929$ minutes.

Solved also by J. Anglesio (France), J. E. Dawson (Australia), P. G. Kirmser, J. H. Lindsey II, R. Reynolds & M. Martinez, and P. Straffin.

A Matter of Adjustment

10558 [1996, 902]. *Proposed by Zhang Chengyu, Hubei University, Wuhan, China.* Let p be a prime, and let k be a positive integer. Let a_1, a_2, \dots, a_{p^k} be any p^k integers. We define the *adjustment* of these integers to be the p^k integers b_1, b_2, \dots, b_{p^k} , where $b_j = a_{j+1} + a_{j+2} + \dots + a_{j+p}$, interpreting subscripts modulo p^k . For example, if $p = 2$ and $k = 2$, one adjustment of 1, 1, 3, 4 gives 4, 7, 5, 2. Prove that after p^k adjustments of a_1, a_2, \dots, a_{p^k} , the list consists entirely of integers divisible by p .

Solution I by Thomas Jager, Calvin College, Grand Rapids, MI. We prove a stronger statement. Given an integer vector $v = (v_1, \dots, v_{p^k})$, define the v -adjustment of $(a_1, \dots, a_{p^k})^T$ to be $(b_1, \dots, b_{p^k})^T$, where $b_j = v_1 a_{j+1} + v_2 a_{j+2} + \dots + v_{p^k} a_{j+p^k}$, again treating subscripts modulo p^k . As a transformation, the v -adjustment is represented by the matrix $A = v_1 S + v_2 S^2 + \dots + v_{p^k} S^{p^k}$, where S is the permutation matrix for a cyclic shift by