PROBLEMS AND SOLUTIONS

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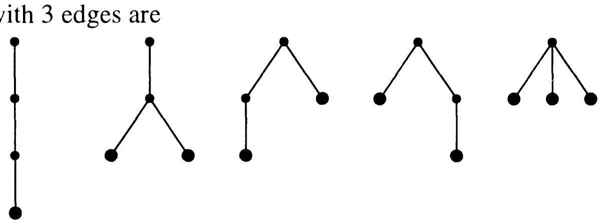
with the collaboration of Paul T. Bateman, Mario Benedicty, Paul Bracken, Duane M. Broline, Ezra A. Brown, Richard T. Bumby, Glenn G. Chappell, Randall Dougherty, Roger B. Eggleton, Ira M. Gessel, Bart Goddard, Jerrold R. Griggs, Douglas A. Hensley, John R. Isbell, Robert Israel, Kiran S. Kedlaya, Murray S. Klamkin, Fred Kochman, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Richard Pfiefer, Cecil C. Rousseau, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Charles Vanden Eynden, and William E. Watkins.

Proposed problems and solutions should be sent in duplicate to the MONTHLY *problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before March 31, 2000; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk (*) after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

10753. *Proposed by Louis Shapiro, Howard University, Washington, DC.* An *ordered tree* is a rooted tree in which the children of each node form a sequence as opposed to a set. The 5 ordered trees with 3 edges are



The number of ordered trees with n edges is the nth Catalan number $\binom{2n}{n}/(n+1)$. Therefore, if one draws each of the ordered trees with n edges, one draws a total of $\binom{2n}{n}$ nodes. Prove that exactly half of these nodes are end-nodes (i.e., leaves with no children).

10754. *Proposed by Paul Bracken, Université de Montréal, Montréal, PQ, Canada.* Let $\zeta(s) = \sum_{k=1}^{\infty} k^{-s}$, and let $\rho(s,n) = \sum_{k=n+1}^{\infty} k^{-s}$. Show that for positive integers $s \geq 2$,

$$\sum_{k=1}^{\infty} \frac{\rho(s,k)}{k} = \frac{s}{2}\zeta(s+1) - \frac{1}{2}\sum_{k=1}^{s-2} \zeta(s-k)\zeta(k+1).$$

10755. *Proposed by Jiro Fukuta, Motosu-gun, Gifu-ken, Japan.* An arbitrary circle O is drawn through vertices B and D of a convex quadrilateral $ABCD$. Let O_1 be the circle tangent to lines AB and AD and tangent to O internally at a point of O on the opposite side of line BD from A. Let O_2 be the circle tangent to lines CB and CD and tangent to O internally at a point of O on the opposite side of line BD from C. Let R_1 and R_2 be the radii of circles O_1 and O_2, respectively, and let r_1 and r_2 be the radii of the incircles of triangles ABD and CBD, respectively. Prove that the quadrilateral $ABCD$ is inscribable in a circle if and only if $r_1/R_1 + r_2/R_2 = 1$.