

10711



Florian Luca

The American Mathematical Monthly, Vol. 106, No. 2. (Feb., 1999), p. 166.

Stable URL:

<http://links.jstor.org/sici?sici=0002-9890%28199902%29106%3A2%3C166%3A1%3E2.0.CO%3B2-D>

The American Mathematical Monthly is currently published by Mathematical Association of America.

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/maa.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

The JSTOR Archive is a trusted digital repository providing for long-term preservation and access to leading academic journals and scholarly literature from around the world. The Archive is supported by libraries, scholarly societies, publishers, and foundations. It is an initiative of JSTOR, a not-for-profit organization with a mission to help the scholarly community take advantage of advances in technology. For more information regarding JSTOR, please contact support@jstor.org.

PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Daniel H. Ullman, and Douglas B. West**

with the collaboration of Paul T. Bateman, Mario Benedicty, Paul Bracken, Duane M. Broline, Ezra A. Brown, Richard T. Bumby, Glenn G. Chappell, Randall Dougherty, Roger B. Eggleton, Ira M. Gessel, Bart Goddard, Jerrold R. Griggs, Douglas A. Hensley, Richard Holzsager, John R. Isbell, Robert Israel, Kiran S. Kedlaya, Murray S. Klamkin, Fred Kochman, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Richard Pfeifer, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Charles Vanden Eynden, and William E. Watkins.

Proposed problems and solutions should be sent in duplicate to the MONTHLY problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before July 31, 1999; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk () after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

10711. *Proposed by Florian Luca, Universität Bielefeld, Bielefeld, Germany.* A natural number is *perfect* if it is the sum of its proper divisors. Prove that two consecutive numbers cannot both be perfect.

10712. *Proposed by Paul Deiermann, Lindenwood University, St. Charles, MO, and Rick Mabry, Louisiana State University, Shreveport, LA.* Let $f(x)$ and $g(y)$ be twice continuously differentiable functions defined in a neighborhood of 0, and assume that $f(0) = 1$, $g(0) = f'(0) = g'(0) = 0$, $f''(0) < 0$, and $g''(0) > 0$.

(a) For sufficiently small $r > 0$, show that the curves $x = g(y)$ and $y = rf(x/r)$ have a common point (x_r, y_r) in the first quadrant with the property that, if (x, y) is any other common point, then $x_r < x$.

(b) Let $(t_r, 0)$ denote the x -intercept of the line passing through $(0, r)$ and (x_r, y_r) . Show that $\lim_{r \rightarrow 0^+} t_r$ exists, and evaluate it.

(c) Is the continuity of f'' and g'' a necessary condition for $\lim_{r \rightarrow 0^+} t_r$ to exist?

10713. *Proposed by Juan-Bosco Romero Márquez, Universidad de Valladolid, Valladolid, Spain.* Given a triangle with angles $A \geq B \geq C$, let a , b , and c be the lengths of the corresponding opposite sides, let r be the radius of the inscribed circle, and let R be the radius of the circumscribed circle. Show that A is acute if and only if $R + r < (b + c)/2$.

10714. *Proposed by Jet Wimp, Drexel University, Philadelphia, PA.* For $a \in (-\pi/2, \pi/2)$, define

$$c_n(t) = \frac{1}{e^{at} \cos a} \left(\frac{d}{da} \right)^n (e^{at} \cos a)$$

for every nonnegative integer n , so that $c_n(t)$ is a monic polynomial of degree n . Let G_n denote the $(n + 1)$ -by- $(n + 1)$ determinant $|c_{j+k}(t)|_{j,k=0,1,\dots,n}$. Evaluate G_n .

10715. *Proposed by Roger Cuculière, Clichy, France.* Choose $u_0 > 1$, and define $u_{n+1} = u_n + \ln u_n$ for $n \in \mathbb{N}$. Find a closed-form expression a_n such that $\lim_{n \rightarrow \infty} (u_n - a_n) / n = 0$.