Effects of Calculus Reform: Local and National

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1. INTRODUCTION. Over the past decade, the large-scale effort to reform calculus instruction has been a prominent feature of mathematics education. It arose as an energetic response to criticism of the calculus curriculum that culminated in the 1987 conference *Calculus for a New Century*. Substantial support from the National Science Foundation stimulated development and implementation of new calculus curricula at many institutions, including the University of Connecticut.

The instructional practices of calculus-reform programs differ markedly from those that had persisted for decades (some would say, centuries). It is only natural for faculty to question whether the new modes *really* improve the approach that in their own education worked successfully. Some observe little if any improvement in conceptual understanding among students from reform courses, and even complain of *lessened* ability to use computational techniques. They contend (for instance, see [**29**]) that much of what passes for reform really amounts to "dumbing down" a formerly demanding but honest grounding in the power of calculus to a sloppy, imprecise, even misleading shadow of the true nature of the subject. Reformers counter that rote "plug-and-chug" hand symbolic calculation is as intellectually stultifying to teach as to learn, and that their teamwork-fostering technological tools and compelling connections to a broad spectrum of "real-life" issues motivate students to become active learners. They assert that their students emerge from calculus with superior understanding and greater capacity to use its methods successfully, and hence better prepared to complete degrees in mathematics, science, and engineering.

Passionate advocacy of such positions can make for lively lunch-room entertainment, but provides little objective basis for shaping an optimal calculus curriculum. The present paper examines the impact of calculus reform at one medium-sized state university. It also considers the effect of reform at several other institutions. It concludes with a discussion of broader national implications of those outcomes.

The following section describes the features of Connecticut's reform project and includes some representative examples of its activity. For more details, consult [**14**], [**15**], and the instructor guides for the latter [**16**]. Ensuing sections discuss data from common final examinations at Connecticut, results of a five-year longitudinal study of students who took traditional and reform versions of calculus during the project's first year, and results from similar analyses at other sites. The concluding section discusses the national picture, and the kind of further studies appropriate to appraising the impact of calculus reform.

2. THE UNIVERSITY OF CONNECTICUT PROJECT. In 1989, with support from the National Science Foundation Instrumentation and Laboratory Improvement Program, the University's Research Foundation, and the State of Connecticut

High-Technology Program, the Department of Mathematics opened a new computer laboratory. This facility made it possible to expand computer integration from the honors sequence to main-track calculus. Such expansion was a major recommendation of the MAA Committee on Calculus Reform and The First Two Years (CRAFTY) after its site visit the previous year. CRAFTY examined the five-year-old approach in the honors course as one early model for reforming calculus instruction. A brief description of the resulting project follows.

One class hour per week transformed to a computer-laboratory period, and an existing problem-discussion period evolved into a group problem-solving session. In both those settings, students work in self-selecting groups of 3-to-5, without referring to books or notes. In each, the instructor provides some guidance to student exploration and problem-analysis. In the problem hour, students attack a range of conceptual and computational questions, many of which are typical examination problems but a significant number of which are more challenging. In the computer laboratory, students pursue investigation of (generally new) topics by using local programs in the way their laboratory manuals suggest. The first semester, those programs are graphical and numerical True BASIC routines. Second-semester activity includes Maple worksheets and Mathematica notebooks. The remaining two weekly meetings resemble traditional lectures, but students have a more active role in discussion of examples. They often work together (with the aid of graphing calculators) to analyze and investigate problems or properties, and suggest how the instructor should proceed.

At the very first laboratory session, students encounter the function f with formula

$$f(x) = \begin{cases} x\sin\dfrac{1}{x} & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}. \tag{1}$$

A zooming program lets them examine the graph of f near the origin, and leads them to conjecture that it has a limit there; later, they use the sandwich theorem for limits to justify that conclusion. In a subsequent problem-solving hour, they work with similar functions for which graphing calculators provide enough information to suggest continuity or discontinuity. Again, they need to apply appropriate theory to support their conclusions.

Local linearity plays a central role in discussion of differentiation, and the graph of the function f with formula (1) indicates its non-differentiability at $x = 0$. A group project asks them to investigate the function g in which $x^2 \cos(1/x)$ replaces $x \sin(1/x)$ in (1). Another outgrowth of local linearity is the approach to implicit differentiation, which uses Euler's method to construct both tables of values and approximate graphs of functional equations $F(x, y) = 0$ near a known (initial-value) point (x_0, y_0). This provides a first experience with numerical solution of differential equations, as well as a ready tool for sketching the graph of equations such as $x^{2/3} + y^{2/3} = 1$.

Initial exposure to area and definite integration occurs in the laboratory, through interaction with a graphical/numerical program that presents the interactive screen in Figure 1. The goal is both to foster conceptualization of $\int_a^b f(x)\,dx$ as a limit of sums and to provide a means of computing the area under the graph of a continuous nonnegative function f over an interval $[a, b]$. Numerical integration thus emerges as a natural and effective means for evaluating definite integrals. This also sets the scene for the fundamental theorem as a remarkable link between

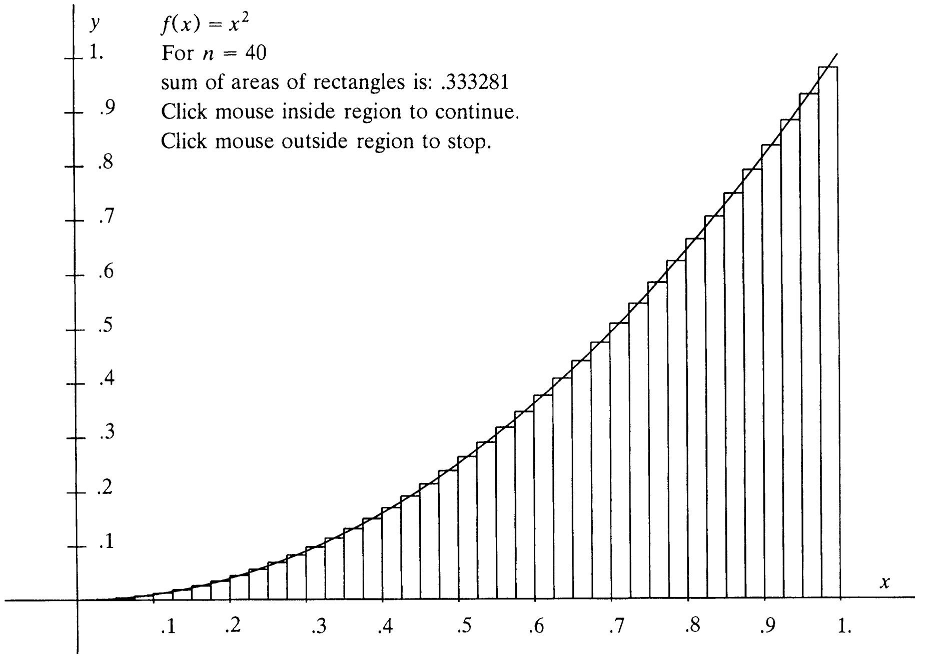

Figure 1

a limiting process and antidifferentiation. Students use numerical summation to estimate $\int_0^1 dx/(1+\cos x)$, and the corresponding laboratory project considers the accuracy of the right-endpoint approximation $R_n(f)$ and midpoint approximation $M_n(f)$. Students investigate $|I - R_n(f)|/h$, $|I - M_n(f)|/h$, and $|I - M_n(f)|/h^2$, where $I = \int_a^b f(x)\,dx$ is known, and discuss the order-of-magnitude accuracy those ratios suggest.

In the second semester, log-log and semilog plotting illustrate the usefulness of logarithmic and exponential functions in modeling, as most students are actually using those tools in their chemistry laboratory work. Mathematica's `Apart` command removes the algebraic tedium from complex partial-fraction decomposition. Numerical summation appears again, to highlight a perceptible difference between the series $\sum_{n=1}^{\infty} 1/n$ and $\sum_{n=1}^{\infty} 1/n^2$. Laboratory activity explores $s_{2n} - s_n$, and the ensuing project asks for an explanation of *why* for the harmonic series that difference not only does not tend to 0 but in fact seems to converge to $\ln 2$.

Years of teaching experience convinced the project faculty that student perceptions of the importance of material derives in large measure from testing. That prompted them to include conceptual (that is, non-computational) questions regularly not only on discussion worksheets, homework, and group projects but also on quizzes and examinations. For example, the first hour exam in 1989 contained the following true-false questions.

- If $\lim_{x \to c} f(x) = L$ then $\lim_{x \to c^-} f(x) = L$
- If f is continuous at $x = c$ then it must be differentiable there
- If f is continuous on $I = (2,3)$ then it must have a maximum value on I
- If f is not continuous at $x = c$ then it cannot be differentiable there
- If f is not defined at $x = c$ then $\lim_{x \to c} f(x)$ cannot exist

The common final examination that term showed the graph of a nonnegative function f on a grid over an interval $[a, b]$ and provided five intervals from which the students had to choose the one containing $\int_a^b f(x)\,dx$. A bonus question asked for a function f and a number a such that $\int_a^x f(t)\,dt = \tan x - 1$. The next year's first hour exam asked whether $\lim_{x \to 0} x^3 \cos(1/x)$ exists, and for justification of the answer. The 1991 final examination provided grids on which to sketch the graphs of a function f and its derivative given information about the signs of f' and f'', limiting information for f, and some function values of f and f'.

3. COMMON FINAL EXAMINATION PERFORMANCE. Every semester from Fall, 1989 through Spring, 1994, all first-year calculus students at the University's main campus—whether in experimental or traditional sections—took a common final examination. There was also a common text for all sections: [**12**] for the first three years and then [**13**]. With the exception of Fall, 1991, when an instructor of a traditional section prepared the common final examination alone, members of the calculus-reform team who taught experimental sections participated with the other instructors in making up the final examination. There were *never* computer-specific questions, but there were always both conceptual questions like those in Section 2 and traditional computational and procedural items. In an effort to measure any possible trade off between conceptual understanding and computational skills, the second-semester examinations tended to put greater emphasis on the latter to balance the heavier conceptual flavor of the first-semester examinations. Table 1

TABLE 1

Term	Standard Section Mean	Experimental Section Mean	Overall Standard Deviation
1989 Fall	59	68	21
1990 Spring	53	56	18
1990 Fall	51	59	18
1991 Spring	59	69	16
1991 Fall	68	71	17
1993 Fall	57	62	18
1994 Spring	62	63	17

presents the data from the results of the common examinations. Although the *size* of the difference is somewhat variable, for every semester the mean score in the experimental sections was higher than that in the standard sections. This suggests that better conceptual understanding in the project sections did not seem to come at the expense of weaker hand-computational skills.

For 1989–90, part of the analysis for a longitudinal study (see Section 4) showed no significant difference between students in the experimental and traditional sections in terms of such success predictors as high-school rank and SAT scores. Such analysis was not carried out for intervening years, but in [**5**] Mary Ann Connors of the University of Massachusetts reports on a comprehensive analysis of the final examination data from 1993–94. That was the first year in which the computer-integrated calculus sequence existed as a separate course (instead of experimental sections within a single first-year calculus sequence). That Fall the two courses were of nearly equal size, a very different distribution from that in Fall, 1989, when 90% of the students were in traditional sections. Enrollment in the two versions was not random: students were free to elect either, and, although rare, switching between them was possible. Most enrollees were entering freshmen, and despite distribution of full descriptions of the two variants of the course, at the first meetings most students were not aware of any format differences. Analysis of SAT Mathematics and Verbal scores again showed no significant differences between the two populations. Males entered both courses with significantly higher SAT math scores than the females.

Connors determined that the difference in final-examination performance was significant ($p < 0.034$), and that the performance of both males and females from the computer-integrated course was superior to that of their counterparts from the traditional version. Her analysis further revealed that in the computer-integrated course the mean score for female students (61%) was almost identical to the male-student mean (61.4%). Moreover, those female students had a mean SAT Math score (576) lower than both the females (596) and the males (601) in the traditional course. Yet on the final examination they outperformed both those groups, whose respective mean scores were 56.6 and 57.6. As Connors observed, this suggests that the female students benefited more from the computer-integrated experience than did the males. Analysis of performance within the respective courses further supports that. Male students in the computer-integrated course, for instance, had a mean SAT mathematics score of 625, but a mean final examination score just four-tenths of one percent higher than the female mean.

4. CALCULUS AS A PUMP TO TECHNICAL MAJORS. Consistently higher performance of students from reformed sections on concept-rich common final examinations suggested meaningful short-term benefit from the new approach, but to gauge the longer-term effect of the new instructional mode the authors focused on a basic theme of *Calculus for a New Century*. The welcoming message [**21**] from Dr. Frank Press, President of the National Academy of Sciences, to *Calculus for a New Century* cited the role of calculus as a giant filter that knocked many students, especially minorities and women, out of the pipeline to the technical work force of the 21st century. In what was to become the rallying cry of calculus reform, Dr. Robert White, President of the National Academy of Engineering [**28**] followed those remarks with a call to transform calculus from a filter into a pump.

To measure persistence and success in majors in the mathematical, physical, and engineering sciences, the authors first surveyed the requirements of all the science and engineering major programs at the University. From that study they compiled a set of 32 key courses with first-year calculus as prerequisite. Those 32 courses are essential to completion of majors in the fields of interest, and they require students to use the quantitative, analytical, and problem-solving skills that calculus aims to impart. The post-calculus careers of all students who enrolled in first-semester calculus in the Fall of 1989 ($n = 579$) were tracked for the subsequent five academic years. The population was divided into two cohorts for the purposes of this analysis: those who took at least one semester of computer-integrated calculus and those who took two semesters of traditional calculus during 1989–90. The data provided a list of the key courses in which each student subsequently enrolled. Among all students who took first-year calculus in 1989–90, approximately 59% of males took at least one such course, and approximately 43% of females did so. A detailed analysis of each student's performance in all such later courses was performed. A general linear model was fitted to the number of subsequent courses, and the results were checked for consistency with other methods (Mann-Whitney-Wilcoxon and Savage) and by means of transformations such as square-root and logarithmic.

With Mathematics SAT score, enrollment in the project's sections of calculus was one of only two factors that correlated significantly with persistence in technical majors among all students and among male students. For female students, it was the *only* statistically significant factor the study found to correlate with such persistence.

SAT scores, high-school class ranks, and other predictors of success in technical majors were compared both for students who took one or two semesters of the experimental version of calculus and for those who took two semesters in the traditional format. There were no significant differences in SAT scores or class rank, although in both categories the mean was *slightly* higher for students in the experimental sections. However, there was a significant difference (t-test $p < 0.0162$) between the two cohorts in the number of key post-calculus major courses completed. Among students in the traditional sections, the mean number of such courses was 3.50. Among students in the computer-integrated sections, it was 4.95, that is, more than 40% higher. Under the general linear models procedure with SAT scores and high-school rank as covariates, the p-value was 0.0580.

The mean grade earned in those courses was *not* significantly different between the two groups, indicating that students from the traditional sections who persisted in technical majors seem to have acquired adequate preparation from their first-year calculus courses.

Further analysis revealed additional interesting information. That study considered several factors, including SAT Verbal score, SAT Math score, total SAT score, high-school rank, and various combinations of two or more of those in relation to post-calculus persistence in technical fields. For all students, the SAT Math score was a very significant predictor of persistence in such majors ($p < 0.0001$). However, restricted to females it was not significant. In fact, the *only* significant such factor ($p < 0.0267$) for them that this study identified was enrollment in an experimental section of first-year calculus. By contrast, among males only the SAT Math score was a significant ($p < .0024$) factor in persistence.

5. RESULTS FROM OTHER SITES. Encouraging as the foregoing results at Connecticut may be, in themselves they provide information about the experience of just a single institution. The authors investigated the question of how common the Connecticut experience might be at calculus-reform sites. They solicited data from many sources and examined the literature for reports of studies that addressed such areas as persistence in technical fields, performance on common examinations and in later courses, and similarities and differences in outcomes between males and females.

The institution with the most similar approach to Connecticut's is Dartmouth, where True BASIC originated. While the main thrust of [**1**] is a description of Dartmouth's approach and presentation of sample materials, it does mention an experiment in 1988, a year before Connecticut's longitudinal study started. Dartmouth gave precisely the same *traditional* final examination in its first-term introductory calculus course as it had the previous year, when the degree of computer use in the course was substantially lower. It found no differences in performance between the two classes, which led to the Hippocratic conclusion that at least its computer integration appeared to have done no harm.

At the United States Naval Academy, a study compared results of common final examinations for sections that used the calculus-reform approach of the Harvard

Consortium to those for sections that used a traditional text and approach [**19**]. Students were randomly assigned to the sections, with no switching possible. The final examination included a common block of ten multiple-choice equations of both conceptual and computational types. Students from the reform sections scored better overall on nine of those ten questions, with statistically significant disparities in six of them. There was little or no underperformance on traditional computational questions, but significantly better performance on conceptual questions. These results are consistent with the conclusion that the reform mode could foster better learning, especially of conceptual areas of calculus.

Baylor University, another reform site that adopted the Harvard Consortium approach, also appraised the effect by using a common final examination for reform and traditional students [**25**]. Its analysis incorporated an index that divided students by success predictors (SAT Math, ACT Math, and local calculus placement exam scores). Even though *all* common exam questions came from the traditional course's topical coverage and assignments, mean scores among all levels of students in reform sections surpassed those of the traditional sections. On only two of 20 items was the mean score higher in traditional sections.

A study at the United States Merchant Marine Academy analyzed grades of students in traditional and reform versions of calculus throughout the sequence during the years 1990–91 (traditional) and 1991–92 (reform) [**22**]. Overall, the reform-course students earned consistently higher grades than did their predecessors in the traditional version. For students with weaker SAT and calculus-readiness examination scores, that improvement was striking. By contrast, students with the best readiness-exam scores actually did slightly worse in reform calculus, perhaps reflecting the lessened prominence of algebraic manipulation in the latter.

The C^4L (Calculus, Concepts, Computers, Cooperative Learning) program at Purdue has used a two-pronged approach to evaluate its impact. Besides qualitative research on student learning, the C^4L project conducted a longitudinal study similar to Connecticut's over the period 1988–91 [**23**]. Among the variables it studied were the number of post-calculus mathematics courses students took and the grades they earned. Like Connecticut's study, it found that project students took more of those courses, with no significant differences in grade performance.

The University of Illinois at Chicago compared grade performance in subsequent technical courses among students who took a traditional version of calculus in 1994–95 to that of students who took a reformed version of calculus (again, the Harvard approach) in 1995–96. Results of that study showed that students from reformed calculus performed significantly better in physics courses taken immediately after calculus, with diminishing differences in later courses [**2**].

SUNY Stony Brook compared percents of first-semester students who continued into the second semester of calculus before and after adoption of a reformed calculus program. Results from the study show significant differences over two three-year periods (1988–91 and 1992–95) [**20**]. In the first three years, the "yield" (percent of first-semester students who continued to the second semester) was, respectively, 36.4%, 52.6%, and 52.5%. Over the next three years, those figures improved to 63.1%, 59.8%, and 63.4%. One caveat: markedly higher course grades in the reformed course could be a significant stimulant to the improvement, and it is difficult to compare grading standards in the two quite different settings.

Michigan's reform project (Harvard materials) looked at a similar retention pattern. Results showed significant increases the first two years (when project section grades were also significantly higher), but a reverse in the third year when that grade disparity disappeared [**4**]. Significant attitudinal differences on the part of the students in the reformed version were also identifiable. Those in the reform project's sections showed more positive attitudes about calculus generally and their course experience in particular.

The Duke project (Project CALC) carried out one of the most comprehensive assessment studies, which measured attitude, subsequent enrollment patterns, retention of knowledge, and other factors. No short summary here can do justice to the copious data in that project's final evaluation [**3**]. Suffice it to note that results of the retention tests show a nearly uniform significant advantage in favor of the project students. Their attitudes, problem-solving, and conceptual understanding consistently surpassed those of traditional-section students. However, performance on computational skills was inferior, by a margin that fell just short of statistical significance. Project students again took significantly more technical courses, but students from traditional sections had a slightly higher grade-point average in those courses. Significantly more project students than traditional ones took more than two post-calculus courses.

What about the gender-specific aspects of the Connecticut project? The positive results for females in the Connecticut study are consistent with reports of similar effects in projects that incorporate graphing-calculator technology; see [**7**] and [**18**]. An interesting question that comes from such data: what aspects of technology enhance female learning in quantitative subjects? One possible theory from informal discussions is the prominent role of cooperative learning in many calculus-reform projects. Computer-laboratory sessions and related projects lend themselves naturally to group work, which several studies (among them [**11**]) have correlated with improved performance by females in science and mathematics courses.

Finally, an earlier article in this MONTHLY reported about calculus reform at Oklahoma State University [**17**]. That study found that—as at the Merchant Marine Academy, SUNY Stony Brook, and Michigan—grades in the reform version of calculus were higher than in traditional sections, which in the Oklahoma State study were contemporaneous. Little difference in subsequent enrollment patterns *within the calculus/differential equations sequence* was observed. However, the study did not encompass a sufficiently long time span to measure persistence in and completion of required courses for technical majors, as in the Connecticut, Purdue, and Duke studies. However, its results were consistent with those from the Merchant Marine Academy study in one respect: lower grades in traditional sections of Calculus II for students from reform sections of Calculus I. It reported insufficient data for higher-level course performance to permit meaningful conclusions, but stated that fewer students from the reform sections earned C or better grades in differential equations or linear algebra. It concluded that traditional-section students tended overall to do better in subsequent mathematics courses, but gave no information about the statistical significance of their performance.

The preponderance of evidence from the Connecticut study—as well as the evidence cited in this section—is consistent with the conclusion that the impact of calculus reform has been positive. If the "filter-to-pump" goal is an appropriate assessment standard, then the results from Connecticut and other calculus-reform sites suggest that calculus with modern computational and pedagogical features can promote better end-of-course mastery, significantly improve the flow rate into the technical work force, and foster more gender diversity in that flow. Note the caution in these conclusions: *is consistent with* and *suggest*, rather than *imply* or *prove*. Unlike mathematics, appraisal of pedagogical change is a highly inexact science, lacking not only proofs but even generally accepted rules of inference.

The positive content of the previous paragraph notwithstanding, it is only fair to mention that at many institutions the fruits of calculus reform have not included substantially higher performance and persistence [**9**]. Experimenters whose results fall short of hopes and expectations are certainly less likely to write up and disseminate accounts of their work than those with more pleasing performance data to report. There is also considerable belief in the *Hawthorne effect*, that is, that expenditure of time and attention on a project aiming to improve outcomes leads to actual improvements, which to at least some extent result from the attention itself rather than the project's methods. On the other hand, the atmosphere of passionate debate mentioned in Section 1 certainly is receptive to a well-designed and well-documented study showing predominantly *lower* performance and/or persistence by students in a reformed approach to calculus. The authors are not aware of any such study.

6. GENERAL NATIONAL IMPACT. On a procedural level, there is clear evidence of substantial change in the teaching, learning, and testing of calculus since the 1980's. The appearance and spread of graphing calculators and powerful desktop, laptop, and now notebook and subnotebook computers has noticeably stimulated greater emphasis on the graphical and numerical aspects of calculus. The popularity of projects and written reports has also brought a new verbal dimension to the subject. Even current editions of texts that early reformers cited as contributors to the calculus crisis incorporate (and advertise) numerical, graphical, and verbal components. Calculator and computer supplements, which were virtually nonexistent at the start of the calculus-reform movement, have now become pervasive. It is in fact challenging to identify a current calculus text that in 1987 would have been labeled "traditional."

National examinations reflect national norms about mastery of the subjects they test. Prior to the calculus reform movement, the Advanced Placement calculus program banned calculators. The current AP exams *require* them! A similar situation will shortly exist in the new Graduate Record Examination's Mathematical Reasoning Test for students planning to pursue graduate study in engineering, the physical, mathematical, or computer sciences, economics, or some areas of the life sciences [**26**]. This test presumes a year of calculus, and the questions in its new version probe understanding of concepts and their reflection in graphical or applied settings, areas of particular prominence in calculus reform.

The foregoing sections raise a natural question: what national impact has calculus reform actually had? The rest of this section addresses that question.

More than 500 mathematics departments at postsecondary institutions nationwide are currently implementing some level of calculus reform. These "reformed" courses enroll approximately 300,000 students each year, about 32% of the total national calculus enrollment [**27**]. Such growth of a movement only a decade old suggests that the influence of calculus reform will continue to spread. In fact, many institutions are now initiating programs to improve learning in a wide range of science, mathematics, engineering, and technology courses. These include projects that eliminate the traditional boundaries between these disciplines by means of an integrated teaching approach. Such programs, which respond to the new requirements students face in an increasingly technical and multidimensional workplace, represent a fundamental change in the philosophy that has long guided the structure of undergraduate education.

At present, there are few studies that document the impact of these efforts on student learning, faculty and student attitudes, and the general environment at undergraduate institutions. There has been considerable study of student learning in calculus, e.g. [**6**], [**8**], [**9**], [**10**], and [**24**]. To make meaningful judgments about the value of reform efforts, it is necessary to understand not only how students learn, but also how different environments influence their ability to learn. More studies are essential if the academic community is to understand the implications of this change in philosophy on learning within and across disciplines, throughout a student's experience at the undergraduate level, and beyond.

One such study is part of a larger effort by the National Science Foundation to evaluate the impact of reform in science, mathematics, engineering, and technology education at the undergraduate level [**9**]. It investigated the effect of calculus reform on: student achievement, attitudes, and retention; the implementation of mechanisms that have shown promise in improving the learning environment (e.g., student-centered learning, multiple methods of delivery, and alternative measures of assessment); and the general educational environment. Although the study yielded mixed results for student achievement, several trends did emerge. For example, approximately 50% of the institutions conducting studies on the impact of technology reported increases in conceptual understanding, greater facility with visualization and graphical representations, and the ability to solve a wider variety of more difficult problems, without any loss of computational skills. Another 40% reported that students in classes with technology had done at least as well as those in traditional courses. The impact of long-term projects and group work on student achievement is less clear. However, a pattern was discernible regarding the type of student likely to excel in this environment. This includes students who are "above average" in mathematics, students who do not perform well on traditional tests, and engineering majors.

Projects and group work also seem to affect grade distribution, although not in consistent ways. For example, one institution reported that projects made the final grade distribution more bi-polar, with very few "C" students, while another reported that projects were "the great equalizer," with more C's resulting from the subjective nature of grading the projects and concomitant inability to justify grades at the extremes. The effect of projects on course grades also seemed to be influenced by whether the projects were individual or group: the latter seemed to "equalize" the grade distribution more than individual assignments.

The existence of many common elements throughout the majority of the projects implies that the relative success or failure of reform efforts is not necessarily dependent solely upon content, but also on how, by whom, and in what setting the approach is implemented. Consistent reactions of students from a wide variety of institutions point to several key components for a successful reform environment. For example, instructors must communicate to students (and other faculty) the purpose of the changes being made in the calculus course. This is often not as easy as it may seem, because the reasons for the change must strike the students as relevant and important to future success. Unfortunately, many students believe mathematics is a static list of rules and algorithms to be memorized, a barrier to be overcome before they can do "real" work in other disciplines. One of the most important roles of reform efforts, then, is to challenge those beliefs and help students appreciate the many uses of calculus, both within mathematics and in a wide range of other disciplines. Thus far, the most effective means to this end still await identification.

Two major goals of calculus reform have been to revitalize the sequence and to generate discussion within the mathematics community about the nature and content of the calculus course. The lively debate the introduction cites attests to the fact that these goals have indeed been realized! In itself, that constitutes a major accomplishment that should not be overlooked. Are the reform efforts in fact helping students better understand calculus and appreciate the importance of mathematics in their lives? This basic question may never be definitively answered, but it cries out for study if calculus courses—and mathematics in general—are to remain a vital part of the undergraduate curriculum.

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