# What Makes a Great Mathematics Teacher? The Case of Augustus De Morgan

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**1. INTRODUCTION.** It is often said that no one forgets a good teacher. Whether this statement is true or not, almost everyone can recall at least one teacher who influenced some aspect of his or her future study or career. But there are occasional examples of *great* mathematics teachers who instill a remarkable number of their students with a love and enthusiasm for the subject, which has a lasting and profound effect on them, even if they never become practising mathematicians.

The nineteenth century British mathematician Augustus De Morgan (1806–1871) was one such teacher. Although his name is well known to any student of set theory, his chief mathematical legacy arose from his novel research in logic. This research created the first logic of relations, and promoted a symbolic approach to the subject, in which regard he greatly encouraged the work of his friend and contemporary George Boole. [**34**] De Morgan was also interested in algebra [**26**], and his attempts to extend the geometrical representation of complex numbers influenced the discovery of quaternions by his friend William Rowan Hamilton. Mathematical analysis, a subject very much neglected in early nineteenth-century Britain, also occupied much of De Morgan's attention, and he produced notable work on convergence of series. [**20**, pp. 148–9] He also published many research papers on various aspects of the history of mathematics, about which he was a rare authority at the time. [**29**], [**31**]

By the end of his career in the mid-1860s, De Morgan was one of the most influential and highly-regarded mathematicians in Britain, outliving Hamilton and Boole by several years and out-ranking the likes of Cayley and Sylvester (themselves far more original mathematicians) by virtue of his age. But how was this reputation achieved? Why was he so highly regarded?

One obvious reason is curiously often the most overlooked. For virtually his entire career, De Morgan was professor of mathematics at University College London, a radically innovative establishment, which, at its foundation in 1826, was the first university-level institution to be established in England since Oxford and Cambridge in the Middle Ages. There, he single-handedly delivered courses on mathematics to a generation of undergraduates for a third of a century. Because he was in charge of mathematical tuition at the leading higher educational institutional in his nation's capital, he was a formative influence on numerous mathematicians, scientists, and other prominent intellectual figures of the Victorian period.

Unfortunately, there is a pronounced absence of published material pertaining to De Morgan's work as a teacher. We know that he wrote a series of very successful textbooks, and these are useful to some extent; but they shed little light on what he actually taught in his lecture room.

However, there is a major unpublished source of information about De Morgan's teaching. This source was described in the *Encylopædia Britannica* as "a large mass of mathematical tracts which he prepared for the use of his students, treating all parts of mathematical science, and embodying some of the matter of his lectures". [**24**, p. 9] They are preserved in the University of London Library in the form of more than 320 notebooks containing the majority of De Morgan's course material in his own handwriting. The contents of these manuscripts give us a considerable insight into the material covered by mathematics students at the most progressive educational establishment in nineteenth-century England.

In this article, we first investigate the content of De Morgan's mathematical course and examine his teaching methodology; we then consider evidence from his students about how this material came across. Finally, by comparing his teaching style with that of other well known nineteenth-century mathematicians, we highlight some of the ingredients that go into making a great mathematics teacher, and show that these criteria are satisfied in the case of Augustus De Morgan.

**2. DE MORGAN'S PROFESSORIAL CAREER.** Following his birth in India in June 1806, De Morgan spent the majority of his formative years in southwest England, where he received an adequate classical education. In 1823, aged only sixteen, he entered Trinity College, Cambridge, where his mathematical talents were nurtured by his tutors, who included the prominent English mathematician George Peacock. Peacock, Charles Babbage, and John Herschel had founded the short-lived but influential Analytical Society in 1812, [**17**] which helped secure the adoption of Lagrange's algebraic methods of calculus in the Cambridge syllabus, replacing the Newtonian fluxional system, which had been entrenched in Britain for well over a century. [**21**]

De Morgan's Cambridge years coincided with the foundation of a university in London, the only capital city in Europe without such an institution at this time. Indeed, up to this period, Oxford and Cambridge were the only places in England to offer university qualifications, and since they were fully open only to members of the Church of England, denominations such as Catholics or Jews were effectively barred from university degrees. So too were the urban middle classes who, while not poor, were nevertheless financially incapable of supporting their offspring through courses of study away from home. The establishment of University College London (originally titled "London University") in 1826 was a radical solution to this problem, made all the more so by its explicit secular character and progressive programme of studies.

Equally daring was its choice of founding professor of mathematics. De Morgan was appointed to the position in February 1828, scarcely a year after his graduation from Cambridge, aged only twenty-one. [**30**] But not all went smoothly. After opening for lectures the following October, the new university was plagued by financial troubles and petty personal disputes. After a professorial colleague was dismissed in 1831, De Morgan immediately resigned on principle. Five years later, however, he was invited to return after the premature death of his replacement. He was to remain for a further thirty years.

His final departure was occasioned by the college's (non-)adherence to its policy of religious equality; indeed, he remains the only professor in the history of University College to have resigned *twice* on matters of principle. For De Morgan, the college's refusal to appoint a candidate to the vacant chair of philosophy on the grounds of his being a controversial Unitarian minister was a betrayal of its founding principles. He resigned his professorship on 10 November 1866, giving his last lecture in the summer of 1867. He never returned, refusing even a request from his former students to sit for a bust to be placed in the college library,

**Figure 1.** Augustus De Morgan pictured in 1866.

explaining that, as far as he was concerned, "our old college no longer exists". [**15**, p. 360]

**3. THE COURSE.** During the period of De Morgan's professorship, his mathematics course formed a central component of the college's curriculum. It was intended to constitute part of the students' first two years, during which time they would also study such subjects as Latin, Greek, and natural philosophy (i.e., physics). Since school education was not yet compulsory and the school leaving age was, on average, around fourteen, the students at University College in De Morgan's day were substantially younger than they are now. In general, they ranged from 15 to 18, usually leaving the college to begin vocational training, employment or, in the case of the exceptional students, more advanced study at Oxford (if they were classically inclined) or Cambridge (if mathematically).

In mathematics the students were divided into classes corresponding to the first and second years of undergraduate study, with each class being further divided into lower and higher divisions. The first year (or junior) course was designed to contain "what is most essential for those who are intended for practical professions, such as Civil Engineers, &c.", [**33**, p. 42] while the senior class was intended for those capable of tackling more advanced topics. However, due to the range of materials available, the course was "confined principally to those parts of the subject which are necessary for the study of Natural Philosophy". [**33**, p. 42] The following outline indicates that this definition was a very broad one:

JUNIOR CLASS, LOWER DIVISION:

i) Arithmetic and the arithmetical theory of proportion
ii) Euclid, Books 1–4
iii) 6th Book of Euclid
iv) First book of Solid Geometry in Euclid
v) Algebra, arithmetically considered, up to equations of the first degree.

JUNIOR CLASS, HIGHER DIVISION:

i) Euclid, Books 5 and 6
ii) First book of Solid Geometry in Euclid
iii) A review of the principles and operations of arithmetic
iv) Algebra (including the nature and use of logarithms)
v) Plane trigonometry (including mensuration).

SENIOR CLASS, LOWER DIVISION:

i) Spherical trigonometry
ii) Conic sections
iii) Applications of algebra to geometry
iv) Higher parts of algebra
v) Differential and integral calculus.

SENIOR CLASS, HIGHER DIVISION:

> Extension of subjects in the Lower Senior Class. "Subjects which all must learn who wish to become analysts, whether for Engineering or any other pursuit." [**39**, pp. 6–7], [**40**, pp. 7–8], [**41**, pp. 19, 35]

However, De Morgan was at pains to point out that this plan should not be regarded as a definitive declaration of intentions. As he said in his inaugural lecture of 1828, "I shall not consider myself bound to carry the class through the whole of what is contained in it if it shall appear that their interest will be more effectually consulted by my confining myself to the more prominent parts of it." [**6**, f. 45] As far as he was concerned, it was quality of knowledge that mattered more than quantity.

In order to achieve this, De Morgan highlighted the two principal methods whereby his students could acquire mathematical knowledge: "The first is by diligent study in the retirement of the closet; the second, by haunting the benches of the lecture-room, and picking up what may chance to fall." [**13**, p. 14] Lectures alone, he maintained, were insufficient to bring the student to the appropriate level of understanding. Moreover, student lecture notes, while important, were no substitute for a full treatise; indeed, he compared the information obtained from listening to a lecture to the comprehension achieved from reading a book at speed.

De Morgan thus regarded the role of lectures as merely providing students with assistance in difficulty and guidance on relevant reading.

In order to enlarge this oral instruction, he prepared a vast quantity of handwritten tracts on all aspects of his course, which were then placed in the University College library for his students to refer to. They were designed to supplement not only the lecture material, but also the wide reading that De Morgan expected his students to undertake.[1] The surviving notebooks (written between 1843 and 1866) are well over three hundred in number, each featuring De Morgan's legible handwriting. De Morgan also had an idiosyncratic habit of pasting in printed material that he considered particularly relevant; these insertions often consisted of an appropriate paper, usually by himself. These tracts reveal that he taught much more than was indicated in the published syllabi and exam papers. What now follows is a survey of the material contained in the existing tracts to give some idea of what a student of mathematics at University College could have expected to study under De Morgan 150 years ago.

TABLE 1. De Morgan's surviving tracts

| CLASS | NUMBER OF NOTEBOOKS |
|---|---|
| Lower Junior | 10 |
| Higher Junior | 70 |
| Lower Senior | 109 |
| Higher Senior | 138 |

**3.1 The Lower Junior Class.** Of the 327 surviving notebooks, only ten contain material designed for the use of students in De Morgan's lower junior class. Not only was the subject matter far less extensive than in succeeding classes, but also, in the mode of tuition adopted by the Professor for this class, oral lectures occupied a very small place, the majority of the time being devoted to giving written exercises and answering students' questions. Furthermore, for much of the relevant material at this introductory level, existing textbooks were perfectly adequate, such as his own *Elements of Arithmetic* (1830) and *Elements of Algebra* (1835), as well as numerous editions of Euclid's *Elements*.

As well as giving alternative presentations of material that could be found in the students' books, the tracts dealt in some considerable depth with matters with which most textbooks (even De Morgan's) did not concern themselves. One of the most fascinating tracts (#110–comprising three notebooks) was designed to be read before the student opened the first page of Euclid. Entitled "Notions preliminary to Geometry", it illustrates De Morgan's desire for his students to be acquainted from the very start with the philosophical and epistemological issues relating to the subject. More significantly, it demonstrates his belief that a thorough grounding in logical notions and processes was essential for the students'

[1]In the absence of photocopiers, many would copy the contents of these tracts wholesale, as evinced by a large volume in University College Library containing a student's transcription of 33 of them: University College London Archives, MS.ADD.6, "Mathematical Tracts by Professor De Morgan, copied from the original Manuscripts in the Library of University College London by John Power Hicks 1849–1851".

understanding of geometrical arguments.[2] As he said:

> The principles on which geometrical propositions are established belong to the totally distinct and equally simple science of logic; and since geometry without logic would be absurd, it is desirable that the principles of the latter science should be studied with precision previously to employing them upon the former. [**9**, pp. 238–9]

De Morgan was wont to complain about the lack of contact between the disciplines of mathematics and logic: "Geometers have seldom been very *formal* logicians; and their patent of exemption was signed by Euclid." [**7**, p. 435] One of the principal sources of confusion when initiating students into the study of geometrical demonstrations was the distinction between a proposition and its converse. So, for example, the statement that 'all equilateral triangles are equiangular' was often taken to imply that 'all equiangular triangles are equilateral'. "These errors," said De Morgan, "should be guarded against beforehand, by exercising the pupil in simple deductions, such as are to be found in every syllogism, taking care that all terms used have reference to objects with which they are familiar. It should be illustrated to them that the truth of an argument depends on two distinct considerations, the truth of the premises, and the manner in which the conclusion is deduced from them." [**8**, pp. 272–3]

The next step before coming to actual geometrical demonstrations was to introduce his students to the concept of a proof. "A proposition", he wrote, "may be proved in two ways: Directly, by showing that it *is true*. Indirectly, by showing that the contradiction *is false*." [**46**, f. 8] Since the latter was conceptually the most difficult for the beginner, this tract was principally concerned with this mode of procedure, which "forces an absurd result out of the contradiction, and therefore forces the denial of the contradiction, or the affirmation of the proposition". [**46**, f. 9] Proof by contradiction, was, in De Morgan's opinion, rendered far more intelligible by the early study of logic, and mastery of the technique was a vital skill to acquire before tackling Euclid.

**3.2 The Higher Junior Class.** On entering De Morgan's higher junior class, his students were expected to be fully familiar with Euclidean deductive reasoning up to the fourth book of the *Elements*. But the fifth book, introducing the complex ideas of ratios and proportion, often caused the most problems. De Morgan noted that, owing to its highly convoluted presentation, "it has been customary for mathematical students among us to read the Fifth Book of Euclid; frequently without understanding it". [**10**, p. iii] For this reason, he substituted arithmetical notions of proportion instead of the traditional geometrical ones, a simplification that helped to make the subject far more intelligible to his junior students than if he had left them to study it unaided.

Despite his almost instinctive mathematical abilities, De Morgan was fully aware of the need to eliminate as many barriers as possible to the beginner's

[2] His writings on geometrical education provide the first published evidence of De Morgan's interest in logic, although at this point it was utilised purely as a pedagogic tool. He later elaborated his ideas in a short book for his students entitled *First Notions of Logic(preparatory to the study of geometry)*, published in 1839. This was later incorporated as the first chapter of his *Formal Logic* in 1847, by which time his interest in logic had transcended its utility merely as an aid to geometry, and was manifesting itself in the publication of research papers concentrating more on the intrinsic nature of the subject itself.

understanding of unfamiliar mathematical topics. A further aid to the students' geometric cognition was his rejection of perspective drawings in favour of three-dimensional models. As he explained to the audience of his introductory lecture:

> Considerable obstacles generally present themselves to the beginner, in studying the elements of Solid Geometry, from the practice which has hitherto uniformly prevailed in this country, of never submitting to the eye of the student, the figures on whose properties he is reasoning, but of drawing perspective representations of them upon a plane. ... I hope that I shall never be obliged to have recourse to a perspective drawing of any figure whose parts are not in the same plane. [**6**. ff. 50–1]

Having dwelt extensively on Euclidean-related matters, the higher junior class would have then turned their attention to a recapitulation of the rules and procedures of arithmetic before being initiated into abstract algebra. The lower junior class would have already practised linear equations, but these were treated more as features of a universal arithmetic than a general algebra. Now the class was ready to learn the distinction between the two: "In *Arithmetic* every symbol of magnitude ... represents a *number*, and nothing but a number ... *Algebra* employs the *symbols*, the *language* and the *rules* of arithmetic. But ... no letter *a* has its full meaning described until we are told both its *value* and its *sign*." [**50**, ff. 1, 4]

De Morgan's algebra was a long way from the highly-evolved structural algebra of today. Yet it was to play a significant role in the development of abstract algebra, building on earlier work by his friend and former tutor, George Peacock. [**27**] However, whereas in Peacock's algebra the symbols were generally understood to represent numbers or operations, De Morgan would deliberately keep them abstract. "Thus," he wrote "*addition* is to be, for the present, a sound void of sense. It is a mode of combination represented by $+$; when $+$ receives its meaning, so also will the word *addition*." [**14**, p. 101] It was an area to which he was to devote much research, although, since it was too advanced for his junior classes, he deferred its discussion until his pupils had reached the senior level.

Now that they were familiar with algebraic terminology and ideas, the students were ready to progress to the solution of quadratic, cubic, and higher order equations. It was at this stage that students would have first come across the binomial theorem, leading immediately to the study of series, both finite and infinite. This turn led them to convergent and divergent series, resulting in their introduction to one of the most crucial mathematical concepts, recently reinstated in analysis: the limit.

In Cambridge during the second decade of the nineteenth century, the Analytical Society had been instrumental in replacing Newton's fluxional calculus with the algebraic method of Lagrange, [**16**] thus rejecting a system based (albeit very dubiously) on the notion of a limit. This concept had been reformulated by Cauchy in the early 1820s, but was not immediately accepted in France or elsewhere. De Morgan's *Elements of Algebra* was the first English work to contain a definition of the continuity of a mathematical function using limits. His subsequent treatise on *The Differential and Integral Calculus* (1842), the most comprehensive English work on the subject for over a generation, was entirely grounded on the concept of limits.

But, he said, it was meaningless to ask what a limit was since "'What *is* the limit' is the same question as 'What is the exact expression for that which does not admit of exact expression'." [**51**, f. 2] What then was the point of introducing limits in the first place? "We introduce them because we cannot do without them, being all the time perfectly willing to do without them if any one will show us how." [**51**, f. 3] Yet despite these reservations, De Morgan was nonetheless firmly convinced of their epistemological soundness and, as one of the first in Britain to publish and teach mathematics using limits, his work helped establish this concept as the basis of modern mathematical analysis.

The use of limits also formed the basis of his introduction to logarithms as well as his teaching of trigonometrical analysis, where he employed limits to analyse the various properties of expressions such as $\sin x/x$ and $(1 - \cos x)/x^2$. His tracts on trigonometry for this class began with the usual problems of plane trigonometry such as finding values of angles and sides given certain information, progressing to questions involving multiple angles and inverse functions.

These were the topics studied by De Morgan's higher junior class, as specified by the published syllabus. However, the existing tracts reveal that students were also given instruction in other areas, the first being interest and annuities. Interest, both simple and compound, was covered by De Morgan in his *Elements of Arithmetic* [**12**, pp. 150–60], but his tracts extended this treatment to include the rudiments of acturial mathematics, introducing the students to the complex calculation of annuities based on mortality tables. Permutations and combinations were also covered, leading directly to elementary problems in probability theory. Interestingly, for this subject, De Morgan relied on a popular algebra primer written by a former student, Isaac Todhunter, [**35**] who by the 1850s had become a successful textbook author.

Additional geometrical topics were also begun in this class, albeit at a fairly introductory level, the first being algebraic geometry. At this stage, problems set primarily involved either tracing curves or finding the intersection of two lines by solving simultaneous equations. Their initiation into projective geometry reached a slightly higher stage, proceeding as far as Pascal's and Brianchon's theorems. They and much more besides, would be repeated in full in his lower senior class.

**3.3 The Lower Senior Class.** By the time they entered the lower division of De Morgan's senior class, the majority of his students would have completed at least one year of mathematical study. That year would have provided the students with a study programme of considerable intensity. However, this pales in comparison with the level of material covered during the following year, as illustrated by the number of relevant notebooks still in existence: in comparison to the 80 such documents relating to the junior classes, there are no fewer than 247 notebooks concerning material covered by the two divisions of the senior class.

According to published sources at least, the lower senior course began with an introduction to spherical trigonometry. Again, De Morgan's tracts on this topic supplemented both his lectures and a book on the subject—in this case, a small textbook he had written in 1834. His tracts included further explanation and examples of various points, including statements and proofs of the standard formulae for spherical triangles, and problems such as finding areas, inscribing and circumscribing circles, and supplemental triangles.

Compared to just five items on spherical trigonometry, the number of individual notebooks containing material relating to conic sections is well over twenty; moreover, De Morgan's treatment often varies from tract to tract. To begin with, the conics would have been defined purely geometrically. De Morgan would then introduce the closely-related topic of projective geometry, although, judging from the higher junior tracts, his students would already have received some introduction to the subject by this time. De Morgan's justification was that "the method of projections establishes the more general and more difficult properties of the conic sections with greater ease than the ordinary methods". [**49**, f. 1] His projective geometry largely consisted of an analysis of various properties and peculiarities of projective figures, such as colinearity and involution, with all demonstrations relying on neatly drawn diagrams and Euclidean-style proofs.

Once the class had reached a certain level of proficiency in projective geometry, De Morgan would employ algebraic geometry to give alternative demonstrations of similar—and, in some cases, the same—results. Having already defined straight lines and circles algebraically in the higher junior class, he began this level with a discussion of the general second degree equation $ay^2 + bxy + cx^2 + dy + ex + f = 0$, and considered the curves generated by its different variations. In such a way, he was able to give yet another introduction to the conic sections, extending the treatment to include algebraic treatments of results originally proved using projective geometry.[3]

At this stage the class would have reached a fairly advanced level of algebra; indeed, by this time, their algebraic exercises included multiplying and dividing polynomials, and solving cubics using Cardano's and Ferrari's methods. Among other algorithms taught by De Morgan in the theory of equations was Horner's method,[4] a procedure for approximating roots of equations with no exact solution. He later described his motivation for introducing this method, and the results his students obtained after applying it to the equation $x^3 - 2x = 5$:

> In 1831, Fourier's posthumous work on equations [**18**, pp. 209–17] showed 33 figures of solution, got with enormous labour. Thinking this is a good opportunity to illustrate the superiority of the method of W. G. Horner, not yet known in France, and not much known in England, I proposed to one of my classes, in 1841, to beat Fourier on this point, as a Christmas exercise. I received several answers, agreeing with each other, to 50 places of decimals. In 1848, I repeated the proposal, requesting that 50 places might be exceeded: I obtained answers of 75, 65, 63, 58, 57, and 52 places. [**5**, p. 292]

It is here that we begin to detect a new feature in De Morgan's teaching: a desire to acquaint the more advanced pupils with recent mathematical developments. Tract #25, for example, contains material that, while ostensibly concerned with the theory of equations, would nowadays be considered as part of complex analysis, being straight from the pages of recent works by Cauchy and Argand. It also includes several new proofs of the existence of a root of every equation, including a paper by De Morgan on the subject, pasted in the back as usual, although his advice to students was: "Read Argand first, and then examine Cauchy's". [**43**, f. 17]

The lower seniors would also have been presented with many of the latest results in modern analysis, especially in their study of infinite series. But it was only after they had been given a thorough grounding in algebraic and analytic operations, especially regarding the meaning and significance of limits, that they were initiated into the subject of the differential calculus. From the tracts, we can

[3]Much of De Morgan's treatment of conic sections in his tracts on algebraic geometry was taken directly from George Salmon's *Treatise on Conic Sections*, Hodges and Smith, Dublin, 1847.

[4]Named after William George Horner (1786–1837), a school-teacher from southwest England, due to a paper he published in 1819.

be sure that a fair number of students encountered problems; a particular difficulty concerned the differential coefficient of a function—a problem still encountered by students today.

> A beginner sees $(1 + x^2)^3$, and remembering that $x^3$ gave $3x^2$, he writes down $3(1 + x^2)^2$. He ought to have written $3(1 + x^2)^2 \times 2x$. The truth is that he has correctly answered *a question*,—but not *the question which was asked.* [**45**, f. 17]

De Morgan's initial teaching of integration proceeded no further than finding areas under curves. However, there is evidence that he began elementary instruction on differential equations in this class, although this involved little more than defining basic notions such as the order of an equation, the integrating factor, and how to find general and singular solutions. Such an introduction would have been of little use to those who chose to end their mathematical studies at this point. But these final subjects were to provide a background for the detailed course of study reserved for students who proceeded to De Morgan's higher senior class.

**3.4 The Higher Senior Class.** Attending De Morgan's lectures as far as his lower senior class would have enabled the average student to pass the B.A. examination at the University of London, as well as to move on to the study of natural philosophy in the college. However, for those exceptionally capable (and keen) students who perhaps wished to try for an M.A. degree, it was advisable to enter University College's highest mathematical class. This course was obviously the most technically demanding and, although the class would never have been huge, was one to which De Morgan clearly devoted much time and attention.

This is evinced by the 138 notebooks he wrote for this class, more than for any other division of his students. This high number of tracts is explained by the fact that fewer, if any, elementary textbooks were available on the topics of his higher senior lectures. For much of this section of the course, the most useful work would have been his *Differential and Integral Calculus*, since the subject dominated the material covered. Other areas were also treated, such as further theory of equations, three-dimensional geometry, and probability theory, but their study was vastly outweighed by the amount of time devoted to calculus-related topics.

Chief among these was the study of differential equations, briefly introduced in the lower senior class. As with all of De Morgan's tracts on subjects of some complexity, those dealing with the first principles cover each aspect in careful detail. It is quite obvious from the sheer number of notebooks relating to the various types of equation (around thirty) that De Morgan was anxious that his student's should obtain as much experience and practice of solving them as possible. He even wrote an entire tract containing model solutions to questions on the subject from University of London examination papers. The chief application of differential equations in De Morgan's higher senior tracts was to the study of curves and surfaces, where the subject matter is almost entirely based on the differential geometry contained in Gauss's *Disquisitiones Generales circa Superficies Curvas* of 1827.

The class was also introduced to a second form of differentiation in order to facilitate the subsequent study of mechanics. This was the calculus of variations. Much of the material contained in the tracts is also presented in his *Differential and Integral Calculus* [**11**, pp. 446–75], such as the famous brachistochrone problem of finding "the curve of shortest descent from one curve to another, a heavy point descending upon the curve (supposed hard) by the action of gravity, with no velocity at the commencement". [**44**, f. 4] But his treatment of the subject, while thorough, was not exhaustive; for example, he directed the more advanced students to "the Memoir of Poisson on the Calculus of Variations, in the twelfth volume of the Memoirs of the Institute". [**11**, p. 454]

In addition to the study of these 'pure' mathematical subjects, De Morgan also managed to include a few items of applied mathematics. Indeed, more time was spent on mathematical applications in the higher senior class than in any other—although the overall proportion was still minute. One subject considered was probability theory, which the students had studied—in its pure form—in the higher junior class. De Morgan would now introduce them to its applications, most notably its use in error theory, a precursor of what would now be called mathematical statistics. De Morgan's teaching of this subject was also heavily influenced by the work of Gauss a few decades before. This is hardly surprising since the main topics in this area, such as the weight of observations and the method of least squares, were all introduced by Gauss. Thus once again, De Morgan can be seen to be acquainting his students with (fairly) recent work on a new and rapidly growing area of mathematical research.

Less recent—but certainly still applied—mathematics is contained in two notebooks on the subject of dynamics. Strictly speaking, this would have been taught by the professor of natural philosophy, but De Morgan's treatment was entirely mathematical, dealing purely with theoretical problems involving the derivation of equations of motion for particles travelling under certain conditions. Moreover, throughout these tracts, he is at pains to stress the distinction between the abstract mathematical notions of velocity and acceleration on the one hand, and the physical phenomena (e.g., force, pressure, and attraction) that cause them. Thus, for example:

> When, as is usual in books on mechanics, *acceleration* is much confounded with *force measured by the acceleration it produces*... —called *accelerating force*—the *centrifugal acceleration*, a law of space, gets the name of *centrifugal force*, whether there be such a force in action or not. [**52**, f. 16]

His motivation for thus trespassing on materials within the domain of mathematical physics was his belief that "the want of sufficient attention to this distinction puts some difficulties in the way of beginners in dynamics". [**52**, f. 1] In other words, he thought that if his students received an adequate notion of velocity and acceleration independently of any physical consideration of the properties of matter, they would be better equipped to understand the subject of dynamics when they came to study natural philosophy.

Having been given a thorough grounding in most areas of contemporary mathematical science, even proceeding far enough in certain subjects to have become acquainted with several aspects of recent research, the student would have several options for further study. Although the concept of a graduate research student did not exist in Britain at this time, the higher senior class would almost certainly have served as a good starting point for those aspiring for an academic career in mathematics, since it not only provided guidance for those aiming for mathematical honours, but also those trying for a London M.A. or preparing to embark on a course of study at Cambridge.

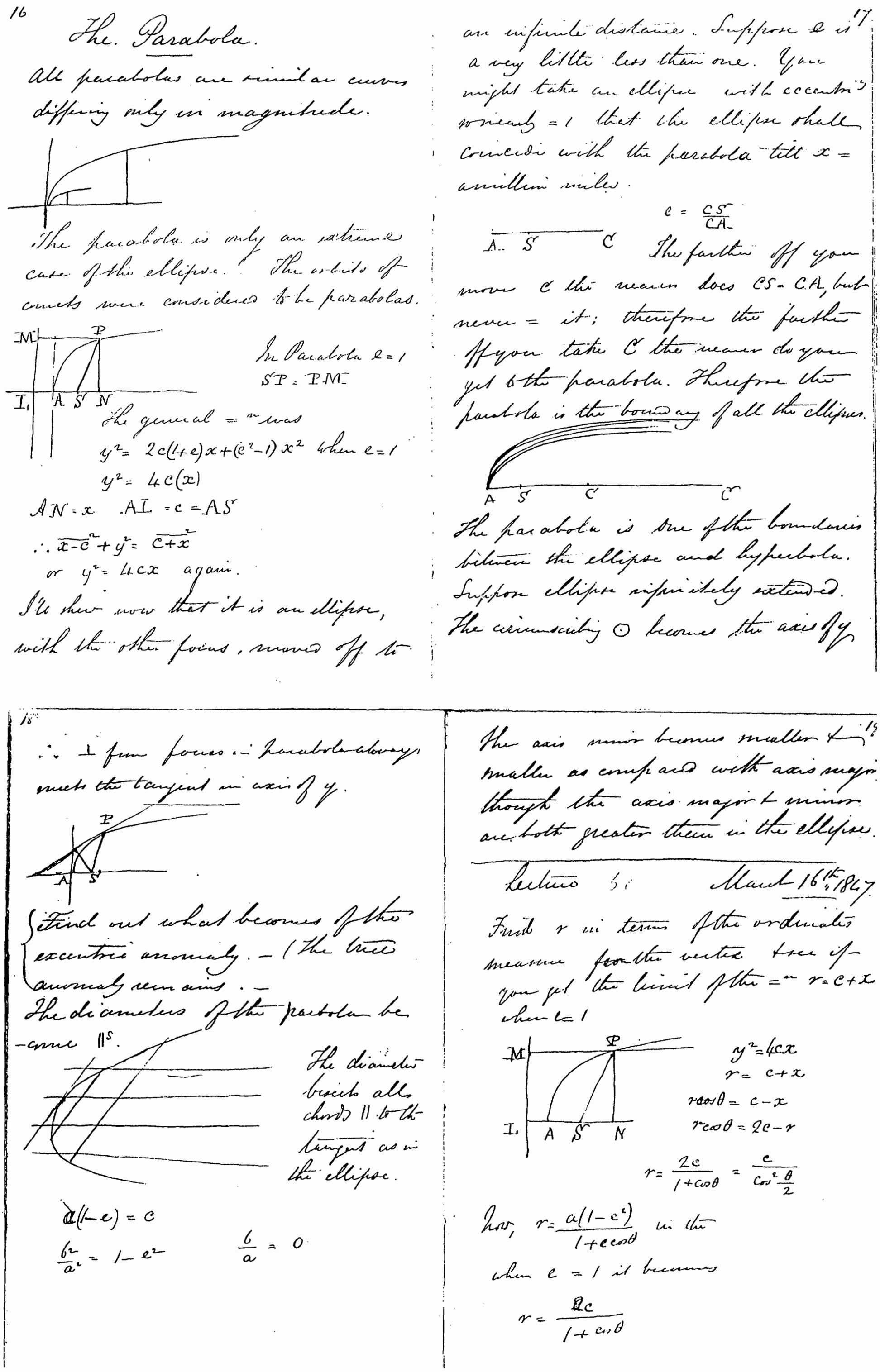
**3.5 Overview.** In fact, viewing the surviving mathematical tracts as representative of De Morgan's entire syllabus, one is impressed not just by the level to which mathematics was taught, but also by the range of topics to which the students were exposed by their professor. To be sure, there is nothing unusual in his basic course structure, whereby the subject is developed from arithmetic and Euclid through the standard branches of algebra, geometry, trigonometry, and calculus; but this is hardly an original feature. Rather, it is the additional, less prominent topics, absent from the course outlines and found only in the tracts, which give the course its variety and make it particularly distinctive. The result is a mathematical course of considerable scope and breadth.

De Morgan's course was as advanced as it was varied. Indeed, it extended almost as far as an undergraduate course could at the time, since developments in many branches (especially analysis and algebra) were transforming the subject almost as it was being taught. This is reflected in the fact that the tracts were constantly being updated. A good example is found in notebooks relating to the theory of equations, one of the most rapidly-expanding areas at this time, which yielded new subjects in the form of complex analysis and group theory. Although never taught by De Morgan, many of the results that contributed to this latter development are contained in a tract written in 1855, featuring "selections from what has been recently done in the higher parts of the theory of equations". [**48**, f. 1] This featured substantial extracts from the second edition of Serret's *Cours d'algèbre supérieure* (1854), the third edition of which, published in 1866, was to mark the first appearance of Galois theory in a textbook.

De Morgan's mathematical tracts give us far more information about the style and structure of his teaching than any printed syllabus or exam paper of the time. But ironically, these manuscripts, invaluable though they undoubtedly are, still do not tell us *exactly* what went on in his lecture room. By virtue of the fact that they were written explicitly to *supplement* the students' notes obtained from the Professor's lectures, the tracts can give us only a general impression of what the students would have been taught in person. Fortunately, however, there are three further sources of information that can bring us one step closer to understanding just what it was like to study mathematics under De Morgan.

**4. STUDENT-AUTHORED ACCOUNTS.** Two of these sources are extracts from the private writings of two eminent students, the journalist and constitutional author Walter Bagehot and the mathematical economist and logician William Stanley Jevons. However, the third is perhaps the most valuable, for two reasons. Firstly, because it was written by a student of more average ability; and secondly, because it is the student's original college notebook, in which he transcribed De Morgan's lectures as they happened. This comparatively academically undistinguished student was one John Golch Hepburn, also destined to achieve no particular eminence following his graduation. However, Hepburn's notebook provides us with a unique insight as to what the student would have experienced in De Morgan's lecture room 150 years ago.

The manuscript contains notes from 21 of De Morgan's lower senior lectures on algebraic geometry and the differential calculus, delivered between 11 March and 13 May 1847. The first begins with a study of the ellipse, considering aspects such as area and conjugate diameters. The next lecture, on 13 March, deals with Kepler's Laws, orbit-time calculations, and an introduction to parabolae. Hyperbolae and asymptotes are treated five days later. By 27 March, the emphasis was on tangents and chords to conic sections. After an absence from two or three lectures, Hepburn's notes resume on 16 April, when sections of cylinders, cones, and



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The. Parabola.

All parabolas are similar curves differing only in magnitude.

The parabola is only an extreme case of the ellipse. The orbits of comets were considered to be parabolas.

In Parabola $e = 1$

$SP = PM$

The general $=^n$ was

$y^2 = 2c(1+e)x + (e^2-1)x^2$ when $e = 1$

$y^2 = 4c(x)$

$AN = x \quad AI = c = AS$

$\therefore \overline{x-c}^2 + y^2 = \overline{c+x}^2$

or $y^2 = 4cx$ again.

I'll shew now that it is an ellipse, with the other focus, moved off to

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an infinite distance. Suppose $e$ is a very little less than one. You might take an ellipse with eccentricity $= 1$ that the ellipse shall coincide with the parabola till $x =$ a million miles.

$e = \frac{CS}{CA}$

A S C The farther off you move C the nearer does $CS = CA$, but never $=$ it; therefore the further off you take C the nearer do you get to the parabola. Therefore the parabola is the boundary of all the ellipses.

The parabola is one of the boundaries between the ellipse and hyperbola. Suppose ellipse infinitely extended. The circumscribing ⊙ becomes the axis of y

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$\therefore \perp$ from focus in parabola always meets the tangent in axis of y.

{Find out what becomes of the excentric anomaly. — (The true anomaly remains. —

The diameters of the parabola become $\|^s$.

The diameter bisects all chords $\|$ to the tangent as in the ellipse.

$a(1-e) = c$

$\frac{b^2}{a^2} = 1 - e^2 \qquad \frac{b}{a} = 0$

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The axis minor becomes smaller & smaller as compared with axis major though the axis major & minor are both greater than in the ellipse.

Lecture 5. March 16th 1847

Find $r$ in terms of the ordinates measured from the vertex & see if you get the limit of the $=^n$ $r = c + x$ when $e = 1$

$y^2 = 4cx$

$r = c + x$

$r\cos\theta = c - x$

$r\cos\theta = 2c - r$

$r = \frac{2c}{1+\cos\theta} = \frac{c}{\cos^2\frac{\theta}{2}}$

Now, $r = \frac{a(1-e^2)}{1+e\cos\theta}$ in the

when $e = 1$ it becomes

$r = \frac{2c}{1+\cos\theta}$

**Figure 2.** John Hepburn's undergraduate notes on De Morgan's Lectures from March 1847.

spheres were under discussion. Less than a week later the students were being introduced to the differential calculus.

De Morgan clearly approached the new subject at some considerable speed since, on its first day, he was teaching derivatives of fundamental expressions and the product and quotient rules, yet, by 24 April, two days later, he was differentiating $x^x$. By the end of the month, physical notions such as velocity had been introduced, with tangent/normal and max/min problems brought in on 1 May. Maclaurin's theorem was proved for convergent series in the next lecture, followed by Taylor's and Lagrange's theorems, together with related problems. By 8 May, the class had been introduced to the calculus of finite differences, the notes concluding with an introduction to the calculus of operations.

Reference to section 3.3 confirms that Hepburn's notes correspond very closely to topics dealt with in De Morgan's tracts for his lower senior class, but perhaps more remarkable is how rapidly the Professor propelled his students through the subject. His introduction to the calculus took him a little over two weeks, consisting of just seven lectures. In that time, he discussed first principles, including foundational concepts such as limits, as well as derivatives of functions, fundamental rules, and elementary applications, before moving on to some crucial results in analysis. It is little wonder that he provided tracts for his students to augment their lecture notes!

De Morgan's homework problems were numerous and far from trivial. A few examples from Hepburn's lecture notes indicate the standard of De Morgan's homework questions at the lower senior level:

> Determine area of parabola as extreme case of area of ellipse. Suppose axis major become $>$ & $>$ ; $e$ being nearer & nearer $= 1$. [**38**, f. 25]
>
> Required the [Maclaurin] developments of $\varepsilon^{ax}$, $(1 + x)^n$, $\sin x$, $\cos x$, $\tan x$, and $\varepsilon^{\cos x}$ to $8^{\text{th}}$ power at least. [**38**, f. 193]
>
> Try to give a geometrical proof of the ratio of two magnitudes wh$^{\text{h}}$ vanish is the same thing as the ratio of their diff. Coeff$^{\text{ts}}$. [**38**, f. 197]

In addition to such problems, Hepburn's notes are permeated with references to recommended reading. Perhaps the most intriguing citation, contained in the lecture on the foundations of the calculus, was "See *Leipzig Acts* 1684".[5] [**38**, f. 137] Thus it would certainly appear from this text that De Morgan's course was not for the faint-hearted, yet perhaps the only detail absent in the document is any indication of how difficult the student actually found it. For this information we are obliged to refer to our two remaining sources, which fortunately shed some considerable light on this question.

The sources are the diaries and correspondence of Walter Bagehot and Stanley Jevons, who attended De Morgan's lectures during the 1840s and 1850s, respectively. From both it would appear that mathematics under De Morgan was stimulating but never easy. Thus we find Bagehot writing in 1843: "De Morgan has been taking us through a perfect labyrinth lately; he was quite lost by the whole

[5]This refers to the 1684 edition of the journal *Acta Eruditorum Lipsienium*, which contained the first publication by Leibniz on the differential calculus.

class for one lecture, but we are, I hope, getting better, and more gleg[6] at the uptake. We have been discussing the properties of infinite series, which are very perplexing." [**1**, p. 118] And, as he approached his final exams, Bagehot's workload intensified: "I have been reading some of the Theory of Numbers, which De Morgan says is the best exercise for the head possible, and certainly is a hard stretch for my reading powers and memory." [**1**, p. 159]

Stanley Jevons experienced De Morgan's teaching a decade later than Bagehot, and at two different periods. These are recorded in detail in the diary and correspondence written by him during his college years. It is thus in his memoirs that perhaps the fullest and most candid account of experiences as a student of De Morgan can be found. These examples give an evocative description of one man's study of mathematics at University College under the tuition of Augustus De Morgan:

> In mathematics we are just beginning the theory of equations, and during the last week have got through Descartes', Fourier's, and Sturm's theorems of the limits of the roots of equations. They are the most truly difficult things we have come to, and I do not thoroughly understand them yet. [**23**, p. 29]

> ... one learns more and more to adore De Morgan as an unfathomable fund of mathematics. We were delighted the other day when, in the higher senior, he at last appeared conscious that a demonstration about differential equations, which extended through the lecture, was difficult; he promised, indeed, to repeat it. But then one is disappointed to find that the hardest thing he gives in any of his classes is still to him a trifle, and that the bounds of mathematical knowledge are yet out of sight. [**23**, p. 150]

**5. CONCLUSION: A COMPARISON OF CONTEMPORARIES.** It is not only through contemporary student accounts that we can determine the calibre of De Morgan's teaching. In later years, many of his students recalled the effect he had on them. More than half a century after experiencing his teaching in the late 1840s, the English historian Thomas Hodgkin wrote:

> Towering up intellectually above all his fellows, as I now look back upon him, rises the grand form of the mathematician, Augustus De Morgan, known, I suppose to each succeeding generation of his pupils as 'Gussy'. A stout and tall figure, a stiff rather waddling walk, a high white cravat and stick-up collars in which the square chin is buried, a full but well chiselled face, very short-sighted eyes peering forth through gold-rimmed spectacles; but above all such a superb dome-like forehead, as could only belong to one of the kings of thought: that is my remembrance of De Morgan, and I feel in looking back upon his personality that his is one of the grandest figures that I have known. [**2**, p. 80]

Hodgkin was not the only non-mathematician on whom De Morgan made a substantial impact. Reminiscing in 1921, the lawyer James Bourne Benson affirmed that "De Morgan [was] looked upon with awe" [**3**, f. 3] by the undergraduates of his day. The distinguished chemist Sir Henry Enfield Roscoe went further

[6] A Scots word (found in the poetry of Robert Burns) meaning astute, quick, keen, or alert.

still, opining that De Morgan was more than "merely a mathematician and a unique teacher; he was one of the profoundest and subtlest thinkers of the nineteenth century". [**32**, p. 25] "As a teacher of mathematics," wrote Jevons, "De Morgan was unrivalled. ... De Morgan's writings however excellent, give little idea of the perspicuity and elegance of his *viva voce* expositions, which never failed to fix the attention of all who were worthy of hearing him," [**24**, p. 8]

Jevons was later to acknowledge the profound effect of De Morgan on his intellectual development, and it is clear that the careers of many other former students were also influenced in some way by De Morgan's teaching. Francis Guthrie, the originator of the Four-Colour Conjecture, became a professor of mathematics in South Africa; E. J. Routh moved to Cambridge, becoming one of the most successful mathematical coaches in its history; and Isaac Todhunter achieved renown for a highly successful series of textbooks as well as his research into various aspects of the history of mathematics.

This influence over students reminds one of the effect of Karl Weierstrass on many of those who attended his lectures at the University of Berlin in the latter half of the nineteenth century. While Weierstrass had far more first-rate mathematics students than De Morgan (among them Otto Hölder, Adolf Hurwitz, Felix Klein, Hermann Minkowski, Gösta Mittag-Leffler, Hermann Schwarz, and E. H. Moore), the image of a mathematician who "became a recognised master, primarily through his lectures" [**4**, p. 221] is consistent to both.

One could not provide a greater contrast to the didactic methods of De Morgan and Weierstrass than those of their earlier contemporary Augustin-Louis Cauchy. As a professor at the Ecole Polytechnique in Paris from 1816, Cauchy taught mathematics for trainee engineers. Yet, while his nineteenth-century biographer C. A. Valson extols his virtues as a teacher who "never left a subject until he had completely exhausted and elucidated it", [**53**, I, p. 64] the records of the Ecole contain a damning report of 1821 in which it is claimed that "numerous warnings have been given, for 5 years, to Mr. Cauchy to undertake to simplify his methods and to conform exactly to the programmes [of lectures]." [**19**, II, p. 711] Furthermore, it is asserted that "there has sometimes been ... a lack of clarity in his lectures ... " [**19**, II, p. 712]

Cauchy was also admonished by his superiors for "the lack of that aplomb that one must generally suppose of a *professeur* already celebrated in the sciences". [**19**, II, p. 711] In particular, his punctuality was criticised when he arrived for a lecture "10 minutes after the gathering of the Students in the amphitheatre". [**19**, II, p. 711] This characteristic was certainly not shared with De Morgan, who consistently placed great emphasis on the precision of his arrival. Indeed, he was apparently "so punctual and so regular in the performance of his college duties that his passage to and from his classes served as a time-piece to observant students". [**28**, p. 115] Moreover, during his time at University College, he became one of its most conscientious and respected professors, lecturing from 9 to 10am and 3 to 4pm every day except Sundays. Indeed, according to Roscoe, "the trouble he took with students was extraordinary". [**32**, p. 25]

De Morgan's innate mathematical propensity was enriched by strong communicative skills, which, together with a talent for presenting complex ideas in an intelligible form and a pithy lecturing style, resulted in the ability to captivate his audience irrespective of the topic he was treating. It would also appear from more than one source that, as with other areas of his instruction, in order to foster correct notions in his students, De Morgan's keen sense of humour was often employed as a pedagogic tool:

> One thing which made his classes lively to men who were up to his mark, was the humorous horror he used to express at our blunders, especially when we took the conventional or book view instead of the logical view. The bland "hush!" with which he would suppress a suggestion which was simply stupid, and the almost grotesque surprise he would feign when a man betrayed that, instead of the classification by logical principles, he was thinking of the old unmeaning classification by rule in the common school-books, were exceedingly humorous, and gave a life to the classes beyond the mere scope of their intellectual interests. [**15**, pp. 97–8]

Here again, De Morgan's formal but good-humoured style contrasts with that of some of his contemporary counterparts. It has been said that "it was only gradually that Weierstrass acquired the masterly skill in lecturing extolled by his later students. Initially his lectures were seldom clear, orderly, or understandable." [**4**, p. 221] Moreover, claimed Felix Klein (himself a highly successful lecturer), the "imposing" personality of the great man "gave his lectures a distinctly uncongenial, authoritarian quality." [**25**, p. 163]

De Morgan was able to balance congeniality with firm discipline to maintain a serious but convivial atmosphere in his lecture room. It was not a skill all of his fellow lecturers were able to achieve. James Clerk Maxwell was arguably one of the foremost British mathematical physicists of the time when he lectured at King's College London in the early 1860s. Yet, remarkable though his scientific credentials may have been, "as a teacher of raw youths,... he did not prove to be a success... and it seems not unlikely that the students were too much for him." [**22**, p. 247] Maxwell's inability to maintain order in his classes was exacerbated by additional shortcomings as a lecturer. A recent biographer explains: "The evidence is that, as a teacher, he had unusual difficulties. His delivery was poor. He could control neither the speed of his thoughts nor the flights of his mind.... Very likely only the occasional, particularly brilliant student could follow his lectures." [**36**, p. 100]

The same could be said of the teaching of James Joseph Sylvester, himself a former student of De Morgan, who, while an outstanding pure mathematician, was by no means a clear lecturer. In fact, as De Morgan later recalled. "When he was with us [as professor of natural philosophy at University College from 1837–41] he was an entire failure: whether in lecture room or in private exposition, he could not keep his team of ideas in hand." [**37**] Yet, forty years later, the very qualities that had made him unsuccessful as an undergraduate lecturer proved inspiring to his graduate students at Johns Hopkins University. As one later recalled: "One could not help being inspired by such teaching, and many of us were led to investigate on lines which he touched upon." [**25**, p. 81] Thus, a style of lecturing far less structured and meticulously arranged than that of De Morgan, was no less successful in holding the attention of the able students.

So what can we conclude from these comparisons of teaching characteristics of De Morgan and his contemporaries? What are the distinguishing features of a great teacher? First of all, the lecturer must be capable of delivering tuition by means of a clear and systematically planned series of lectures to form a structured and intelligible course. Clearly, De Morgan was amply equipped to do this, while Sylvester, Cauchy, and even Weierstrass to begin with, were not. Secondly, he must be able to maintain order in the lecture room so as to keep the attention of the class focused on the subject material. Here again, De Morgan satisfies this criterion, whereas others, most notably Maxwell, do not.

But finally, one must be able to *inspire* one's students with a love and fascination for their subject. De Morgan did this by concentrating on inculcating a deep understanding of fundamental principles rather than a mere skill with techniques and manipulation. Of course, Weierstrass, Klein, and Sylvester achieved success, but this was largely attained by teaching *graduates* whose interest in the subject was already strong. De Morgan's achievement lies in the fact that he was able to persuade so many undergraduates (even those who took the subject no further) of the beauty and allure of mathematics.

Without question Cauchy, Weierstrass, Klein, Maxwell, and Sylvester were all great mathematicians. Some (Weierstrass and Klein in particular) were great teachers as well. But whether De Morgan could also be called a "great" mathematician is debatable. True, he played a major role in the development of symbolic logic and its introduction into mainstream mathematics; he also contributed to the growth of abstract algebra, and promoted the use of Cauchy's limit-based approach to the calculus, but his mathematical output, while by no means trivial, hardly merits the term "great". As a researcher he is clearly not in the same league as those already listed. But, for his work in the lecture room, he certainly deserves a place in the first rank, possessing in abundance all the attributes of a memorable and effective educator. Indeed, if De Morgan's mathematical reputation had to rest on any one achievement, it would have to be as a teacher to whom the term "great" could truly be applied.

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