

10772



William C. Waterhouse

The American Mathematical Monthly, Vol. 106, No. 10. (Dec., 1999), p. 963.

Stable URL:

<http://links.jstor.org/sici?sici=0002-9890%28199912%29106%3A10%3C963%3A1%3E2.0.CO%3B2-P>

The American Mathematical Monthly is currently published by Mathematical Association of America.

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/maa.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

The JSTOR Archive is a trusted digital repository providing for long-term preservation and access to leading academic journals and scholarly literature from around the world. The Archive is supported by libraries, scholarly societies, publishers, and foundations. It is an initiative of JSTOR, a not-for-profit organization with a mission to help the scholarly community take advantage of advances in technology. For more information regarding JSTOR, please contact support@jstor.org.

PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Daniel H. Ullman, and Douglas B. West**

with the collaboration of Paul T. Bateman, Mario Benedicty, Paul Bracken, Duane M. Broline, Ezra A. Brown, Richard T. Bumby, Glenn G. Chappell, Randall Dougherty, Roger B. Eggleton, Ira M. Gessel, Bart Goddard, Jerrold R. Griggs, Douglas A. Hensley, John R. Isbell, Robert Israel, Kiran S. Kedlaya, Murray S. Klamkin, Fred Kochman, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Richard Pfiefer, Cecil C. Rousseau, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Charles Vanden Eynden, and William E. Watkins.

Proposed problems and solutions should be sent in duplicate to the MONTHLY problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before May 31, 2000; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk () after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

10767. *Proposed by Bruce Dearden and Jerry Metzger, University of North Dakota, Grand Forks, ND.* For integers $n \geq 2$ and $m > 1$, how many invertible m -by- m matrices are there modulo n ?

10768. *Proposed by Sung Soo Kim, Hanyang University, Ansan, Kyunggi, Korea.*

(a) Show that there is a continuous function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f + g$ is not increasing for any differentiable function g .

(b) Show that there is a differentiable function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f + g$ is not increasing for any continuously differentiable function g .

(c) Show that, for any continuously differentiable function $f: \mathbb{R} \rightarrow \mathbb{R}$, there is a real analytic function g such that $f + g$ is increasing.

10769. *Proposed by Christian Blatter, Zürich, Switzerland.* Determine the minimum number of colors necessary to color the points of a sphere in such a way that points at spherical distance $\pi/2$ (i.e., points that subtend a right angle from the center of the sphere) get different colors.

10770. *Proposed by Călin Popescu, Louvain-la-Neuve, Belgium.* Suppose that m and n are integers with $1 < m < \phi(m) + n$, where $\phi(m)$ is the number of elements in $\{1, 2, \dots, m\}$ that are relatively prime to m . Show that $\sum_{i=1}^n (-1)^i \binom{n}{i} i^m$ is divisible by m .

10771. *Proposed by Mowaffaq Hajja and Peter Walker, American University of Sharjah, Sharjah, U. A. E.* Evaluate $\int_0^1 \int_0^1 \int_0^1 (1 + u^2 + v^2 + w^2)^{-2} du dv dw$.

10772. *Proposed by William C. Waterhouse, Pennsylvania State University, University Park, PA.* For any ordered field K , one can define the derivative of a function $f: K \rightarrow K$ as usual by $f'(x) = \lim_{y \rightarrow x} (f(y) - f(x)) / (y - x)$. Suppose that every $f: K \rightarrow K$ with derivative identically zero is constant. Prove that K is isomorphic to the field of real numbers.