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The American Mathematical Monthly, Vol. 106, No. 10. (Dec., 1999), pp. 915-918.

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Sat Dec 22 05:43:18 2007

You Don't Need a Weatherman to Know Which Way the Wind Blows

Steven G. Krantz

I am moderately well-known as a complex analyst, but I seem to be almost pathologically well-known as an avatar of traditionalist teaching. The latter attribute stems no doubt from my having penned the book *How to Teach Mathematics*. The discussions pursuant to the appearance of that book have caused all of us to rethink our positions. Certainly my ideas have evolved. Have no fear: I still value traditional methods of teaching. But I have come to appreciate many of the reform ideas as well. Nobody wants to be told that the tried-and-true methods that he or she has been using for several decades are no longer valid. But any well-educated person who is capable of critical thinking surely knows that a skill worth learning is also one that is worth rethinking and refining and developing. What do the reformers have to offer that might appeal to such an individual?

Perhaps the most compelling, yet disturbing, assertion that I have heard from the reformers is this: "It's not just that lecturing doesn't work with today's students. In fact lecturing has never worked." Can this be true? Sadly, you and I are ill-equipped to judge. As professional mathematics instructors, we are the survivors in a rather arcane evolutionary process. We were always good at learning—particularly at learning mathematics. Our mathematical and scholarly abilities raised us to such a level that we were relatively immune to what teaching methods were being used, or what personality quirks the teacher had, or what medieval textbook was being foisted upon us. Alas, most students don't fit that mold. It is valid, and appropriate, to pose the question of whether there are teaching techniques that are more effective than lecturing *in teaching an average student of average ability*.

I still lecture; on days of extraordinary hubris, I think I'm pretty good at it. But I endeavor to create the illusion in my classroom that the students and I are actually carrying on a dialogue, that we are developing the ideas together. In my own way, I am enabling my students to engage in group work, and to participate in discovery learning. I may not be a card-carrying reformer, but I have been influenced by the reform tenets.

In the past few years I have become convinced that lower division mathematics should be a laboratory science. Chemists and biologists have known for lo these many years that labs are an effective way to make ideas concrete for the student. They are a way to enable discovery learning. Why has mathematics remained out of the loop?

One obvious reason is that accessible and affordable high speed digital computing has been unavailable until fairly recently. Quality software—that is of interest to the mathematician—did not exist. But things have changed: most math departments are full of computer equipment and also full of exciting new software tools such as `Mathematica` and `Derive` and `Axiom` and `Maple`. Do you find it difficult to explain to your students why the method of Lagrange multipliers works? Or why the gradient of a function of three variables is always orthogonal to the level sets? Or why Simpson's rule converges more rapidly than the trapezoid rule? Couldn't well-constructed computer labs bridge this gap, and help students of average ability to understand why and how mathematics works?

In the past I have been guilty of asking:

- How can students discover mathematical facts if they have no knowledge base and no technical training?
- How can students work in groups when nobody in the group knows what he or she is talking about?
- How can students formulate conjectures if they don't know anything?

These questions are not entirely off-base. But they are a bit cranky. And well-thought-out laboratories may provide at least a partial answer to all of them. A student might discover a mathematical fact if a lab activity is designed to lead him or her to it. Students might discuss and collaborate profitably if (computer-aided) material is put before them that will stimulate such interaction. A highly trained person—say a Ph.D. in mathematics—needs very little grist, and almost no catalyst, to get his mill grinding. A young student needs considerably more, and interaction with the computer can help. It is difficult for a person lacking a highly developed intellectual framework to formulate conjectures; but a good computer lab can help the student to build a short-term framework that will lead to interesting queries.

A good teacher does three things for his/her students:

(1) Sets a pace for the students;
(2) Teaches the students to read;
(3) Engages the students in the learning process.

It is item **(3)** that causes most of us the greatest frustration and discomfort. Why won't our students talk to us? Why don't they show any interest? Why is class attendance so poor? Why is there no sense of curiosity or excitement in the typical calculus classroom?

I'm sorry to say it—I know that nobody wants to hear it—but lectures, in and of themselves, are not by nature engaging or exciting. At least not for eighteen-year-olds. This has been one of the chief messages of the reform movement and, in essence, I think that the message is correct. I have learned to use my own lectures as an effective tool. I fill the room with myself; I get my students to talk to me. Under my guidance, the students shout out conjectures, and they *help me to construct the lesson*. This is a skill that I have honed over more than one quarter of a century of teaching. But it is a great deal of work to develop such a skill. Not all of us are born with such skill or such dedication, nor do we all have the inclination to learn it. A reasonable alternative is to say, "Lectures are not working; let's try something else."

I don't buy in to *that* particular conclusion. I have learned to make my lectures work for me. And they work for my students too. But each mathematics instructor must find his or her own means of getting students involved in the learning process, of helping them to become educated. The reformers have put before us a menu of possibilities—including group work, discovery learning, computer labs, and other techniques too—that are well worth exploring. Take those that appeal, sample some others. Keep the ones that work. And then move on.

One of the more controversial tenets of reform is that we should reduce the role of drill in our classrooms, that we should soft-pedal rigor and theory, and that we should instead concentrate on *concepts*. [Certainly you cannot claim to the world that you have written a reform calculus book unless the word "concepts" appears in your title.] How is a died-in-the-wool traditionalist to come to terms with these notions?

I am convinced that our freshmen are very bright, but they do not have the intellectual equipment to appreciate a genuine mathematical proof. Those who have taken a high school course in Euclidean geometry in the past ten years did *not* have the course that some of us experienced thirty years ago. Modern high school geometry texts minimize proofs (and stress concepts!). You and I have intensive training in the discourse of mathematics. When I write a proof—that I want you to read—then I prepare it in the accepted form that I have been trained to produce, so that you will both appreciate it and believe it. Our freshmen are *not* privy to this discourse.

In a class full of freshmen, I find it appropriate to say "Here is a picture that illustrates why this is true" (when I am explaining, for example, the Fundamental Theorem of Calculus) or "Here is an example that shows why this works" (when I am explaining why $\det(A \cdot B) = (\det A) \cdot (\det B)$ or "Here is an analogy that will help you to believe this formula" (when I am explaining the Chain Rule). You have to speak to people in their own language. For freshmen that language is English. If the math curriculum is well-constructed, then by the time that the student is a junior he or she will have learned *mathematical argot*; at that time we can present such a student with a proof, and he/she will appreciate it (and believe it). Prior to that, we should resist.

Do I teach concepts? Who wouldn't? On the one hand, we teach students *technique*. For instance, when we teach maximum/minimum problems we show them how to actually *do* such problems; on the other hand there is a concept (due to Fermat) behind the technique, and we teach that as well. Concepts without technique are hollow. Technique without drill is meaningless. Most reformers that I know would agree. There is some debate over whether drill or concepts should come first. I leave that to the individual: there are many worthy and productive paths that lead to the same goal.

For many years we have all known, in the backs of our minds, that our students cannot write. They hand in homework assignments that bear scant resemblance to anything more than incoherent gibberish. The reformers—especially the Harvard group—have helped us to realize that writing has a deserved place in the mathematics classroom. And I'm talking about real writing here, with sentences and paragraphs and overall organization. The accident at Three Mile Island occurred in large part because the engineers at that power plant could not communicate their concerns to the governor of Pennsylvania. I wonder how many of those engineers were our calculus students?

Good writing and clear thinking are inexorably linked. Certainly we all want our students to be clear thinkers. One sure way to help them develop in that direction is to teach them to write, to organize their thoughts, to judge their audience, to argue a point. It is just a bit too facile for us to object that all these reform techniques take more time and more effort on the parts of the instructors. Of course they do. Anything worthwhile requires a great deal of effort. Once we have decided that these methodologies are worthwhile, and worth trying, then we can find practical methods for implementing them.

Reform always works in the hands of the reformers. For everyone else, reform is an object lesson and a crucible for experimentation. We will all be better off when we realize that reformers and traditionalists are after the same grail: to enable our students to appreciate and to learn and in the end perhaps to love mathematics. We want to give them the grounding they need in mathematical techniques and concepts so that they can go on to advanced study in any area they might choose to pursue, whether it be engineering or epidemiology or even mathematics. We want them, as part of their education in Western thought, to understand the mathematical method. These realizations should make it easy for reformers and traditionalists to work together. Let us find the means to do so.

STEVEN G. KRANTZ received his B.A. degree from the University of California at Santa Cruz in 1971. He received the Ph.D. from Princeton University in 1974. Krantz has taught at UCLA, Princeton University, Penn State University, and Washington University in St. Louis. He has been a visiting professor at Princeton University, the University of Paris, the University Paul Sabatier, the University of Umea, Uppsala University, the University Autonoma de Madrid, the Mathematical Sciences Research Institute, the Institute for Advanced Study, and Beijing University. Krantz has received the UCLA Alumni Association Distinguished Teaching Award, the Kemper Prize, the Chauvenet Prize of the MAA, and the Beckenbach Book Award of the MAA.
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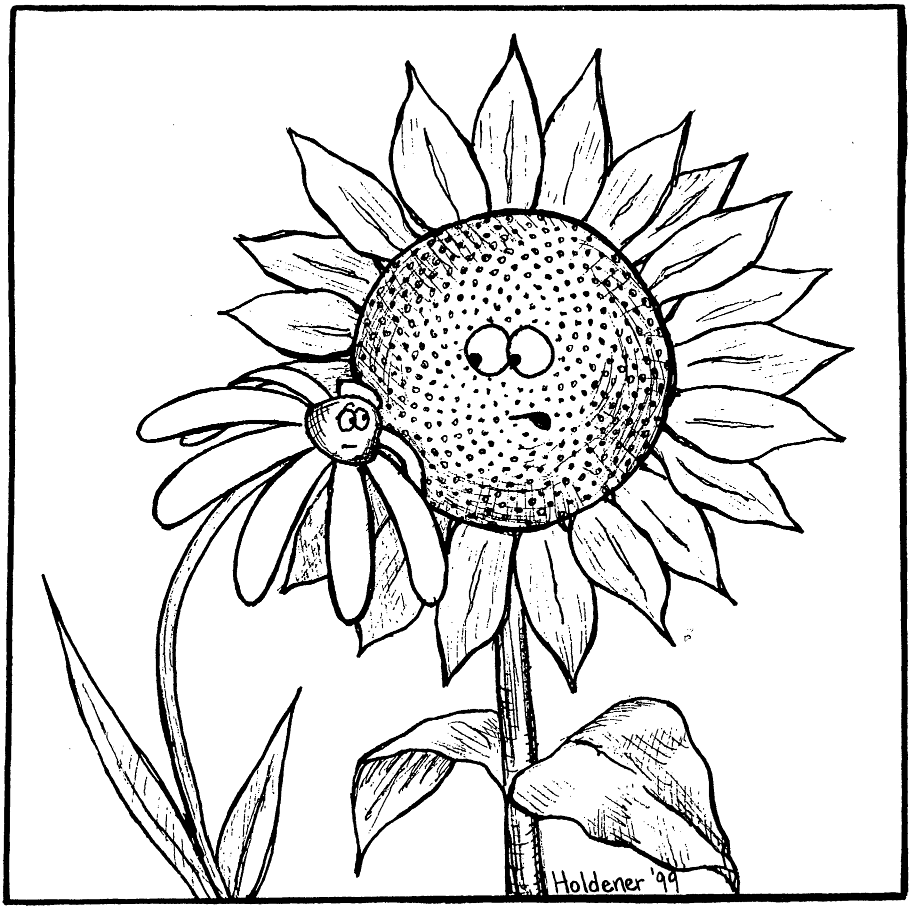

"Ever notice that the number of legs on an animal is always a number from the sequence {0, 2, 4, 6, 8,...}?"

Contributed by Judy Holdener, Kenyon College