The Social Life of Numbers: A Quechua Ontology of Numbers and Philosophy of Arithmetic. By Gary Urton (with the collaboration of Primitivo Nina Llanos). University of Texas Press, Austin, 1997, xv + 267 pp., $35.00 hardcover, $17.95 softcover.

Reviewed by **John Meier** and **Trisha Thorme**

1. Introduction. It is probably best to start with a confession: while we are interested observers of ethnomathematics, we are not researchers in the field. John studies geometric group theory, and Trisha is an anthropological archaeologist working in the Andes. However, we co-designed a sophomore-level intensive writing course at Lafayette College called *Counting and Culture* that is essentially a course in ethnomathematics. Although neither one of us is an ethnomathematician, as a pair we come close.

Before developing *Counting and Culture*, we presumed that "ethnomathematics" was the anthropology of mathematical activities, primarily in non-Western cultures. We expected ethnomathematicians to explore the mathematics developed and used by various (non-Western) cultures, the cultural values their mathematical activities carry, and the meanings inherent in them. We were aware of some famous texts in the field, such as *Code of the Quipu*, which basically fit this picture. But as we dug deeper into the literature we found a broad mixture of approaches to mathematics in various cultures. Much of the literature is devoted to educational issues, often promoting the preservation of a particular culture's mathematical heritage. This revitalization of indigenous mathematics is important and worthwhile, and can instill pride in the traditions of that culture. But it is often difficult to see how this is ethnomathematics; rather it seems to be a branch of the history of mathematics that laudably is not focused on European issues, and is interested in pedagogical uses. We also found several articles that catalog mathematical activity in traditional cultures without exploring the cultural context of the mathematics. We were, and still are, disappointed in the small number of articles and books that take a serious anthropological look at mathematics.

Producing catalogs at the expense of context is a general problem in the ethnosciences. Astounded at the sophisticated scientific achievements of so-called primitive cultures, we fail to ask the more interesting question of how engineering, astronomy, agronomy, and so forth were integrated into the culture in question. For example, while figuring out the underlying structure of ancient Mesoamerican calendars is interesting, such a study should integrate the anthropological richness of the region; without contextual connections, one wonders if it is really the *Mesoamerican* calendar being studied. Knowing that some ancient or contemporary culture uses a concept that we put in the domain of science becomes more interesting when we understand what cultural domains (such as kinship, religion, or economy) give meaning to the concept in that culture. Decontextualizing such studies removes individuals, their culture, and their agency from the picture.

One of the main strengths of *The Social Life of Numbers* is the deep and carefully presented anthropological analysis that refuses to reduce Andean mathematical activity to the formalism of Western mathematics.

Gary Urton is an Andean anthropologist who established himself as an exceptional ethnoscientist in his earlier work on ethnoastronomy. His study of astronomy and calendars in a Quechua-speaking community near Cuzco (Peru) illuminated Andean peoples' close attention to celestial and terrestrial geography [**4**]. Quechua was the language of the ruling Inka in their polyglot empire, which stretched from Ecuador to Chile along the spine of the mountain chain, and millions of people in the contemporary Andean nations of Peru, Bolivia, and Ecuador speak Quechua as their first language.

The Andes provide an excellent opportunity for the study of non-Western knowledge systems. A mathematical favorite is the Inkas' sophisticated accounting system, the knotted string records called khipus (or "quipus" in the old-fashioned spelling). This example of Inka cultural logic differs substantially from the roots of our own material conventions for record keeping; see [**2**]. The study of khipus and other Andean technologies helps us to understand these ancient and contemporary cultures and to ask new and difficult questions about our own.

2. What's in the book. *The Social Life of Numbers* opens with a careful and thoughtful critique of the field of ethnomathematics. For someone interested in the field, or for any mathematician interested in connections between mathematics and the humanities, the chapter on Anthropology and the Philosophy of Arithmetic is required reading. Urton unwinds the various threads that have been woven together into "ethnomathematics", showing how the field has been muddled through lack of care in distinguishing educational, philosophical, and anthropological concerns.

> Rather, it is clear that by the use of the label *ethno*mathematics, what is considered to be susceptible to cultural influence are the *conceptions* of numbers and the *philosophy* of mathematics. In other words, to my mind at least, ethnomathematics is actually concerned with *ethnophilosopy*. The elision of "philosophy" in the word ethnomathematics masks the idea, firmly held by most practitioners of ethnomathematics, that while philosophy may be influenced by culture, mathematics is not. [p. 7, emphasis in original]

Much of *The Social Life of Numbers* would fall under the heading of ethnoarithmetic. That is, Urton studies the philosophy of arithmetic (specifically, ontological concerns) indigenous to Quechua-speaking regions of the Andes.

In the subsection Anthropology's Potential Contribution to Mathematics, Urton directly addresses the troubling question of how ethnomathematics can contribute to mathematics. Clearly ethnomathematics can contribute to anthropology's goal of studying the diversity of human culture, but Urton also pursues the converse: Mathematicians ought to take more than a casual look at the anthropology of their field.

If ethnomathematics were to contribute to mathematics, one would generally expect the "contribution" to be some new relation previously unknown in the West. Urton suggests that there are other ways ethnomathematics can be of service. If mathematical philosophies vary across cultures, can an anthropological study of how these philosophies integrate mathematics into a cultural context help mathematicians clarify the philosophical grounding of their field? It is a very good question.

While a phrase such as "mathematics is a cultural construct" may not raise an eyebrow amongst our colleagues in the humanities, it does directly contravene mathematicians' inherent Platonism. As Urton states (p. 17), "the view that mathematical truths do not—*cannot*—differ cross-culturally is a central tenet of

the Platonist vision of the reality of numbers, shapes, sets, etc." While the neo-Platonic viewpoint of most mathematicians has come under attack in the philosophy of mathematics, perhaps an anthropological approach will have a greater impact on how we do mathematics. Our attachment to Platonism is almost surely more cultural than philosophical.

Urton explores the meaning of numbers and arithmetical operations in Quechua-speaking communities. Instead of taking as given the universality of numbers and counting, he looks at the origin and culturally specific significance of numbers. This ontology reveals the social nature of numbers, counting, and sets. Through examining ordinal sequences such as ears of corn, the tines of a pitchfork, and most importantly, a mother and her offspring, Urton concludes (p. 95) that "it is only by coupling the number series with the organizing forces of social relations that ordinal sequences emerge as meaningful, rational expressions of the relations and forces organizing and uniting the members of a group." This discovery of relatedness as the driving force of numbers and sets is one of the most important points in the text.

This leads to Urton's explication of the creation and maintenance of balance and harmony as the underlying rationale for arithmetical operations. He terms this *rectification.*

> Simply stated, rectification is grounded in the idealistic, philosophical, and cosmological notion that all things in the world—from material objects to habits, attitudes, and emotions—ought to be in a state of balance, or equilibrium.... In the contexts that we are interested in, the major forms of rectification include addition, subtraction, multiplication, and division. While each of these concepts and practices has a variety of labels in different mathematical contexts, collectively they are grouped together as corrective actions aimed—each in a slightly different way from the others—at achieving rectification in a circumstance of imbalance, disequilibrium, and disharmony. [pp. 145–146]

More examples, such as weaving and the cultural meaning of odd and even numbers, further illustrate this idea. This insight accords well with what we know about Andean society, particularly the paradigm that balance is created out of the melding of opposites.

In the penultimate chapter, Urton turns his eye toward historical and archaeological sources. He makes a convincing case that one can see the "arithmetic of rectification" and the association between numbers and the model of a mother with her age-graded offspring (or at the very least, the central role of sets of five) in ancient Andean civilizations. He rightly views the conquest of the Inka by the Spanish as an important test case for fairly radical ideas of Bishop who argues that mathematics was an important tool in the imposition of Western culture [3]. Urton presents convincing evidence that this was indeed the case for the Spanish conquest.

> ...(M)uch of what was initially imposed and subsequently became embedded as elements of everyday practices of colonial administration in the Americas rode in on the back of a new political arithmetic that gave coherence and authority to the Spanish colonial adventure. [p. 196]

It is clear from studying khipus that the Inka used a base ten number system. So it is not the number system itself that was imposed by the Spanish, rather it was the symbols used by the Spanish, and more importantly, the Spanish philosophy of arithmetic.

A striking piece of evidence of this imperialism can be found in a collection of drawings of Inka kings dressed in tunics decorated with Hindu-Arabic numerals.

This clearly indicates not only Spanish influence, since such designs did not exist before the invasion, but also the importance of these symbols. "After, perhaps, the God of Christianity, numbers—that is, the symbols that were used by Spanish administrators for keeping census and tribute records—represented one of the most powerful instruments of colonial rule in the Andes" (p. 205). Urton concludes by arguing that one of the most important, and yet quite subtle, conflicts that occurred was between Andean and Spanish conceptions of arithmetic operations.

3. Critique. *The Social Life of Numbers* is an important contribution to anthropology, mathematics, and ethnomathematics. By cracking open the very definition of ethnomathematics, Urton has clarified muddled issues and directly addressed the vexing issue of why either of the fields from which ethnomathematics is mixed should care about results of the mixture. This case study in the anthropology of number philosophy and ethnoarithmetic does not lose sight of the anthropological context of the mathematics. It is also an important addition to the ethnography of the Andes.

Regrettably, we can't claim this is a flawless text. As Urton confesses, *The Social Life of Numbers* is weak on the mathematics. His arguments are primarily concerned with arithmetic, certainly an important and often neglected aspect of mathematics, but certainly not the only mathematics developed in the Andes. Other accessible examples can be found in the geometry of textiles and architecture. If one is to look for ontological foundations for arithmetic within a culture, it is reasonable to believe that such foundations would be equally interesting for other aspects of mathematics. Urton anticipates this criticism:

> ...(A)ny mathematically sophisticated reader who has stayed with me to this point must be asking: "So, where's the mathematics?" Just so that that reader will not be under the impression that I think that there *is* anything mathematically complex or fancy about the foregoing, I want to state clearly that I do not think that. I leave it to those who are truly mathematically inclined, *and who have a firm grasp of the Quechua language and culture as experienced in the Andes*, to expand on the information and issues raised in this study... in order to illuminate and advance our knowledge about what I firmly believe to be a true depth and complexity in Quechua numerical ideas and arithmetic and mathematical practice. [p. 214, emphasis in original]

A related criticism is that this book is only the first half of a larger project. "I must admit that I did not take up this study from a profound, single-minded interest in numbers and mathematics" (p. 1). Urton is ultimately interested in a study of khipus as proto-writing; this book was in some sense forced upon him because the field of ethnomathematics had not prepared the ground for the latter study. We suspect that Urton's insights into khipus will be of more interest to a general reader and will provide further evidence for the importance of understanding Andean philosophies of arithmetic.

Mathematicians will be happy that Urton's writing is never overburdened with jargon, and that he provides good general insights and commentaries throughout the text. For example, after describing the use of the fingers as a counting tool in the Quechua (for example) base ten numerical system, he cautions: "But we should not allow the fact of the ubiquity of the relationship between fingers and numbers to lull us into seeing this as a trivial relationship, for we will find in these data explicit links among numbers, the classification and organization of the fingers, and ideas about human reproduction, age, gender, and kinship relations" (p. 73). Mathematicians will also be impressed by how well Urton has absorbed the history and philosophy of mathematics.

> To the degree that the introduction into European mathematics of the Hindu-Arabic numeral for zero precipitated a philosophical and theological crisis, at the same time that it allowed (if not sparked) tremendous advances in abstract mathematics, it appears that the Quechua do not today, nor did they in the past, present themselves with the conditions for such a crisis by *naming* the empty set with a cardinal number. We are no doubt also confronted here with one of the implications of the absence of a system of written numerals in the Andes—that is, of *making* a mark to indicate nothingness in a system of writing signs on a two-dimensional surface, as opposed to *not making* any knots in a piece of string to indicate the same (absence of) value. [p. 50]

On the other hand, the book's organization is weak. For mathematicians used to the orderly development of ideas, this can be quite annoying. For example, after discussing the complicated number memory necessary for being a *Mama*—a master weaver—Urton relates how weavers receive their pay and immediately hand it to someone else to count. While the incredible thread manipulations involved in weaving prove that Mamas can count, they cannot count *money* (p. 136). This observation relates to the theme of what is countable and what is not, a main part of the book to this point, but also to the historical subjugation of the Andean region after the Spanish invasion. This historical dimension, which involved the introduction of money, and thus the imposition of "a new set of values that lay behind the conception of numbers and the manipulation of those numbers in arithmetic and mathematical operations" (p. 196), is addressed much later.

Of course, anytime one is sufficiently bold as to write for two different audiences, there are some criticisms on the proportions used in the blending. On the one hand, Urton's mode of argument may be a problem for a mathematician. Since the field does not lend itself to airtight proofs, Urton relies on the standard anthropological technique of illustrating his concepts in various contexts. For example, the numerous illustrations of "mother as the origin of numbers" may tire mathematicians but are sure to satisfy anthropologists. On the other hand, anthropologists are not used to giving arithmetic this much thought.

In total, this book is an important contribution to ethnomathematics, both in the research presented and in Urton's careful analysis of the field. We look forward to his study of khipus and further insights about the indigenous mathematics of the Andes.

REFERENCES

1. Marcia Ascher, *Ethnomathematics: A Multicultural View of Mathematical Ideas*, Brooks/Cole, 1991.
2. Marcia Ascher and Robert Ascher, *Code of the Quipu: A Study in Media, Mathematics and Culture*, University of Michigan Press, 1981; reprinted as *Mathematics of the Incas: Code of the Quipu*, Dover, 1997.
3. A. Bishop, Western Mathematics: The Secret Weapon of Cultural Imperialism, *Race and Class* **32** (1990) 51–65.
4. Gary Urton, *At the Crossroads of the Earth and the Sky: An Andean Cosmology*, University of Texas Press, 1981.

Lafayette College, Easton, PA 18042
meierj@lafayette.edu

Department of Anthropology, Cornell University, Ithaca, NY 14853
tat2@cornell.edu