



Fermat's Last Theorem for Gaussian Integer Exponents

John A. Zuehlke

The American Mathematical Monthly, Vol. 106, No. 1. (Jan., 1999), p. 49.

Stable URL:

<http://links.jstor.org/sici?sici=0002-9890%28199901%29106%3A1%3C49%3AFLTFGI%3E2.0.CO%3B2-U>

The American Mathematical Monthly is currently published by Mathematical Association of America.

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/maa.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

The JSTOR Archive is a trusted digital repository providing for long-term preservation and access to leading academic journals and scholarly literature from around the world. The Archive is supported by libraries, scholarly societies, publishers, and foundations. It is an initiative of JSTOR, a not-for-profit organization with a mission to help the scholarly community take advantage of advances in technology. For more information regarding JSTOR, please contact support@jstor.org.

NOTES

Edited by Jimmie D. Lawson and William Adkins

Fermat's Last Theorem for Gaussian Integer Exponents

John A. Zuehlke

In this note, we observe that Wiles' Theorem [2] on the impossibility of

$$x^n + y^n = z^n$$

for x, y, z positive rational numbers with integer exponents $n \neq \pm 1, \pm 2$ can be generalized to the case of Gaussian integer exponents $\nu = n + im$ without additional exceptions. The proof uses the Gelfond-Schneider Theorem [1], according to which α^β is transcendental for β algebraic but not rational and α algebraic $\neq 0, 1$.

The proof almost fits into the margin. In fact, from

$$x^\nu + y^\nu = z^\nu, \text{ with } \nu = n + im, \quad m \neq 0$$

it follows by taking the complex modulus squared that

$$x^{2n} + 2x^n y^n \cos \theta + y^{2n} = z^{2n}, \text{ with } \theta = m \log(x/y),$$

so $\cos \theta$ is rational. Since, for any real number θ whatsoever there is the identity

$$e^{2i\theta} - 2 \cos \theta e^{i\theta} + 1 = 0,$$

it follows for the particular θ that

$$e^{i\theta} = (x/y)^{im}$$

is algebraic. Then the Gelfond-Schneider Theorem, with $\alpha = x/y$ and $\beta = im$ forces $x = y$. Therefore

$$(z/x)^\nu = 2,$$

forcing $z = x$ similarly, contradicting $y \neq 0$.

We remark that the generalization holds, with the same proof, for exponents $\nu = n + im$, with n an integer and m a real algebraic number.

REFERENCES

1. A. Baker, *Transcendental Number Theory*, Cambridge University Press (2nd edition), Cambridge 1979.
2. A. Wiles, Modular Elliptic Curves and Fermat's Last Theorem, *Annals of Math.* **141** (1995) 443-551.

Columbia University, New York, New York 10027
jaz@cpw.math.columbia.edu