Why Dickson Left Quadratic Reciprocity Out of His *History of the Theory of Numbers*

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In a 1993 letter to the *Notices of the American Mathematical Society*, Irving Kaplansky called attention to an astonishing omission in the history of mathematics [**27**]. "Everybody knows," Kaplansky asserted, "that Dickson's *History of the Theory of Numbers* covers all of number theory up to about 1918. Right?" "Wrong," he answered solidly, "[t]ry looking up quadratic reciprocity." Kaplansky *is* right. Leonard Eugene Dickson's monumental compendium of the history of number theory excludes the history of quadratic reciprocity, the "crown jewel of elementary number theory" [**27**]. Why? Why did Dickson leave this celebrated number theoretic result out of his *History*? Kaplansky offers a brief explanation: "he farmed it out to a student" [**27**]. Again, Kaplansky is right, on some level at least.

In this paper, we offer further insight into this perplexing omission. In the process, we reveal an entirely new perspective on Dickson and unfold yet another example in the history of mathematics where extra-mathematical factors contribute to the development of mathematics. The history of Dickson's *History* actually begins in the last decade of the nineteenth century.

While Dickson pursued a Ph.D. at the young Chicago from 1894 to 1896, the then group-theoretically minded E. H. Moore inspired him to write a thesis on (what we would call) permutation groups [**15**]. Although group theory would remain among Dickson's research interests throughout his career, he would add finite field theory, invariant theory, the theory of algebras, and number theory to his repertoire of research interests. In the spring of 1900, just a few months past his twenty-sixth birthday, the Chicago Mathematics Department invited Dickson to join them as an assistant professor. From this position, Dickson made significant contributions to the consolidation and growth of the algebraic tradition in America [**23**]. Specifically, Dickson spent forty years (all but the first two) of his professional career on the faculty at Chicago where he directed 67 Ph.D. students, wrote 18 books and roughly 300 manuscripts, served as editor of the *American Mathematical Monthly* and the *Transactions of the American Mathematical Society*, and guided the American Mathematical Society as its President from 1916 to 1918 [**3**].

Yet, this mathematical workhorse, who played billiards and bridge by day and did mathematics from 8:30 to 1:30 a.m. every night [**1**, 377], interrupted his thriving pure mathematical career for nearly a decade to write a three-volume, 1500 page historical account of the theory of numbers. As he explained it himself, he undertook this project because "it fitted with my conviction that every person should aim to perform at some time in his life some serious useful work for which it is highly improbable that there will be any reward whatever other than his satisfaction therefrom" [**17**, 2:xxi]. Although he viewed it as "highly improbable," this altruistic mission paid handsome rewards for Dickson as this historical study ultimately led to his celebrated work in the arithmetics of algebras [**23**].

Dickson's description of this historical undertaking as "serious useful work," however, proved more than accurate. This was no hastily written history of number theory. On the contrary, Dickson had planned both the content of his project and the precise method he would follow to present the details of his study. He revealed the scope of his plans when he explicitly stated his bold intention to "give an adequate account of the entire literature of the theory of numbers" [**17**, 1:iii]. As for his method, the following excerpt from Dickson's *History* reveals both the thoroughness of his study and the historiographic view he maintained throughout this work. On the development of the theory of perfect numbers, he included, for example, that

> Hrotsvitha, a nun in Saxony, in the second half of the tenth century, mentioned the perfect numbers 6, 28, 496, 8128.
>
> Abraham Ibn Ezra (1167), in his commentary to the Pentateuch, Ex. 3, 15, stated that there is only one perfect number between any two successive powers of 10.
>
> Rabbi Josef b. Jehuda Ankin, at the end of the twelfth century, recommended the study of perfect numbers in the program of education laid out in his book "Healing of Souls."
>
> Jordanus Nemorarius (1236) stated (in Book VII, props. 55, 56) that every multiple of a perfect or abundant number is abundant, and every divisor of a perfect number is deficient. He attempted to prove (VII, 57) the erroneous statement that all abundant numbers are even.
>
> Leonardo Pisano, or Fibonacci, cited in his Liber Abbaci of 1202, revised about 1228, the perfect numbers
>
> $$\tfrac{1}{2}2^2(2^2-1)=6, \quad \tfrac{1}{2}2^3(2^3-1)=28, \quad \tfrac{1}{2}2^5(2^5-1)=496$$
>
> excluding the exponent 4 since 2^4-1 is not prime. He stated that by proceeding so, you can find an infinitude of perfect numbers [**17**, 1: 5].

In 1500 pages, Dickson never swerved from this comprehensive, facts-only style of writing. This strict style, in the opinion of the number theorist D. N. Lehmer, made "the book... not so much a history as a list of references from which a history of the theory of numbers might be written" [**28**, 131–132].

To be sure, the reviews of this historical text indicate that Dickson made minor errors in his account. The operative word here is minor—he did not omit major contributions to number theory from his compendium—save the one under discussion. In fact, the reviews of this masterpiece suggest that Dickson accomplished his historical endeavor with the same prowess as his work in pure mathematics. As Robert Carmichael, a number theorist who read the proof sheets for the entire second and third volumes, expressed it in his review for this MONTHLY,

> To give an adequate account of the entire literature of so vast a subject and one of such long history as the theory of numbers is an undertaking of enormous magnitude; and it is carried through in this work with a marvelous success in the presence of which one must pause in admiration. Henceforth this history will be indispensable to all investigators in the theory of numbers... It is a piece of work for which one cannot find a parallel in the whole of scientific history [**5**, 397; 403].

Dickson's *History* remains the classic reference on number theory up to 1918. It provided—and provides?—an "indispensable" source for those lacking adequate library facilities [**5**, 397]. In particular, as Dickson intended, the many "amateurs" interested in mathematics benefited from this (reputably) comprehensive, available account of number theory [**17**, II: xx] and [**5**, 397].[1] As for the professional mathematician, Lehmer emphasized "the greatest need for just such a piece of work to promote efficiency among the professional workers in this field and to prevent them from wasting their time on problems that have already been adequately treated, and also to suggest other problems which still defy analysis" [**28**, 132]. Lehmer made this point in his review of volume I of Dickson's *History*. The research mathematician would gain much more than "efficiency" by the time all three volumes appeared in print.

Dickson's "systematic" study of Diophantine Analysis for the second volume of his *History*, for example, provided him with a unique, sweeping perspective on this area of mathematics. From this vantage point, Dickson could assert that "[s]ince there already exist too many papers on Diophantine Analysis which give only special solutions, it is hoped that all devotees of this subject will in future refrain from publication until they obtain general theorems on the problem attacked if not a complete solution of it. Only in this way will the subject be able to retain its proper position by the side of other virile branches of mathematics" [**17**, 2: xx]. Dickson, in no uncertain terms, made this assertion with authority. Who better than a prominent research mathematician studying the "disjointed elements" of Diophantine Analysis, could so confidently declare in essence that "[i]deas rather than computations are needed in this field"?[2] Dickson's firm grasp on the past allowed him to see what would lead to a prosperous future for Diophantine Analysis.[3] Interestingly, he himself would devote the final fifteen years of his mathematical career focused on establishing a general result in Diophantine Analysis. But we have gotten ahead of our story. Simply put, Dickson made the history of number theory work in very utilitarian ways—far beyond serving solely as a reference volume—for the research mathematician.[4]

Even still, Dickson's purportedly complete history of the theory of numbers lacks the quintessential topic of elementary number theory, the law of quadratic reciprocity. This law relates the solvability of the congruences $x^2 \equiv p \pmod{q}$ and $x^2 \equiv q \pmod{p}$ for p and q distinct, odd primes. Specifically, if p or q is of the form $4k + 1$ (for $k \in \mathbf{Z}$), the two congruences are both solvable or both not solvable. If p and q are both of the form $4k + 3$ (for $k \in \mathbf{Z}$), one of the congruences is solvable and the other is not. In terms of the Legendre symbol, for

[1] In the letter to President R. S. Woodward of the Carnegie Institution, where he first put forth the idea of a History of Number Theory [**13**], Dickson described his "aim to make a volume indispensable to the specialists, but also a magnet to draw hold the attention of those non-specialists who desire to secure a connected scientific account of the subject." This theme of appealing to professional and amateur alike appears throughout Dickson's correspondence with the Carnegie Institution regarding his *History*.

[2] Both quotations are from [**6**, 72–73]. This emphasis on general results could easily be viewed as a "Dicksonian trademark." In his development of the definition of an algebra, Dickson sought the definition that yielded the theory with the widest applications. Similarly, in his work on the arithmetic of algebras, Dickson built his definition of an integral element, the crucial concept in the theory, using a "strategy of enlargement" as employed by Kummer and Gauss rather than defining an integral element on a case-by-case basis. See [**22**] and [**23**, 139–143; 152].

[3] In [**7**, 262], Carmichael emphasized the value of such forecasting when he wrote that "[w]hen a master, with the work of the past well in mind, tries to see the trend of the future, his judgement will be a matter of interest whether or not the direction of progress turns out to be such as he anticipates. It may even throw some light on the difficult question as to the way in which new discoveries arise."

[4] Dickson's *History* inspired—in the most general sense of the word—one prominent twentieth-century mathematician. Richard Guy purchased Dickson's *History* when he was about seventeen and found it "better than getting the whole works of Shakespeare and heaven knows what else" [**2**, 136].

p and q distinct, odd primes,

$$\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\left(\frac{p-1}{2}\right)\left(\frac{q-1}{2}\right)}.$$

This law, as Dickson described it himself, "is doubtless the most important tool in the theory of numbers and occupies the central position in its history. Its generalizations form a leading topic, past and present, in the theory of algebraic numbers" [**18**, 30]. Since the development of algebraic number theory grew, in large part, out of efforts to generalize quadratic reciprocity, it seems all the more unusual that a supposedly comprehensive *History of the Theory of Numbers* included no discussion of this area.[5]

Why, then, did Dickson exclude an account of "this most important" tool from his *History*? The historical record suggests that Dickson did not intend for this omission to occur. In his closing remarks in the preface to volume II, Dickson refers to a Volume III as the "concluding" volume in the series [**17**, 2: xii]. Volume III appeared in 1923, "promptly" prepared, as Dickson described it in the preface, "owing to the favorable reception accorded to the first two volumes of this history" [**17**, 3: iii]. Early in the text of this third volume, nestled in his history of binary quadratic forms, Dickson points us forward to a *fourth* volume [**17**, 3: 3]. In this parenthetical remark, Dickson indicated his plan to include the quadratic reciprocity law in the fourth volume. But, of course, as we know now, the fourth volume never appeared. What happened to the fourth volume?

The fourth volume involves Albert Everett Cooper, a University of Chicago graduate student from 1924–1926. Cooper had come to Chicago from the University of Texas where he had earned three degrees and taught as an instructor in the mathematics department for 5 years [**9**, vita following dissertation text]. Arriving as he did in the fall of 1924, Cooper met a Dickson who had just spent over a decade collecting references on number theory, "digesting" them, and writing them up in a suitable historical account of the subject, which ultimately grew from one to two to three volumes and spanned some 1500 pages [**14**]. This same Dickson had recently managed to secure publication for volume III of his *History* despite a change in presidency at the Carnegie Institution. He had just published his work in the arithmetics of algebra. He had now begun to focus his research interests on number theoretic topics, increasingly inclined toward problems related to extensions of Waring's Theorem. Perhaps more to the point, Cooper met a Dickson who had promised the Carnegie Institution, the mathematical world, and himself a comprehensive account of number theory, and, thus far, had failed to include information on the history of quadratic residues or reciprocity laws.

Cooper earned his Ph.D. in mathematics in the spring of 1926 under Dickson's guidance, with his historical dissertation "A Topical History of the Theory of Quadratic Residues" [**9**]. The title of Cooper's dissertation alone suggests a connection with Dickson's larger historical undertaking. Indeed, Cooper wrote this

[5]In [**16**], Dickson claimed that "the study of this challenge problem [Fermat's Last Theorem] and the general law of reciprocity of higher residues led [Ernst Eduard] Kummer to invent his ideal numbers, out of which grew the general theory of algebraic numbers, one of the most important branches of modern mathematics [p. 161]." This article seems to represent an expanded version of a similar discussion in his *History*, 2, pp. xviii–xix, 739–740. In [**21**, 324], Harold M. Edwards advances the view that the study of higher reciprocity laws and not Fermat's Last Theorem led Kummer to his study of what we know as cyclotomic integers and, ultimately, his ideal numbers. Edwards developed this idea more thoroughly in [**20**, 79–81].

dissertation with the intention that it appear as a chapter in the fourth volume of Dickson's history. Cooper cited earlier volumes of "this History," as he referred to it, more than twenty times in his 98-page thesis. In Cooper's section on "Number and Distribution of Residues," for example, Cooper asserted that "Dirichlet's fundamental formula for the distribution, in half, quarter, and eighth intervals, of the quadratic residues of a positive odd integer P stated in terms of the class number of a binary quadratic form of negative determinant are *quoted in vol. III, page 101 of this History*" [**9**, 53–53A, our emphasis].

More importantly, at least relative to the study at hand, Cooper's dissertation did not include a history of the law of quadratic reciprocity. Cooper did not overlook this "principal" theorem, however. Cooper titled his dissertation appropriately; he wrote a historical account of the theory of quadratic residues. On at least sixteen occasions in his dissertation, in fact, Cooper referred to a specific chapter on quadratic reciprocity in the purported fourth volume. "For proof [of G. Zolotareff's "unique method" of evaluation of $\left(\frac{k}{p}\right)$ where $k, p \in \mathbf{Z}$, p a prime, and p does not divide k]," Cooper wrote in his dissertation for example, "see chapter on quadratic reciprocity, this History, vol. IV" [**9**, 35]. He ended the text of his dissertation pointing to the "chapter on the law of quadratic reciprocity" and leaving spaces to assign both this chapter a number and the results of eight mathematicians page numbers within this chapter. Thus, Cooper wrote his dissertation to form a chapter on quadratic residues in Dickson's fourth volume, perhaps in the same spirit as G. H. Cresse's chapter on the class number in volume III.[6] Dickson's fourth volume, as Cooper understood it, would contain a separate chapter on the history of quadratic reciprocity. Dickson must have had this same understanding; he, after all, approved Cooper's thesis.

Dickson did more than simply approve Cooper's thesis at the end; he also gave him a boost at the beginning. Cooper described Dickson as an advisor who "not only furnished the entire body of original references from which the topical history was written, but also took a great deal of personal interest in the preparation of the material. My appreciation is particularly due Professor Dickson" [**9**, vita, immediately following dissertation text]. Although Cooper probably intended that this comment serve as an acknowledgement to his thesis advisor, this remark indicates that Dickson had amassed a collection of references on the history of quadratic residues by the time Cooper arrived at Chicago in 1924. H. S. Vandiver further substantiates this claim in a letter he wrote to Cooper on 5 November, 1925. "I'm sending you under separate cover," Vandiver wrote in response to Cooper's request for copies of his articles relating to quadratic residues, "copies of my articles which touch on quad-residues .... Perhaps you know that Dickson collected references of quad. residues while writing his *History*" [**32**]. Since Dickson collected quadratic residue references, surely he must have intended to include them, in some way at least, in his *History*. Although it may not have been his initial plan when he conceived of his *History* in 1911, Dickson ultimately entrusted the history of quadratic residues to a graduate student, namely, Albert Cooper.

[6]George Hoffman Cresse earned a Chicago Ph.D. in 1918 under Dickson's guidance with the historical dissertation "On the Class Numbers of Binary Quadratic Forms" [**30**]. A revised form of this dissertation appeared as chapter VI in the third volume of Dickson's *History*. The official title page of the third volume of Dickson's *History* reads, "History of the Theory of Numbers, Volume III, Quadratic and Higher Forms, By Leonard Eugene Dickson, Professor of Mathematics at the University of Chicago, With a Chapter on the Class Number, by G. H. Cresse."

Perhaps the more compelling question raised by this evidence surrounds the publication of Cooper's work. What happened to Cooper's thesis work on quadratic residues? And where is the chapter on quadratic reciprocity?

After Cooper completed his Ph.D. at Chicago, he rejoined the University of Texas mathematics faculty. A year later, in 1927, Dr. Harry Y. Benedict, a former classmate from Dickson's undergraduate days at Texas, assumed the presidency of that institution. Dickson gave his view on the election of Benedict to the presidency of the University of Texas in the alumni publication, *The Alcalde*. As Dickson saw it, "[t]he election of Dr. Benedict as President of the University of Texas is particularly fortunate. All are familiar with his success as dean, due to his unerring judgement, rare talents as an executive, and deep affection for the University. But I wish to emphasize the fact that the man having all these essential qualities is also a scientist [astronomer]. This is the age of science ... " **[4]** (and partly quoted in [**29**, 233]). Within months of penning this favorable response to Benedict's election, Dickson personally appealed to Benedict to finance a mathematical publication. That publication? The fourth and final volume of the *History of the Theory of Numbers*.

The letter to Benedict came on the heels of a two-month-long exchange of correspondence between Dickson and the Carnegie Institution regarding the publication of the fourth volume of Dickson's *History*. On 1 December, 1927, Dickson wrote the Carnegie Institution regarding the recently expressed interest of G. E. Stechert & Co. to reprint volumes I and II of Dickson's *History*. Dickson advised "allowing Stechert to reprint" and explained that "[w]e think that even if Carnegie Inst[itutio]n could afford to reprint, it would be wiser for it to spend same sum to print (short) vol IV of the History and so complete the series and to spend the balance on a History of the Solution of *Equations*" **[11]**. The administrative secretary of the Carnegie Institution, W. M. Gilbert, replied that the Institution would both prefer Stechert to reprint the two volumes and "be glad to have an opportunity to issue your fourth and final volume ... " **[25]**.

Dickson, however, had more on his mind than reprinting his *History* and publishing his fourth volume. Apparently, since the first of December he had outlined a new plan in a series of letters to Gilbert and John C. Merriam, the president of the Carnegie Institution. "My suggestion is as follows," Dickson wrote to Gilbert and Merriam at the end of that December,

> [l]et the Institution abandon not only the reprinting of Vols I & II, but also the printing of Vol. IV. Instead, let the Institution take on half the burden of publishing an entirely new work on the Theory of Numbers (which will meet all legitimate needs of a History, but also attend to the more important needs of presenting the whole theory of numbers as a science, with emphasis on methods). As I wrote before, the University of Chicago Press is committed to publishing *one* of the two necessary volumes... .
>
> The new work will be incomparably superior to the old History; will be what is needed permanently in this field; and will be a fitting sequel to my 35 years research in this field **[12]**.

Thus Dickson himself proposed that the Carnegie Institution no longer plan to publish the final, fourth volume of his *History*, but rather, agree to publish one of two new forthcoming volumes on Number Theory. Dickson felt that "an abbreviated Vol. IV might be published by some agency other than the Institution (& I would so undertake)" **[12]**. Apparently, over the course of the next month, Dickson determined that his old classmate Benedict and the University of Texas might just prove the best possible path of publication for volume IV.

"My dear Benedict," Dickson began in his letter of 27 January, 1928, "Dr. Cooper has now practically complete his historical ms on 'Quadratic Residues and Reciprocity Law.' He has done a fine job" [**10**]. This opening sentence indicates that between the spring of 1926, when Cooper earned his Ph.D. from Chicago, and January of 1928, Cooper had written "the chapter" on the history of quadratic reciprocity. Dickson explained to Benedict that he had originally planned to include Cooper's historical manuscript on quadratic residues and reciprocity law in the final, fourth volume of his *History*. If he carried out this plan, however, he would have to include a supplement containing corrections and additions to the first three volumes. (In [**17**, 3: iv], Dickson asked readers to send "errata or omissions, which will be published later as a supplement.") "There are," as Dickson tersely explained it, "reasons against my undertaking [a supplement]." "Also," Dickson added, now shifting the focus from himself to Cooper, "the objection that such a vol. IV would not be wholly the work of Cooper" [**10**]. Although Dickson did not cite the source of this "objection," he seemed to imply that Cooper (and, presumably, the University of Texas by association?) would not receive the credit he deserved if his work appeared as chapters in Dickson's fourth volume. It would be better for Cooper, Dickson emphasized, if he published his manuscript separately, perhaps, as Dickson suggested, with a subtitle to indicate that it formed the fourth volume of his *History*.

Dickson told Benedict that the "problem," as he referred to it, had further complications. Specifically, by this time, the first two volumes were out of print. Dickson had planned to "replenish" the information in volumes I & II by writing a 2-volume historical and expository account of the Theory of Numbers. As Dickson presented it to Benedict, the University of Chicago agreed to print one of these volumes and the Carnegie Institution agreed to print the other. There was a condition, however. The Carnegie Institution would print one of these two new historical and expository accounts of number theory only if relieved of any responsibility for printing a fourth volume in the original historical series. It seems Dickson gently twisted the details to attempt to secure publication for both his two new works on Number Theory and "his" volume IV.

Having spelled out all of these details, Dickson proposed "the following best plan to serve the interests of Mr. Cooper and mathematics [and himself?]: Let the University of Texas publish a book by Dr. Cooper on the History of Quadr[atic] Residues and Reciprocity Law. This would close up the gap now existing from lack of vol. IV of my History and would take the place of the latter" [**10**]. Although Dickson promised sales for a book by Cooper and urged Benedict to follow the lead of other large universities who aided in the publication of serious work done by their faculties, we know Benedict did not agree to publish this work of Cooper's as the subtitled fourth volume in the series. We know Benedict did not agree to publish the fourth volume, but we do not know why. His reply to Dickson remains lost.[7] Benedict replied to hundreds of requests, including those of garden clubs

[7]Here, by lost we mean that the copy of the letter Benedict sent Dickson is not in any of the "natural" or even some of the "unnatural" collections in the Texas Archives. Of course, Benedict could have responded to Dickson by phone. As Albert Lewis points out in [**29**, 207], in the 1920's university presidents began responding to more sensitive and controversial issues by telephone. In these cases, the president usually noted "Answered by telephone" at the bottom of the original letter. Dickson's letter bears no such annotation.

and small girls schools in south Texas; he must have replied to this letter. Since Dickson burned his papers upon his retirement, he probably destroyed the original copy of Benedict's response.

After January of 1928, Dickson apparently never mentioned volume IV in his correspondence with the Carnegie Institution again. Cooper, however, sent a telegram to the Institution in June of 1929, announcing that "I have ready the manuscript for the fourth volume." He queried further, "How many copies do you think should be printed?" [**8**] The secretary of the Division of Publications of the Carnegie Institution sent Cooper his advice but added that "[i]t seems to me that, with the information given here, Dr. Dickson would be the best one to decide this matter" [**26**]. Although Cooper and Dickson exchanged scores of papers, notes, and communiques of various forms on the history of quadratic reciprocity, they did not seem to leave any traces of a discussion of the publication of this material [**31**].

The organized and polished pieces of this collection, however, appear to represent the page proofs of a book written in the same spirit and style of the first three volumes of Dickson's *History*. (Perhaps these form the fourth volume Cooper referred to in his telegram?) The more loose and ordinary fragments (written on "scrap" paper, scrawled on the back of notices from the registrar, etc.) seem to provide brief summaries of and references to various articles on quadratic residues and reciprocity. The collection contains notes and papers written by both Cooper and Dickson. Cooper, however, maintained the collection. It seems unlikely that Dickson would have continued to supply Cooper with the information if he intended to write this portion of the history himself. Moreover, as Dickson indicated in his letter to President Benedict, Dickson could not have written the main section on the history of quadratic residues because Cooper had already done it. So either Dickson intended to publish the history as a collaborative effort with Cooper or, more likely, he saw Cooper as publishing it himself.

The question now becomes "why did Dickson not forge ahead with the fourth volume in some form?" It was totally uncharacteristic of Dickson to leave his work incomplete, as it were. He had a plan, at least in late 1927 and early 1928, to see this material to press. Cooper still persisted with ideas of publication as late as June of 1929.[8] But the work never appeared. Perhaps, by the late 1920's, with his historical project more than fifteen years old and his research program devoted almost exclusively to Waring's Problem, Dickson found himself completely occupied with other mathematical endeavors. Maybe his interest waned in the historical text, maybe he mentally turned over the fourth volume and its publication to Cooper, or, maybe, his "astonishing supply of energy" finally evaporated.[9]

Whatever the case, our historical study leads to an intriguing observation. Leonard Dickson sits squarely in the center of this episode in the history of (American) mathematics. Yes, Leonard Dickson, the prolific mathematician who had a reputation for completeness, for high standards, for excellence—even to the point of being impolite when insisting upon these standards [**24**, 13–14]. And, yet, this same Dickson had to come to terms with the non-appearance of this signifi-

[8]Aside from the aforementioned telegram from Cooper to the Carnegie Institution in June of 1929, we have no other record of Cooper's attempt(s?) to publish this material.

[9]Interestingly, in [**7**, 259], Carmichael subtly hinted at the possibility of Dickson running out of steam when he wrote that "[t]he reviewer ventures to predict that the favorable reception of the third volume will give the author still more reason for proceeding promptly with the fourth if his astonishing supply of energy is holding out well enough to leave him still susceptible to such influence."

cant result in number theory. This set of events in the history of mathematics certainly sheds new light on Dickson the mathematician and the man.

In the end, then, once again, the history of mathematics teaches us that mathematics—and mathematicians, for that matter—are more than they appear. In particular, mathematical and extra-mathematical factors impinge upon the development and publication of mathematics, and the history thereof. In this case, clearly, the extra-mathematical factors, in the form of authorship priority, publishing contracts, and finances, outdistanced the mathematical factors and resulted in a highly unusual omission.

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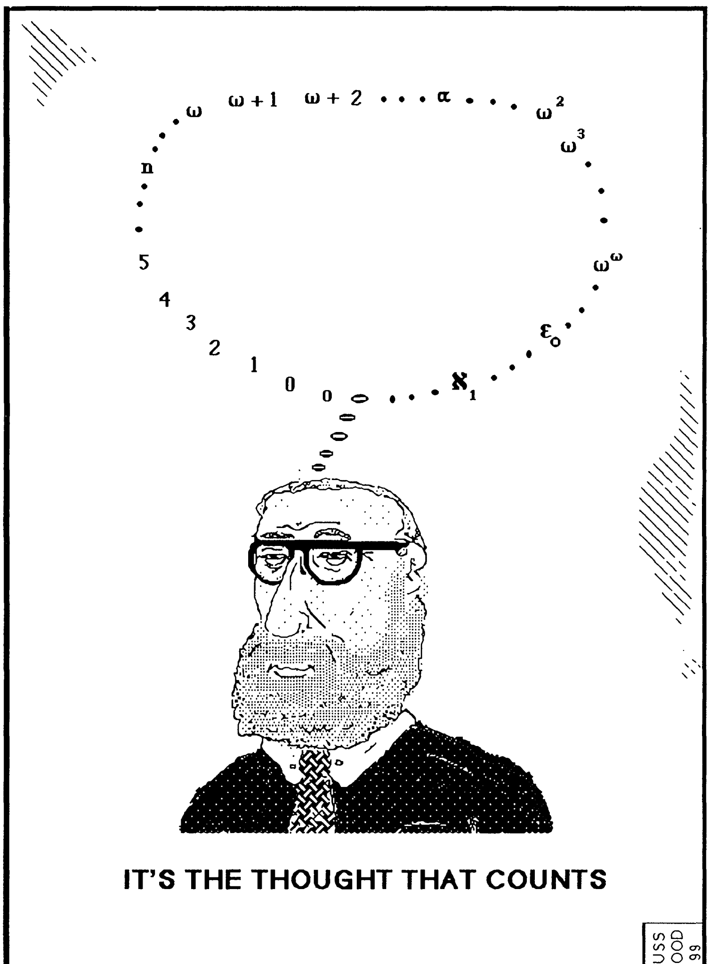

IT'S THE THOUGHT THAT COUNTS

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