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PROBLEMS AND SOLUTIONS

Edited by **Gerald A. Edgar, Daniel H. Ullman, and Douglas B. West**

with the collaboration of Paul T. Bateman, Mario Benedicty, Paul Bracken, Duane M. Broline, Ezra A. Brown, Richard T. Bumby, Glenn G. Chappell, Randall Dougherty, Roger B. Eggleton, Ira M. Gessel, Bart Goddard, Jerrold R. Griggs, Douglas A. Hensley, John R. Isbell, Robert Israel, Kiran S. Kedlaya, Murray S. Klamkin, Fred Kochman, Frederick W. Luttmann, Vania Mascioni, Frank B. Miles, Richard Pfeifer, Cecil C. Rousseau, Leonard Smiley, John Henry Steelman, Kenneth Stolarsky, Richard Stong, Charles Vanden Eynden, and William E. Watkins.

Proposed problems and solutions should be sent in duplicate to the MONTHLY problems address on the inside front cover. Submitted problems should include solutions and relevant references. Submitted solutions should arrive at that address before April 30, 2000; Additional information, such as generalizations and references, is welcome. The problem number and the solver's name and address should appear on each solution. An acknowledgement will be sent only if a mailing label is provided. An asterisk () after the number of a problem or a part of a problem indicates that no solution is currently available.*

PROBLEMS

10760. *Proposed by Bruce Reznick, University of Illinois, Urbana, IL.* A function $f: \mathbb{N} \rightarrow \mathbb{C}$ is completely multiplicative if $f(1) = 1$ and $f(mn) = f(m)f(n)$ for all positive integers m and n . Find all completely multiplicative functions f with the property that the function $F(n) = \sum_{k=1}^n f(k)$ is also completely multiplicative.

10761. *Proposed by Fred Galvin, University of Kansas, Lawrence, KS.* Let G be a graph with n vertices. For each vertex v , let $f(v)$ be the maximum cardinality of an independent set of neighbors of v . Show that $\sum f(v) \leq n^2/2$, where the sum is taken over all vertices of G .

10762. *Proposed by Leroy Quet, Denver, CO.* Let $x_1 = 1$, and for $m \geq 1$ let $x_{m+1} = (m + 3/2)^{-1} \sum_{k=1}^m x_k x_{m+1-k}$. Evaluate $\lim_{m \rightarrow \infty} x_m/x_{m+1}$.

10763. *Proposed by Jean Anglesio, Garches, France.* Let ABC be a triangle; let O be its circumcenter, H its orthocenter, I its incenter, N its Nagel point, and X, Y, Z its excenters. Let S be defined so that O is the midpoint of HS , and let T denote the midpoint of SN . It is known that the orthocenter and the nine-point center of triangle XYZ are I and O , respectively. Prove that

(a) the circumcenter of triangle XYZ is T ; and

(b) the centroid of triangle XYZ is the centroid of SIN .

10764. *Proposed by Ray Redheffer, University of California, Los Angeles, CA.* Let $A = (a_{ij})$ be a real n -by- n matrix, and let x and y be real n -vectors satisfying $Ax = y$. Suppose that

$$\sum_{j \neq i} \max\{a_{ij}, 0\} < y_i \leq a_{ii} + \sum_{j \neq i} \min\{a_{ij}, 0\}$$

for all $i \in \{1, 2, \dots, n\}$. Show that $x_i > 0$ for all $i \in \{1, 2, \dots, n\}$.

10765. *Proposed by Peter J. Ferraro, Roselle Park, NJ.* Let f_n be the n th Fibonacci number, defined by $f_1 = f_2 = 1$ and $f_{n+2} = f_{n+1} + f_n$ for $n \geq 1$. Fix positive integers k and n with $n \geq 2k + 1$. Prove that $\lfloor \sqrt[k]{f_n} \rfloor - \lfloor \sqrt[k]{f_{n-k}} + \sqrt[k]{f_{n-2k}} \rfloor$ is 0 unless f_n is a k th power, when it is 1.