



Principal Ideals in Noetherian Rings: 10534

Paul Arne Ostvoer; Robert Gilmer

The American Mathematical Monthly, Vol. 106, No. 3. (Mar., 1999), p. 265.

Stable URL:

<http://links.jstor.org/sici?sici=0002-9890%28199903%29106%3A3%3C265%3APIINR1%3E2.0.CO%3B2-1>

The American Mathematical Monthly is currently published by Mathematical Association of America.

Your use of the JSTOR archive indicates your acceptance of JSTOR's Terms and Conditions of Use, available at <http://www.jstor.org/about/terms.html>. JSTOR's Terms and Conditions of Use provides, in part, that unless you have obtained prior permission, you may not download an entire issue of a journal or multiple copies of articles, and you may use content in the JSTOR archive only for your personal, non-commercial use.

Please contact the publisher regarding any further use of this work. Publisher contact information may be obtained at <http://www.jstor.org/journals/maa.html>.

Each copy of any part of a JSTOR transmission must contain the same copyright notice that appears on the screen or printed page of such transmission.

The JSTOR Archive is a trusted digital repository providing for long-term preservation and access to leading academic journals and scholarly literature from around the world. The Archive is supported by libraries, scholarly societies, publishers, and foundations. It is an initiative of JSTOR, a not-for-profit organization with a mission to help the scholarly community take advantage of advances in technology. For more information regarding JSTOR, please contact support@jstor.org.

10722. Proposed by Richard F. McCoart, Loyola College, Baltimore, MD.

(a) In how many ways can $2n$ indistinguishable balls be placed into n distinguishable urns, if the first r urns may contain at most $2r$ balls for each $r \in \{1, 2, \dots, n\}$?

(b) Suppose that $0 \leq m \leq n$. In how many of the ways enumerated in part (a) are exactly m urns empty?

10723. Proposed by Christopher J. Hillar, Yale University, New Haven, CT. Let p be an odd prime. Prove that $\sum_{i=1}^{p-1} 2^i \cdot i^{p-2} \equiv \sum_{i=1}^{(p-1)/2} i^{p-2} \pmod{p}$.

10724. Proposed by Serge Tabachnikov, University of Arkansas, Fayetteville, AR.

(a) Let P be a convex plane polygon with vertices A_1, \dots, A_n , and let l be a continuous transverse field of directions along the boundary ∂P . (This means that through every point $X \in \partial P$ there passes a line $l(X)$ that intersects the interior of P and depends continuously on X .) Let α_i and β_i be the angles between the line $l(A_i)$ and the adjacent sides $A_i A_{i-1}$ and $A_i A_{i+1}$, respectively. Assume that $\prod_1^n \sin \alpha_i = \prod_1^n \sin \beta_i$. Prove that the lines $l(X)$ cover the interior of P twice, that is, every interior point of P belongs to at least two of these lines.

(b) Suppose $n \geq 3$, and let P be a convex polyhedron in n -dimensional space. As in (a), a continuous transverse line field l is given along the boundary ∂P . This field has the property that for every $(n-2)$ -dimensional face E of P there exists a hyperplane $\pi(E)$ such that all the lines $l(X)$ with $X \in E$ belong to $\pi(E)$. Prove that the lines $l(X)$ cover the interior of P twice.

SOLUTIONS

Principal Ideals in Noetherian Rings

10534 [1996, 510]. Proposed by Paul Arne Østvær, Oslo University, Oslo, Norway. Suppose that R is a Noetherian ring in which all maximal ideals are principal. Show that all ideals in R are principal.

Solution by Robert Gilmer, Florida State University, Tallahassee, FL. If $M = (m)$ is a maximal ideal of R , then M/M^2 is a vector space over the field R/M of dimension at most 1. Hence there are no ideals of R properly between M and M^2 . From this it follows (R. Gilmer, *Multiplicative Ideal Theory*, Queen's Papers Pure Appl. Math. **90** (1992), Theorem 39.2) that $R = D_1 \oplus \dots \oplus D_n \oplus S_1 \oplus \dots \oplus S_m$ is a finite direct sum of Dedekind domains D_i and special primary rings S_i . To show that each ideal of R is principal, it suffices to show that the D_i and S_i have this property. For S_i this is part of the definition of a special primary ring (Gilmer, p. 200). Moreover, D_i inherits from R the property that each of its maximal ideals is principal, and a Dedekind domain is a principal ideal domain whenever all of its maximal ideals are principal.

Editorial comment. D. D. Anderson mentions a stronger result that appears in R. Gilmer and W. Heinzer, Principal ideal rings and a condition of Kummer, *J. Algebra* **83** (1983) 285–292: If R has the ascending chain condition on principal ideals and each maximal ideal of R is principal, then every ideal of R is principal.

Solved also by Mahalal'el ben keinan (Israel), F. Calegari (Australia), J. E. Dawson (Australia), T. H. Foregger, O. Moubinool (France), S. Sertöz (Turkey), and M. Tabaâ (Morocco).

A Telescoping Constraint

10566 [1997, 68]. Proposed by Gerry Myerson, Macquarie University, Australia. Let S be a finite set of cardinality $n > 1$. Let f be a real-valued function on the power set of S , and suppose that $f(A \cap B) = \min\{f(A), f(B)\}$ for all subsets A and B of S . Prove that

$$\sum (-1)^{n-|A|} f(A) = f(S) - \max f(A),$$