PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Daniel H. Ullman,** and **Douglas B. West**
with the collaboration of Paul T. Bateman, Mario Benedicty, Paul Bracken, Duane M. Broline, Ezra A. Brown, Richard T. Bumby, Glenn G. Chappell, Randall Dougherty, Roger B. Eggleton, Ira M. Gessel, Bart Goddard, Jerrold R. Griggs, Douglas A. Hensley, Richard Holzsager, John R. Isbell, Robert Israel, Kiran S. Kedlaya, Murray S. Klamkin, Fred Kochman, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Richard Pfiefer, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Charles Vanden Eynden, and William E. Watkins.

Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before July 31, 1999; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

10711. *Proposed by Florian Luca, Universität Bielefeld, Bielefeld, Germany.* A natural number is *perfect* if it is the sum of its proper divisors. Prove that two consecutive numbers cannot both be perfect.

10712. *Proposed by Paul Deiermann, Lindenwood University, St. Charles, MO, and Rick Mabry, Louisiana State University, Shreveport, LA.* Let $f(x)$ and $g(y)$ be twice continuously differentiable functions defined in a neighborhood of 0, and assume that $f(0) = 1$, $g(0) = f'(0) = g'(0) = 0$, $f''(0) < 0$, and $g''(0) > 0$.
(**a**) For sufficiently small $r > 0$, show that the curves $x = g(y)$ and $y = rf(x/r)$ have a common point (x_r, y_r) in the first quadrant with the property that, if (x, y) is any other common point, then $x_r < x$.
(**b**) Let $(t_r, 0)$ denote the x-intercept of the line passing through $(0, r)$ and (x_r, y_r). Show that $\lim_{r\to 0^+} t_r$ exists, and evaluate it.
(**c**) Is the continuity of f'' and g'' a necessary condition for $\lim_{r\to 0^+} t_r$ to exist?

10713. *Proposed by Juan-Bosco Romero Márquez, Universidad de Valladolid, Valladolid, Spain.* Given a triangle with angles $A \geq B \geq C$, let a, b, and c be the lengths of the corresponding opposite sides, let r be the radius of the inscribed circle, and let R be the radius of the circumscribed circle. Show that A is acute if and only if $R + r < (b + c)/2$.

10714. *Proposed by Jet Wimp, Drexel University, Philadelphia, PA.* For $a \in (-\pi/2, \pi/2)$, define

$$c_n(t) = \frac{1}{e^{at}\cos a}\left(\frac{d}{da}\right)^n \left(e^{at}\cos a\right)$$

for every nonnegative integer n, so that $c_n(t)$ is a monic polynomial of degree n. Let G_n denote the $(n+1)$-by-$(n+1)$ determinant $\left|c_{j+k}(t)\right|_{j,k=0,1,\ldots,n}$. Evaluate G_n.

10715. *Proposed by Roger Cuculière, Clichy, France.* Choose $u_0 > 1$, and define $u_{n+1} = u_n + \ln u_n$ for $n \in \mathbb{N}$. Find a closed-form expression a_n such that $\lim_{n\to\infty} (u_n - a_n)/n = 0$.