PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Daniel H. Ullman,** and **Douglas B. West**

with the collaboration of Paul T. Bateman, Mario Benedicty, Paul Bracken, Duane M. Broline, Ezra A. Brown, Richard T. Bumby, Glenn G. Chappell, Randall Dougherty, Roger B. Eggleton, Ira M. Gessel, Bart Goddard, Jerrold R. Griggs, Douglas A. Hensley, John R. Isbell, Robert Israel, Kiran S. Kedlaya, Murray S. Klamkin, Fred Kochman, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Richard Pfiefer, Cecil C. Rousseau, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Charles Vanden Eynden, and William E. Watkins.

Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before May 31, 2000; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

10767. *Proposed by Bruce Dearden and Jerry Metzger, University of North Dakota, Grand Forks, ND.* For integers $n \geq 2$ and $m > 1$, how many invertible m-by-m matrices are there modulo n?

10768. *Proposed by Sung Soo Kim, Hanyang University, Ansan, Kyunggi, Korea.*
(**a**) Show that there is a continuous function $f\colon \mathbb{R} \to \mathbb{R}$ such that $f + g$ is not increasing for any differentiable function g.
(**b**) Show that there is a differentiable function $f\colon \mathbb{R} \to \mathbb{R}$ such that $f + g$ is not increasing for any continuously differentiable function g.
(**c**) Show that, for any continuously differentiable function $f\colon \mathbb{R} \to \mathbb{R}$, there is a real analytic function g such that $f + g$ is increasing.

10769. *Proposed by Christian Blatter, Zürich, Switzerland.* Determine the minimum number of colors necessary to color the points of a sphere in such a way that points at spherical distance $\pi/2$ (i.e., points that subtend a right angle from the center of the sphere) get different colors.

10770. *Proposed by Călin Popescu, Louvain-la-Neuve, Belgium.* Suppose that m and n are integers with $1 < m < \phi(m) + n$, where $\phi(m)$ is the number of elements in $\{1, 2, \ldots, m\}$ that are relatively prime to m. Show that $\sum_{i=1}^{n}(-1)^i\binom{n}{i}i^m$ is divisible by m.

10771. *Proposed by Mowaffaq Hajja and Peter Walker, American University of Sharjah, Sharjah, U. A. E.* Evaluate $\int_0^1 \int_0^1 \int_0^1 \left(1 + u^2 + v^2 + w^2\right)^{-2} du\, dv\, dw$.

10772. *Proposed by William C. Waterhouse, Pennsylvania State University, University Park, PA.* For any ordered field K, one can define the derivative of a function $f\colon K \to K$ as usual by $f'(x) = \lim_{y\to x}(f(y) - f(x))/(y - x)$. Suppose that every $f\colon K \to K$ with derivative identically zero is constant. Prove that K is isomorphic to the field of real numbers.