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Reform, Tradition, and Synthesis

Thomas W. Tucker

The recent debate in the mathematical community about calculus instruction is not the first struggle between reform and tradition, and it won't be the last. Perhaps the two sides in this case may be much closer to agreement than the rhetoric indicates. My goal is to make a few remarks about some of the issues that have divided the two sides: technology, lecturing, drill, rigor, algebra, choice, and outcomes. For most of these issues, there are stances usually attributed to the traditionalists and reformers. For example, reformers may favor collaborative learning, while traditionalists prefer lectures. I don't think such contrasts are necessarily accurate, and I hope my remarks might initiate a dialogue to reach some middle position, a synthesis of tradition and reform. I should acknowledge that, although I have spent most of the last twelve years in the reform camp (through work in the MAA and in the Calculus Consortium based at Harvard), I am a timid reformer and I make no claims that my views represent anything but my own opinion.

Before I proceed I offer a brief apology for the use of the word "reform," whose connotations are not nice (e.g., "reform school"). I was always a little hesitant to use the word in the early days of calculus reform after the 1986 Tulane Conference, which itself avoided the word as much as possible. Unfortunately, no better term came along and we are stuck with it now. I thought of writing this whole article spelling the word "re-form" instead of "reform," but that seemed too precious. So it's "reform," warts and all.

Technology. I asked my multivariable calculus class yesterday if they knew the sines or cosines of any special angles, whether the numbers 2 or root 3 sounded familiar. Only about a third of the class raised a hand. I know this would not have happened in a calculus class thirty years ago. I mourn the loss of this lovely bit of knowledge. The widespread use of graphing calculators is to blame, of course, and it can get worse. As graphing calculators with symbolic manipulation become more widely used, I share the fears of many mathematicians that a question about the derivative of sine or cosine could draw equally blank looks from my class. If it's in the machine, why memorize it? I sympathize with mathematicians who ban graphing calculators in their classes. When I was on the AP Calculus committee a few years ago, I strongly supported having sections of the test where graphing calculators are not allowed. On the other hand, I also strongly supported allowing them, or even requiring them, on other parts of the test.

Here is why. It pays to heed history: Technology always wins. The world may have been better when people walked instead of driving cars, but that is irrelevant. As long as there is gas, people will drive cars, and what I really care about is that they drive them sensibly. The mathematical world may have been better when people did arithmetic or graphed functions on paper or in their head instead of on a calculator, but that is irrelevant. As long as there are batteries, students will use calculators, and what I really care about is that they use them sensibly. So I allow them in my classes and have learned to appreciate my students' facility and inventiveness. When my students misuse their calculators or something unexpected happens, I have an opportunity to give them some important advice or talk about an interesting mathematical phenomenon. Pretending something doesn't exist is not a good teaching strategy. For many of my students, graphing calculators are as much a part of their intellectual constitution as pencil and paper, and I have to learn to deal with it.

Computers seem to be less an issue. Indeed, I suspect there are some calculus courses that require computer laboratories or assignments but ban graphing calculators. This is actually quite understandable. Most mathematicians spend enough time around computers, in their everyday life or even in their research, that it seems natural to use computers in their teaching as well. On the other hand, most college or university mathematicians have spent no time at all with a graphing calculator and are not inclined to spend the start-up time of an hour or two to learn, especially since they are unlikely to use graphing calculators on a regular basis outside the classroom. A bridge is needed for this gap between mathematics students (and secondary school teachers) on the one side and college faculty on the other. Indeed, I think the role of hand-held devices in mathematics education, from college right down to kindergarten, needs to be studied and discussed far more than at present. For example, is long division with pencil and paper still a necessary skill? No one seems to be willing to entertain the notion that it is not, and until someone does, I don't think there will be an honest discussion.

Lecturing. Let's cut to the chase. Do I lecture? Yes. All the time? Just about. Do I believe that students learn by talking to each other? Absolutely, because I myself learn best by talking with other mathematicians, even when we have little idea what we are talking about. My implementation of collaborative learning is low-key. Once or twice a week, I have a pair of students present a homework problem on the board (they know ahead of time who their partner is and which problem they have to do). In a class of 35, this gets everyone to the board at least once during the semester at the cost of 10 minutes a week. This gets control of the blackboard out of my hands for a few minutes and forces two students to talk to each other outside of class. I also distribute a class list, which includes email addresses, phone numbers, and dorm rooms, so everyone can find someone to hook up with. I never make available a solutions manual so students are forced to talk to someone else when they are confused. The result is that usually a little more than half the students in my classes work on their homework in groups of two or more. I wish it were more and I harangue them as much as I can, but loners may be happier as loners and I can't change that. All I know is that I was a loner myself in my undergraduate courses and couldn't have been more unhappy (mathematically). Sometimes all it takes is a nudge in the right direction.

Classroom formats with little lecturing can be wonderful, but the evolutionary forces that brought us the lecture format havn't gone away. Lectures are here to stay. The real issue is how to get students talking with each other, and there are lots of mechanisms for doing that.

Drill. A colleague of mine has said "There are some things you should do with your spine rather than your brain." I agree. Students should be able to take derivatives of most elementary functions without having to think about it, with their spine. Again, I am delighted that the AP Calculus exam has a multiple choice section where calculators are not allowed and "spinal" manipulations can be tested without interference from calculators that can take derivatives symbolically. To do this, students need drill. The question is determining when you have reached the point of diminishing returns. If I drill my students on differentiation all semester, there will still be some who make mistakes on a four-deep chain rule. In the meantime, think of the other things I could have done.

Another colleague has said "Better rote learning than no learning." I used to agree, mostly because I think memorization is good for the mind. I am not so sure, however, whether this is true in mathematics. The belief that mathematics is just formulas, a belief that studies show American students hold and Japanese students do not, undermines everything mathematics educators are trying to do. Some rote, some drill, fine, but it better be less than half of what is taught and tested, or else it isn't mathematics anymore.

Rigor. When it comes to theory in calculus courses, mathematicians surrendered a long time ago. There is almost no theoretical content at all in the compendium of calculus final exams given in the 1987 MAA Notes Volume, *Calculus for a New Century*. Despite the talk that one can learn mathematics (or any science for that matter) only by doing it, when it comes to theory, students have no hands-on activity. Students may see correct definitions and proofs but they don't do them. I understand why the debate over rigor in calculus instruction has been so bitter: mathematicians have conceded so much since the heights of abstraction reached in the new math era of the 1960's, that they cling to what little formalism remains. I hope instead there is a serious effort to reclaim the high ground.

I think calculus students should do proofs. The word "prove" should appear in problems. One should be careful, however, about what students are asked to prove. In mathematical research, proof is a tool used to answer questions where the issue is in doubt. Asking for an epsilon-delta proof that a certain limit is what we know it must be is guaranteed to irritate and confuse students. Ask instead for proofs in situations where there is doubt. For example: Prove or disprove that if two functions are both concave up on an interval, their sum is concave as well. I know a few other examples (a couple have appeared on AP exams), but many more are needed.

I also think students should write sentences and paragraphs in which they use formal mathematical terminology correctly. The mathematical content does not have to be deep; a full discussion of the graphical behavior of some function is enough. The culture shock that hits mathematics majors in their first theory course is not just the abstraction. It is that arguments are to be written in logically coherent sentences and paragraphs, not strings of equations as usually is the case in a calculus class. At the very least, students should be asked frequently to explain what they think they are doing. Although some reform projects have worked very hard on improving student writing, I hardly think of this as a reform issue. Students need to write.

Algebra. I guess this is the one area where I am most fervently a reformist. Algebra is one of the most powerful intellectual tools known to mankind. Computers could not operate without algebraic representations of functions. Students can, however, get the impression from calculus (and earlier mathematics courses) that algebra and mathematics are synonymous. That is not good. I have already noted how American students seem to think mathematics is just formulas. Far worse, if mathematics is algebra, then it must be irrelevant to most students' lives. Just read the *New York Times* for a month, every page, and tell me how often you encounter an algebraic equation or formula. There is plenty of mathematics there in numbers, tables, graphs, or verbal descriptions, but nary an x or y in sight. I often think that my own algebraic manipulative skills stay honed only because I teach calculus; I certainly don't use those skills much in my research.

Functions in a calculus course should be represented by tables of values, graphs, and verbal descriptions, as well as algebraic formulas. This does not water down the course. Non-algebraic reasoning and communication is not "softer" than algebraic, any more than geometry is softer than algebra. Interestingly enough, the inclusion of non-algebraic viewpoints seems to be one aspect of calculus reform that has gained acceptance. It is often the way new editions of many traditional texts most resemble reform texts, and it has also become part of the guidelines for the construction of many standardized mathematics tests.

Choice. Back around 1990, Peter Lax proposed to the American Mathematical Society the following resolution that might act like a stick of dynamite to break up the logjam in curricular diversity: "Requiring a professor to teach from a common textbook or for a common exam is an abridgment of academic freedom." I remember this sounded awfully revolutionary. I believe Peter was careful to say "professor," and there may have been some weasel words, like "qualified" or "tenured" professor, but still it seemed common sense that some sort of uniformity is needed in a multiple section calculus course taught by professors, post-docs, adjuncts, and graduate students. Nowadays, it is becoming more common to see fewer common exams and even different textbooks in different sections of a calculus course. This is a reasonable compromise when departments (such as my own) cannot reach a consensus on how to teach calculus.

I am still not sure how I feel about this. Diversity is better, I know. Even the most traditional calculus instructor has bemoaned at least once the lack of variety in textbooks. For a few years in the early days of calculus reform, there really was some choice; now there is still some diversity, although less than before as the more radical texts are remaindered by publishers. On the other hand, students are prone to making invidious comparisons, and it is a lot easier for everyone if all sections of a multisection course look the same. Also, making up and grading common exams is a source of departmental camaradie; many reform efforts focus on the social aspects of teaching and learning, and common syllabi and exams build community, both among faculty and among students. In general, it is probably better for a department to reach some compromise consensus for its calculus courses. Allowing each instructor to go his or her own way, with only an agreement over the core content, should be a last resort.

Outcomes. Reform courses have been under pressure to assess their success. You can't say something is better without backing it up with data. I have always viewed this as a red herring. First, traditional courses do almost no assessing of outcomes other than student performance on the final exams; I doubt that the pass/fail rate on a final exam is viewed as a reasonable form of assessment. Second, most reformers end up working 16 hour days to prepare new materials (the usual criticism of reform courses is that they are way too labor intensive) and have little time for extensive assessment. Finally, most of the comparisons I know between traditional and reform courses at the same institution are not controlled experiments: even when the students are assigned randomly to different sections, the instructors are not. When reform courses come out looking better on common exams, perhaps it is because the instructors who choose to teach the reform versions are not typical instructors.

Nevertheless, the call for assessments is useful. It is a good idea to think hard about what students take with them from a course, in terms of not only content but also experience. For example, one should question the choice of content for a first semester calculus course that does not include the exponential and natural log functions; after all, the course is probably terminal for half the class. In terms of experience, one should ask questions of a calculus course that could be asked of any course: Did students have to write? Did they speak to an audience? Did they have an opportunity to work on some significant project independently? Did they acquire a viewpoint or skills that are applicable in a wide variety of circumstances? Did they work with others? Did they have to find and evaluate information for themselves, from a library or the web? The outcomes of a calculus course should be viewed in the context of the entire college curriculum.

Community. It has been observed that one thing reform has accomplished in the last ten years is the creation of a community of mathematicians who share a common interest in mathematics education. The more people who feel they are part of this community, the better. That is why it is so important for both reformers and traditionalists to see their common interest: they both want their students to learn and appreciate mathematics.

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Editor's Note: The preceding article by Thomas Tucker and the following article by Steven Krantz were solicited to present a collegial discourse about calculus reform. Each author was encouraged to comment on positive attributes of the 'other side' and to be honest about problems on 'their side'.