Finally, in the last expression set $l = m - n$.

Editorial comment. William Seaman and the proposer proved that both sides equal the value at $x = -1$ of $\sum_{m=0}^{2n} \left(\frac{d}{dx}\right)^m \left(1 - x^2\right)^n$.

Solved also by J. C. Binz (Switzerland), R. J. Chapman (U. K.), Q. H. Darwish (Oman), J. E. Dawson (Australia), M. Ismail & P. Simeonov (U. K.), M. Omarjee (France), L. Pebody (U. K.), C. R. Pranesachar (India), R. Richberg (Germany), W. J. Seaman, H.-J. Seiffert (Germany), A. Tissier (France), and the proposer.

A Large Bipartite Subgraph

10580 [1997, 270]. *Proposed by Stephen C. Locke, Florida Atlantic University, Boca Raton, FL.* Let G be a simple graph with v vertices and e edges and with maximum degree at most 3. Suppose that no component of G is a complete graph on 4 vertices. Prove that G contains a bipartite subgraph with at least $e - v/3$ edges.

Solution by James M. Benedict and Gerald Thompson, Augusta State University, Augusta, GA. When G is bipartite, the claim holds trivially, so we may assume that the chromatic number of G is at least 3. Since G does not have a complete graph of order 4 as a component, Brooks's Theorem implies that G is 3-colorable. Consider a proper 3-coloring using colors red, white, and blue; we may assume that blue appears least often.

Each blue vertex has at most 3 neighbors, all red or white. In either red or white it has at most one neighbor. After removing that edge, we can change the blue vertex to that color and still have a proper coloring. Doing this for each blue vertex deletes at most $v/3$ edges and produces a 2-colored (that is, bipartite) subgraph.

Editorial comment. Brooks's Theorem states that a graph with maximum degree k has a proper k-coloring if $k \geq 3$ and no component is a complete graph of order $k + 1$ (see for example J. A. Bondy and U. S. R. Murty, *Graph Theory with Applications*, North-Holland, 1976, p. 122). An inductive solution that avoids Brooks's Theorem is also possible.

Solved also by R. J. Chapman (U. K.), C. P. Rupert, P. Tracy, and the proposer.

Solid Angles of a Tetrahedron

10598* [1997, 457]. *Proposed by Jeffrey C. Lagarias, AT&T Research, and Thomas J. Richardson, Bell Laboratories, Murray Hill, NJ.* Let F_1, F_2, F_3, F_4 denote the faces of a tetrahedron. For $i = 1, 2, 3, 4$, let α_i denote the solid angle of the vertex opposite face F_i, where the measure of a solid angle is normalized so that a full solid angle is 1, and let β_i denote the area of F_i, where the unit of area is normalized so that the tetrahedron has surface area 1.

(a) Prove that $\beta_i \geq \alpha_i$.

(b) Generalize to m dimensions.

Solution by John H. Lindsey II, Ft. Myers, FL.

(a) We prove the sharper claim that $\beta_i > f(\pi\alpha_i)$, where $f(\theta) = \sec\theta \tan\theta - \tan^2\theta = 1/(\csc\theta + 1)$. To see that this bound is sharper, note that $\alpha_i < 1/2$, since $1/2$ is the normalized solid angle of a plane and each angle of the tetrahedron lies on one side of a plane. Since $f(0) = 0$, $f(\pi/2) = 1/2$, and $f''(\theta) = \sec^4\theta(\sin\theta - 1)^2(\sin\theta - 2) < 0$, we have $f(\pi\alpha) \geq \alpha$ for $0 < \alpha < 1/2$.

Suppose that a counterexample exists. We relabel and translate to arrange that the counterexample occurs for $i = 1$, the vertex opposite F_1 is the origin O, and the other vertices are xU, yV, zW, where U, V, W are unit vectors and x, y, z are positive. Then

$$\frac{1}{1/\beta_1 - 1} = \frac{\beta_1}{\beta_2 + \beta_3 + \beta_4} = \frac{|(xU - zW) \times (yV - zW)|}{xy|U \times V| + xz|U \times W| + yz|W \times V|}. \tag{1}$$