continus, *Tôhoku Mat. J.* 13 (1918) 300–303. Klee also noted that circular disks are *smooth* (i.e., possess a continuously differentiable parameterization) as well as rotund. For more on smooth tilings, see V. Klee, E. Maluta, and C. Zanco, Tiling with smooth and rotund tiles, *Fund. Math.* 126 (1986) 269–290; V. Klee and C. Tricot, Locally countable plump tilings are flat, *Math. Ann.* 277 (1987) 315–325; and P. M. Gruber, How well can space be packed with smooth bodies? Measure theoretic results, *J. London Math. Soc.* (2) 52 (1995) 1–14.

D. G. Larman, A note on the Besicovich dimension of the closest packing of sphere in R_n, *Proc. Cambridge Philos. Soc.* 62 (1966) 193–195 shows that, in the case of packing of circular disks in the plane, the uncovered set has Hausdorff dimension at least 1.03.

Solved also by G. E. Bredon, P. Budney, J. D. Clemens, J. Cobb, R. Holzsager, A. A. Jagers (The Netherlands), V. Klee, J. H. Lindsey II, O. P. Lossers (The Netherlands), R. Martin (Germany), L. E. Mattics, M. Misiurewicz, I. Namioka, O. Nanyes, C. G. Petalas & T. P. Vidalis (Greece), C. Popescu (Belgium), A. W. Schurle, J. H. Shapiro & T. L. McCoy, A. A. Tarabay & R. Barbara (Lebanon), and the Anchorage Math Solutions Group.

Random Perfect Matchings

10587 [1997, 361]. *Proposed by Joaquín Gómez Rey, Madrid, Spain.* Let K_{2n} be the complete graph on $2n$ vertices. Let P_n be the probability that two random perfect matchings of K_{2n} are disjoint. What is $\lim_{n\to\infty} P_n$?

Solution by José Heber Nieto, Universidad del Zulia, Maracaibo, Venezuela. The limit is $e^{-1/2} \approx 0.60653$. The number of perfect matchings of K_{2n} is $M_n = (2n)!/(2^n n!)$. Given a perfect matching G of K_{2n} and a set J of j edges of G, there are M_{n-j} perfect matchings of K_{2n} containing J. Therefore, the inclusion-exclusion principle yields $\sum_{j=0}^{n}(-1)^j\binom{n}{j}M_{n-j}$ as the number of perfect matchings of K_{2n} disjoint from G. Thus

$$P_n = \sum_{j=0}^{n}(-1)^j\binom{n}{j}\frac{M_{n-j}}{M_n}.$$

Now $\lim_{n\to\infty} P_n$ can be computed by applying Lebesgue's dominated convergence theorem. Let $X = \{0, 1, 2, \ldots\}$, and define a measure μ on X by $\mu(\{j\}) = 1/j!$. Let $f_n\colon X \to \mathbb{R}$ be defined by

$$f_n(j) = \frac{(-1)^j n! M_{n-j}}{(n-j)! M_n} = (-1)^j \prod_{i=0}^{j-1} \frac{n-i}{2n-2i-1}.$$

Then $\lim_{n\to\infty} f_n(j) = (-1/2)^j$. Furthermore, $|f_n(j)| \le 1$, and the constant function 1 is integrable, since $\int_X 1\, d\mu = \sum_{j=0}^{\infty} 1/j! = e$. Therefore,

$$\lim_{n\to\infty} P_n = \lim \int_X f_n\, d\mu = \int_X \lim f_n\, d\mu = \sum_{j=0}^{\infty} \frac{(-1/2)^j}{j!} = e^{-1/2}.$$

Solved also by R. J. Chapman (U. K.), R. DiSario, J. Grossman, J. Labelle, D. Tenny, NCCU Problems Group, and the proposer.

Characterizations of the Medial Triangle

10588 [1997, 361]. *Proposed by Marcin Mazur, The University of Chicago, Chicago, IL.* Let $A_1A_2A_3$ be a triangle. For $i = 1, 2, 3$, let B_i be a point on side $A_{i+1}A_{i+2}$, where subscripts are taken modulo 3.
(**a**) Show that $|A_iB_{i+1}| + |B_iB_{i+1}| = |A_iB_{i+2}| + |B_iB_{i+2}|$ for $i = 1, 2, 3$ if and only if B_i is the midpoint of $A_{i+1}A_{i+2}$ for $i = 1, 2, 3$.
(**b**) Show that $|A_iB_{i+1}| + |A_iB_{i+2}| = |B_iB_{i+1}| + |B_iB_{i+2}|$ for $i = 1, 2, 3$ if and only if B_i is the midpoint of $A_{i+1}A_{i+2}$ for $i = 1, 2, 3$.