Rajnish Kumar, born in 1971, is a Mechanical Engineer working with the Indian Railways. He calls himself a number maniac, but never got cent percent in Mathematics. He loves the subject and after years of exciting adventures with figures and numbers treads upon the path of writing on Mathematics.

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What this book can do for you?

The single biggest fear for all students of mathematics is the cumbersome calculations involving even medium sized numbers. The reason being two fold

1. Lack of time.
2. No intrinsic way for verification of result.

The problem is aggravated more during competitive examinations where speed and accuracy are the essence, and even a slight error can fetch negative marks. This simply is overwhelming because of which lot of escapist tendencies and remorse occurs in potential candidates. This book has been compiled after extensive study of various quick calculation techniques picked up from Prof Tratchenberg's system for speed Mathematics and Vedic Mathematics, and also from tidbits across a spectrum of books. It has been kept very simple and only the very relevant and "truly" QUICK techniques have been mentioned. Most of the exercises have been fully solved highlighting all important steps and potential areas of error. The techniques increase speed and also provide an intrinsic verification of result.

The wonders that you can do with your calculations are

- **Add up 11342+67545+89876+56428+78625+23148 in just 30 seconds!**
- Without jotting down a single digit, mentally multiply 54x67, 89x12, 87x14 etc.
- In single line multiply 89x98, 95x86, 88x99 etc.
- You can even multiply 544x567=308448 in a single step
- 357x732= $2\ 1\ _4 4\ _7 0\ _3 1\ _1 4 \Rightarrow 261324$ Great so easy!! Just one line
- 62÷9 without dividing, don't believe this, continue reading!
- 10113÷88 with only addition and plain single digit multiplication

```
88 ) 101 / 13
      12
       1 / 2
         / 48
     _________
     114 / 81
```

- More surprising 98613÷63 can be found with just division by 6, like this

3									
6	9		8		6		1	/	3
		3		5		5		3	
	1		5		6		5	/	18

Quotient = 1565, Remainder = 18

- Even more interesting 25453÷8829 in just one step

8 8 2 9	2	/ 5	4	5	3
1 1 7 1		2	4	-6	2
1 2 -3 1					
	2	/ 7	7	9	5

Quotient = 2, Remainder = 7795

- You can find square of 97 in just a blink

97^2 = 97-3/09 = 9409

1009^2 = 1009+9/081 = 1018081, unbelievable but possible

- If this does not appeal can you find the square of 7693 in one line, without several steps involving multiplication and addition.

$7693^2 = \ldots 7693 \Rightarrow 49\ {}_{8}4\ {}_{16}2\ {}_{15}0\ {}_{11}7\ {}_{5}4\ 9 \Rightarrow 59182249$

- The only way to find a square root is using a calculator which is not actually allowed upto Intermediate examination and in competition. But there is two-line method to find square root of any number.

Square root of 20457529 is 4523 by this two-line method

	2	0	:	4		5	:	7		5	:	2		9
8			4		4		4		3		1		0	
		4		5		2		3	:	0		0		0

- 12^3 can be calculated without multiplying twice

12^3= 1 2+4 4+8 8 = 1728 in just this much space

- Also 997^3 = 991/027/-027 = 991026973, can you imagine the reduction in calculation time with these steps?

- Look at the number and find the cube root

For 262,144 the cube root is 64 without a single calculation

And for 571,787 cube root is 83, believe this book and you will be a ***calcu-magician***.

- With a little more effort the cube root of 8 to 9 digit numbers can be found, for which there is no standard method in conventional arithmetic. For example,

Cube root of 178,453,547 is 563 in just two steps.

a=3 , c=5 , 178453547-27= 178453520,
$3 \times 3^2 b$ end in 6 thus b=2, so cube root is 563.

Confused! You shall not be once all chapters are studied and problems practiced.

- Did you know

1/19 = . 0 5 2 6 3 1 5 7 8 9 4 7 3 6 8 4 2 1
can be calculated in just this single step

and also

1/29= ………${}_19$ ${}_16$ ${}_15$ 5 ${}_21$ 7 ${}_12$ 4 ${}_11$ ${}_23$ ${}_27$ 9 3 1

= **. 0 3 4 4 8 2 7 5 8 6 2 0 6 8 9 6 5 5 1 7 2 4 1 3 7 9 3 1**

Amazing, that's the power of this book.

There are a few more things to know. All of us have heard of divisibility rules for 2,3,4,5,6,8,9,11 etc. These are generally taught in all schools. But there is actually a rule for any number whatsoever and its really easy, much more than you can imagine.

To find whether 10112124 is divisible by 33 all you have to do is

1011212+40=1011252⇒101125+20=101145⇒10114+50=10164⇒1016+40⇒
1056⇒105+60=165⇒16+50=66 : and you have the answer, with only simple addition and multiplication to do and no cumbersome division till the end.

Using the clichéd statement, you have to learn to believe. There is enough potential in you to become a mathe-magician rather a calcu-magician. All that is needed is wholehearted attempt without the slightest doubt and cynicism on mastering the techniques explained.

The author made a rough estimate that in the arithmetic portion of GRE, GMAT etc., there is possibility of arriving at right solution by intuition in 10% of the cases, and 90% of the time is WASTED in calculations. The book can help reduce this time by as much as 50% for a person with normal arithmetical skills and a person with genius can actually ***be near to paperless in calculations,*** of course jotting down the answer is excluded. This is not a false claim. Geniuses like Shakuntla Devi from India have only mastered such techniques to become calculation wizards.

Happy calculating.

WARNING: Do not attempt to reduce time further as these are the shortest possible ways to calculate, any attempt on that line can be hazardous and self-defeating.

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Scheme of the book

The book contains chapters on following aspects

1. Quick Addition techniques
2. Techniques for memorizing Multiplication tables.
3. Speeding up basic Multiplication.
4. Multiplication of two digits by two digits.
5. Higher order Multiplication Techniques
6. Simple Division
7. Higher order Division
8. Checking the Division
9. Quick Squaring of numbers
10. Quick Square roots
11. Quick Cubing of numbers
12. Quick Cube roots
13. Recurring Decimals
14. Rules for Divisibility for any number - Positive Osculation Method
15. Rules for Divisibility for any number - Negative Osculation Method
16. Quick Averaging of numbers
17. Application of techniques to Algebraic equations.

Quick mathematics. It sounds a bit unusual but it is not. They have been very intelligent and innovative teachers in mathematics who have developed very nice ways of teaching unusual ways of calculations. It is often bewildering for the mother to teach the multiplication tables which later on help in calculations. They can be a special way to train the fresh mind on calculations, which may later on retain the interest and also ease during the final stages of schooling. Most of the competitive examinations do not allow the calculator. Whether it is the famous GRE,GMAT,CAT or any of the banking examination, the students who are slightly poor at calculations become part of the great depression circle. The numbers scare them and the scare prevents them from improving their skills. The scheme in the book may not be replacing any of the conventional techniques but it can certainly add on and complement whatever gaps are found in the conventional technique. It is important not to be overenthusiastic about what is learnt their but adapt the skills in a manner that for a particular situation the technique can be applied and tremendous saving in time is achieved. Nothing can replace the older proven calculation techniques, as they are universal by nature. The techniques depicted in the book are for particular set of number and are really effective in saving time.

Besides the practical utility of the techniques, it will also be very interesting for those who love numbers and their relationships. Each technique is actually a very simple algebraic proof. The proof is not dealt in this book because it is of mere academic interest and does not really help in easing calculations.

The only difficulty with these techniques is that they are too many and often it may be confusing for the student to remember all of them. But there is a way out. The exercises at the end of the chapter should be done at least three to four times before leaving the chapter. This way the technique is ingrained and will like a reflex action of a car driver automatically pressing the correct pedal.

The difficulties arising out of quick calculations are of different nature. The first problem is with the addition of subsequent numbers that sometimes leads to error if there is any break in the concentration. So it is most important to be particularly attentive during the calculation and nothing else should be thought of. Quick calculation does not mean impatience or haste, it should be done very systematically and strictly as per the guidance. This is the shortest possible way to calculate, so if the student again tries to shorten the procedure he would land at making silly mistakes and of course blame the poor book for the misdemeanor.

The second problem is with the addition and subtraction involved. During the course of learning these techniques it is very important to have sound practice of addition and subtraction of two numbers. There should be absolutely no waste of time in adding or subtracting two numbers. This aspect should be thoroughly practiced to perfection.

Hopefully the book will serve its purpose.

CHAPTER 1 Quick Addition Techniques

The basic thing for practice to get accurate calculations is addition. It is the easiest but the most error producing aspect of quick calculations. The addition of two single digit numbers should be just a ***REFLEX Action.*** Following is a method to develop this reflex.

Place all numbers 1 to 9 in a grid.

	5	2	9	4	1	6	8	3	7
1									
2									
3									
4									
5									
6									
7									
8									
9									

Now quickly without wasting any second fill in the columns adding the number in the row and column. ***(Notice the numbers in the column are not in sequence)***. Anyone who is not too good at calculations will make some mistakes. Not to worry, anyone who is not good at calculations will try to be careful and try to relate the calculation to the previous one. So do this again until there is perfection. The ideal time for filling up the grid 150 seconds without a single mistake. Practice this to perfection and then when you see two numbers the addition will be reflex action.

One more similar exercise for perfecting addition is to mark the wrong answers in the grid below within 150 seconds using a stopwatch.

	5	2	9	4	1	6	8	3	7
4	9	7	11	8	5	10	12	3	9
2	7	4	11	6	3	8	5	10	9
9	14	11	18	13	10	15	17	12	16
3	8	5	12	7	4	9	11	6	10
5	10	7	14	9	6	11	13	8	11
6	10	8	15	10	7	11	14	9	13
1	6	3	10	5	2	7	9	4	8
8	13	10	17	12	9	14	16	11	15
7	12	9	16	11	8	13	15	10	14

There is all the possibility that you will loose patience half way, but that's the whole game. Do not loose patience even if you eyes get sort of rolled over. Once this grid is over, you can ask your friend to make a grid for you incorporating some mistakes so that enough practice is done and by being able to recognize errors your capability to make error free additions will improve.

EXERCISE 1.1 Add up these figures mentally and jot down on a paper

Time **10 seconds**

3+4	5+7	7+5	7+8	3+6	9+8	1+5	9+8	8+8	6+4

How many mistakes did you make?

EXERCISE 1.2 Copy and fill up the grid.

Time **30 seconds**

	7	2	6	4	3
4					
2					
9					
3					
5					
6					
1					
8					
7					

How many mistakes did you make? In case you could not complete the grid in 30 seconds, do it again and again till you reach the 30 seconds figure.

Now we shall proceed to some techniques for additions of higher order, where this REFLEX addition will be the only thing required. It is the tendency of most to be very sure of their common mistakes. For example, if you are asked to calculate 6 x 7, the answer would be 42 but while making a continuous calculation it may be done as 61 or 62, thus rendering the whole effort useless.

Adding up numbers like the following example is quite easy but there is enough scope for mistakes as 6+7+7+2 all digits of units place add up to 22, then 2 is carried over to ten's place and so on the addition is done in the conventional way. The grocery shopkeeper can often make mistakes in case he is without calculator thus putting you at a loss most of the times!

```
  586
  657
  897
  542
  456
 ----
 3138
```

Considering the same example we shall learn the Trachtenberg's Addition System.

THE RULE IS ***Do not add more after Eleven.***

While adding up numbers as soon as the total gets more than 11, STOP. Reduce the figure by 11, tick mark and proceed further. Do not get confused, just pay attention carefully.

FIRST COLUMN UNIT'S PLACE		
58**6**		add 6+7=13 drop 11
65**7**	tick	2 is left
89**7**		now add 2+7=9
54**2**	tick	add 9+2=11 drop 11, 0 is left
45**6**		add 0+6=6
		we have 2 ticks and 6 as sum of first column

SECOND COLUMN TEN'S PLACE		
5**8**6		add 8+5=13 drop 11
6**5**7	tick	2 is left
8**9**7	tick	now add 2+9=11 drop 11, 0 is left
5**4**2		add 0+4=4
4**5**6		add 4+5=9
		we have 2 ticks and 9 as sum of second column

THIRD COLUMN HUNDRED'S PLACE		
586		add 5+6=11 drop 11
657	tick	0 is left
897		now add 0+8=8
542	tick	add 8+5=13 drop 11, 2 is left
456		add 2+4=6
		we have 2 ticks and 6 as sum of third column

It is expected that the reader has understood what has been done upto this point. We have simply isolated each column and added separately.

- Whenever total is more than 11
- Subtract 11 from the result ,
- Place a tick
- Proceed further with the result

All this can sound very complex but just be patient and you shall find this is to be quite advantageous while making long calculations.

	5	8	6
	6	5	7
	8	9	7
	5	4	2
	4	5	6
Running total	**6**	**9**	**6**
Ticks	**2**	**2**	**2**

Now prefix 0 to both the rows of running total and ticks

IMPORTANT STEP

ADD THE RUNNING TOTAL AND TICKS WITH RIGHT NEIGHBOUR IN ROW OF TICKS. This addition is based on conventional rules

Observe closely.

Running total	**0**	**6**	**9**	**6**
Ticks	**0**	**2**	**2**	**2**
	0+0+2+1	6+2+2+ 1(carry over) =11 retain 1 carry over 1	9+2+2=13 retain 3 carry over 1	6+2+0 =8 (as there is no neighbour)
Total	**3**	**1**	**3**	**8**

The explanation may seem so long and futile. Nothing to worry, look at the table below. If the procedure is understood then there is no need for any extra calculation and all is done mentally with the only thing to jot down is the tick and final answer.

Let us add the numbers**.** You first try the conventional way. Isn't this boring and very confusing. Now let us try the method explained above.

Well what can make you commit an error is subtraction of 11

- ***simply remove the first digit.***
- ***reduce remaining by 1 e.g. 18, remove 1 and subtract 1 from 8 making it 7.***

		4	6	7	3	8
		3	**5'**	**9'**	**8'**	**5'**
		9'	6	**7'**	5	4
		2	3	1	4	**9'**
		8'	**7'**	6	**8'**	**9'**
		5	4	**3'**	**6'**	7
running total	0	9	9	0	1	9
ticks	0	2	2	3	3	3
adding up	0+0+2+1=3	9+2+2+1=14	9+2+3=14	0+3+3=6	1+3+3+1=8	9+3=12
Grand total	**3**	**4**	**4**	**6**	**8**	**2**

Notice the' marks which have been used as ticks. If you examine the procedure it is all mental. You only need to mark the tick on the number and write the running total.

For practice do the following calculations yourself. The detailed calculation is in the solutions chapter.

EXERCISE 1.3 Add up the following using the rules explained

1. **2897+4356+7843+5434**
2. **3768+5467+9087+8796**
3. **34256+18976+98768+45367+13987**
4. **2341+5467+9088+7009+3219**
5. **14678+45325+58326+90087+76509**

VERIFICATION OF ANSWER

While making large calculations, there is enough scope for mistakes at any stage. There are a few very pin pointed methods of checking any calculation. But we shall discuss the simplest one. This technique is applicable for multiplication, division, subtraction and addition.

Let us first understand what **INTEGER SUM** is. It is defined as the addition of all the numbers of the digit so that only one digit is the final result. For example,

For the number 12, add up 1 and 2 to get number 3. This is the integer sum of 12.

Let us consider a larger number like 462789,

where we start with **4+6 = 10, 10+2=12, 12+7=19, 19+8=27, 27+9= 36**, and finally **3+6=9**, thus **9** is the integer sum of **462789.**

There are shortcuts to this,

Shortcut 1 Observe the number and whenever you find the number 9, do not consider it for integer sum.

Shortcut 2 A group of numbers like 27, 36, 45, 81 which add up to 9, can be ignored.

The reason is when you add 9 to a number its integer sum remains the same. So instead of wasting your effort in adding up 9 to the integer sum, it may best be ignored.

Shortcut 3 At each step keep adding up the number, so that there is always a single digit to add.

Using this scheme let us redo the integer sum of 462789

- ignore 9 and group of 7 and 2 ***(shortcut 1 and 2)***
- we are left with 4,6,8
- 4+6=10, 1+0=1 ***(shortcut 3)***
- now 1+8=9

Taking another example, find integer sum of 437862901

- ignore 9, 6/3,7/2, 8/1
- we are left with 4 and 0, hence integer sum is 4, SIMPLE !

EXERCISE 1.4 Find the Integer sums of following numbers

Time 45 seconds- 1st try
30 seconds- 2nd try

1. **361547980**
2. **376539246**
3. **546372**
4. **900876**
5. **9994563**
6. **918273645**
7. **225162**

Continuing with the topic of verification of answer. The aim of the integer check is to first find the integer sums of all the numbers to be added and then adding up these integer sums, ***to get the integer sum which should be same as integer sum of the result.*** The principle is integer sum of result obtained should be same as this. The following example using the same set of numbers would illustrate this

								integer sum of numbers
		4	6	7	3	8		1
		3	5'	9'	8'	5'		3
		9'	6	7'	5	4		4
		2	3	1	4	9'		1
		8'	7'	6	8'	9'		2
		5	4	3'	6'	7		7
Grand total	3	4	4	6	8	2	9 is integer sum of 344682	9 is the integer sum of this column

Here you find that the integer sum of result and of the constituents is same, hence the result is verified. For exercise 1.3 do the same and verify your answers using this method.

CHAPTER 2 Techniques For Memorising Multiplication Tables

The most difficult aspect of calculations is memorising the multiplication tables. If not much at least up to 20 should be on the fingertips. If one can as explained previously develop reflexes for multiplying numbers like 13 and 8, 12 and 6 etc, then there is tremendous scope for increasing the multiplication speed by three or four times without using any unconventional method which shall be explained later in this book.

The first thing is to find patterns in the multiplication table. Form a grid of all the numbers up to 10 in rows and columns just as was done for the improvement in addition. After the normal grid is practiced, it should be jumbled up, so that random answers emanate. The cramming up of the table should not be like that of a child who forgets the table at the very first instance of disturbance.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

Go through this, and you shall find are certain patterns, which can be remembered to have no confusions regarding these basic one digit multiplication. If you are very quick at small additions and multiplication then it may not be really important remember the tables up to 20. As 13 x 8, 12 x 6, 14 x 9 can be easily be derived. But it is always an advantage to remember these two-digit multiplications also. Complete the exercise in the stipulated time.

EXERCISE 2.1 Form this grid and complete without erasing any answer. Check your time (without any mistake)

Time 75 seconds- 1st try
60 seconds- 2nd try

	4	2	9	7	5	6	3	1	8	10
9										
4										
3										
2										
5										
6										
10										
8										
1										
7										

SOME PATTERNS

Look at the first digits of table of 9, they are increasing from 0 to 9.

The second digit is decreasing from 9 to 0. So no need to memorize this table just write the numbers

0	1	2	3	4	5	6	7	8	9
9	8	7	6	5	4	3	2	1	0

There you have the table of nine

By using the famous deductive technology of mathematical learning. Add to all digits in table of 5 in increasing order starting from top – 2,4,6,8,10,12,14,16,18,20 to get the table of 7.

Here is something for multiplying nine by seven. Count the seventh finger from the right and bend it:

There are six fingers to the right of the bent finger, and three fingers to the left. So we have 9 X 7=63.

Some simple facts

- To multiply a number by five, multiply a half of that number by ten
- When you multiply a number by two, you just add the number to itself
- To multiply numbers that differ by two, multiply the number between them by itself and subtract one.

Interesting property of table of 8

	Integer sum	
8 x 1=8	8	**8**
8 x 2=16	1+6=7	**7**
8 x 3=24	2+4=6	**6**
8 x 4=32	3+2=5	**5**
8 x 5=40	4+0=4	**4**
8 x 6=48	4+8=12=1+2=3	**3**
8 x 7=56	5+6=11=1+1=2	**2**
8 x 8=64	6+4=10=1+0=1	**1**
8 x 9=72	7+2=9	**9**
8 x 10=80	8+0=8	**8**
8 x 11=88	8+8=16=1+6=7	**7**
8 x 12=96	9+6=15=1+5=6	**6**

Do you see the pattern, sequence 9 to 1 repeats.

Similarly for table of 6

	Integer sum		
6 x 1=6	6	**6**	
6 x 2=12	1+2=3	**3**	
6 x 3=18	1+8=9	**9**	Do you see
6 x 4=24	2+4=6	**6**	the pattern,
6 x 5=30	3+0=3	**3**	6,3,9 repeats.
6 x 6=36	3+6=9	**9**	
6 x 7=42	4+2=6	**6**	Its an easy
6 x 8=48	4+8=12=1+2=3	**3**	way to
6 x 9=54	5+4=9	**9**	remember.
6 x 10=60	6+0=6	**6**	
6 x 11=66	6+6=12=1+2=3	**3**	
6 x 12=72	7+2=9	**9**	

CHAPTER 3 Speeding up Basic Multiplication

Well if the multiplication tables can be memorized, there is nothing more convenient. But it does remain a bottleneck for most of us and sometimes a miss in the cramming sequence may lead to disastrous results.

In this chapter, some very nice skills for small number multiplication are being discussed which help in reducing speed,.

There are rules of multiplication for numbers starting from 3 to 12, but few of them are so complex that it is better to follow the conventional method.

- ***The first thing to note is that add a 0 to the left side of the multiplicand.***
- ***The second thing is half of an odd number is unit value only, i.e half of 3 is 1, of 5 is 2 etc.***

RULE OF 6

The rule is to each number add half of the right-hand side digit. In case the digit is odd, add 5 to half of the right-hand side digit.

Add 0 to the left side of 6892

06892 x 6	S T E P S
2	2 is the first number and it has no neighbour, so write 2
52	9 is second number and odd, hence add 5 and half of 2, i.e. we have 15, write 5, carry over 1
352	8 is third number, add half of 9 which is 4, and the carry over 1, i.e. 8+4+1=13, write 3, carry over 1
1352	6 is the fourth number, add half of 8, i.e. 10 add carry over 1, to get 11, write 1, carry over 1
41352 **FINAL ANSWER**	IMPORTANT: 0 is to be considered part of number, hence to 0 add carry over 1 and half of 6, i.e. 3. Write 4

EXERCISE 3.1 **Multiply following numbers with 6 using above rule**

Time 30 seconds

1. **84622**
2. **57793**
3. **568903**
4. **900864**

RULE OF 7

The rule is to *double* each number add half of the right-hand side digit. In case the number is odd, add 5 to half of the right-hand side digit.

Add 0 to the left side of 6892

06892 x 7	**S T E P S**
4	2 is the first number and it has no neighbour, so double 2,write 4
44	9 is second number and odd, double 9 to get 18, add 5 and half of neighbour 2, i.e. 18+5+1=24, write 4 and carry over 2
244	8 is third number, double 8,add half of 9 which is 4, and the carry over 2, i.e. 16+4+2=22, write 2, carry over 2
8244	6 is the fourth number, double 6, add half of 8, add carry over 2, i.e. 12+4+2=18 write 8, carry over 1
48244 FINAL ANSWER	IMPORTANT: 0 is to be considered part of number, hence to 0 add carry over 1 and half of 6, i.e. 3. Write 4

EXERCISE 3.2 **Multiply following numbers with 7 using above rule**

Time 40 seconds

1. **34662**
2. **97093**
3. **569763**
4. **306884**

RULE OF 5

The rule is to add half of the right-hand side digit. In case the number is odd, add 5 to half of the right-hand side digit.

Add 0 to the left side of 6892

06892 x 5	**S T E P S**
0	2 is the first number and it has no neighbour, so 0
60	9 is second number and odd, add 5 to half of neighbour 2, i.e. 5+1=6, write 6
460	8 is third number, add half of 9 which is 4
4460	6 is the fourth number, add half of 8
34460 **FINAL ANSWER**	IMPORTANT: 0 is to be considered part of number, hence to 0 add half of 6, i.e. 3.

EXERCISE 3.3 **Multiply following numbers with 5 using above rule**

Time 20 seconds

1. **34662**
2. **97093**
3. **569763**
4. **306884**

RULE OF 11

This is a slightly different rule.

1. **The last number of the number to be multiplied is written below as right hand digit of answer.**

2. **Each successive number of the multiplicand is added to its right side number.**

3. **The left hand digit of the multiplicand is placed as the first digit of answer**

06892 x 11	**S T E P S**
2	2 is the first number and written as it is.
12	9 is second number , adding 2 to it gives 11, write 1 and carry over 1
812	8 is third number, adding 9 and carry over, we have 8+9+1=18, write 8 and carry over 1
5812	6 is the fourth number, add 8 plus carry over of 1 to get 15, write 5 and carry over 1
75812 **FINAL ANSWER**	IMPORTANT: 0 is to be considered part of number, hence to 0 add 6 and carry over 1 to get 7.

EXERCISE 3.4 **Multiply following numbers with 11 using above rule**

Time 20 seconds

1. **34662**
2. **97093**
3. **569763**
4. **306884**

RULE OF 12 **The rule is to add double each digit and add to its right side neighbour.**

06892 x 12	**S T E P S**
4	double 2 and write 4 as it has no right side neighbour.
04	double 9 and add 2 , to get 18+2=20, write 0 and carry over 2

704	double 8, adding 9 and carry over of 2, we have 16+9+2=27, write 7 and carry over 2
2704	double 6, add 8 plus carry over of 2 to get 12+8+2=22, write 2 and carry over 2
82704 FINAL ANSWER	IMPORTANT: 0 is to be considered part of number, hence to 0 add 6 and carry over 2 to get 8.

EXERCISE 3.5 Multiply following numbers with 12 using above rule

Time 30 seconds

1. **34662**
2. **97093**
3. **569763**
4. **306884**

There are some rules for multiplication of 3,4,8 and 9 also, but they are complex and the conventional method is better.

A nice way to multiply a number by 9 is to write a zero on right side of the number and subtract the number itself from this.

For example,

42671 x 9, add 0 to 42671, to get 426710, and now

$$\begin{array}{r} 426710 \\ -\ 42671 \\ \hline 384039 \end{array}$$

EXERCISE 3.6 Using above rules calculate

Time 180 seconds

1. 34567 x 9
2. 798324 x 11
3. 149675 x 5
4. 577439987 x 5
5. 400987 x 12
6. 432908 x 11
7. 39753 x 6
8. 665778 x 6
9. 900872 x 7
10. 75409006 x 12

The next chapter will deal with higher order multiplication techniques. It is advised to keep practicing till the stipulated time against each exercise is achieved.

CHAPTER 4 Multiplication of two digits

After you have achieved a certain proficiency in multiplication of small digits, it is now time to understand certain techniques for multiplying two digits by two digits.

One technique is crosswise multiplication. The Steps are

1. Multiply the left hand digits and right hand digits. Put them down in the extreme ends respectively.
2. Cross-multiply the digits and put down their sums, carry over to left side digit if required.

43		
x 54		
20	**put down 4 x 5 =20**	*step 1*
12	**put down 3 x 4=12**	*step 2*
31	**multiply cross wise 4 x 4 and 3 x 5 total = 16+15=31**	*step 3*
2322	**add carry over 1 of 12 to 31, write 2 add carry over 3 to 20 , to get 23 thus the answer is obtained**	

This may seem a bit complex at the first instance, but after some practice it gets so easy that you don't even need to write the steps and the correct answer is obtained instantly.

EXERCISE 4.1 Find the products mentally

Time 30 seconds

1. **12 x 13**
2. **67 x 77**
3. **54 x 67**
4. **12 x 14**
5. **53 x 67**
6. **90 x 80**
7. **22 x 11**
8. **87 x 12**
9. **90 x 56**
10. **30 x 89**

NUMBERS CLOSE TO BASE

There are certain cases when the numbers are close to a base. Base means numbers like 10, 100, 1000 etc. There is a very simple way to calculate the product then

The first thing to understand is the COMPLEMENT from a base. We shall be dealing with only two digit numbers in this chapter; hence for all cases the base will be 100.

Complement of a number from a base is the amount by which the number is less or more than the base. For 97, complement is -3 as 100-3=97, for 82 complement is -18 as 100-18=82.

RULE

1. Find the complement of the numbers and write it on right side.
2. Multiply the complements and write down in tens and units form, i.e. 2 should be written as 02.
3. Cross add (since the complement is negative it amounts to subtraction) the complement from the main number; any number can be used as the answer will be same.

Following example would illustrate this,

97	**-3**	100-97=3, complement is 3
x 96	**-4**	100-96=4, complement is 4
	12	Multiply 3 x 4 = 12
93		97+(-4)or 96+(-3) both are equal to 93, write down 93
9312		Thus 9312 is the answer

Let us take another example,

91	**-9**	100-91=9, complement is 9
x 86	**-14**	100-86=4, complement is 14
1 carry over	**26**	Multiply 9 x 14 = 126, write 26 and 1 is carry over
77		86-9=77 write down 77, add 1 to 77
7826		Thus 7826 is the answer

EXERCISE 4.2 Find the products using the complement rule.

Time 30 seconds

1. 98 x 89	**2. 88 x 97**
3. 89 x 99	**4. 95 x 85**
5. 99 x 97	**6. 94 x 93**

Now we shall understand multiplication of numbers that are above the base or ***one is below and other above the base.***

The Rule is same except that the sign is important in complement and there is a slightly different way of arriving at the answer. If the complement is for a number less than the base it is negative written with a dash. In case it is for a number above base, it is written plainly.

Consider this example

91	**-9**	91-100=-9, complement is -9
x 106	**6**	106-100=6, complement is 6
	-54	**ATTENTION HERE** Multiply -9 x 6 = -54
97	**00**	91+6=97, **Attention** put down two zeroes to identify as hundreds.
9646		Now add 9700 to -54, or 9700-54

One more example would suffice in understanding this technique

104	**4**	
x 111	**11**	
	44	4 x 11=44
115	**00**	104+11=115
11544		Place 115 and 44 together

See how simple it becomes, for these particular cases. A definite question in your mind would be what about numbers that are not close to the base. This of course is one deficiency of this technique but if the numbers are close to each other there is a way out.

To calculate numbers away from base we can use multiples or sub multiples of the bases.

IMPORTANT TIP

Whatever base is used 10, 100 or 1000, the calculations involving the complements restrict to number of digits of base. To illustrate, in base of 10, 27 would mean 7 and carry over 2, but in base 100 it would mean 27 and in base 1000 it would mean 027.

With the above knowledge let us calculate

using base 10

8	-2
x 9	-1
7	2

using base 100

99	-1
x 98	-2
97	02

If you have understood what has been done, then proceed further otherwise please go back to previous sections.

Suppose you were given to find 48 x 47, both base 10 and base 100 will prove more cumbersome than the direct method. Now this calculation will be illustrated using both multiple of 10 and sub multiple of 100.

Remember only the left-hand side of the number will have to be multiplied or divided by the multiple. ***The right side that is the product of the complements will remain unchanged.***

Let us take the base as 50 = 100/2

	48	-2
	x 47	-3
	45	06
divide 45 by 2 as 50 =100/2	22.5	
Answer is 2256 **22.5 is actually 22.5 x 100=2250, add 06 to this**	2256	

Let us take another example

	41	-9
	x 46	-4
41-4=37	37	36
divide 37 by 2 as 50 =100/2	18.5	
Answer is 1886 **18.5 is actually 18.5 x 100=1850, add 36 to this**	1886	

Now using 10 x 5 = 50 base for same numbers

	41	-9
	x 46	-4
41-4=37	37	36
write 6 as units digit with 3 as carry over		6
multiply 37 x 5 as 50 =10 x 5	185	
Add 3 to 185 and get 188, so answer is	1886	

Here are some quick examples without full explanation but self evident

base 10 x 7	
67	-3
69	-1
66	3
66 x 7=	**4323**
432	

base 10 x 4	
47	7
39	-1
46	-7
46 x 4=	**1840-7**
184	**1833**

base 10 x 3	
31	1
29	-1
30	-1
30 x 3=	**900-1**
90	**899**

base 10 x 2	
19	-1
18	-2
17	2
17 x 2=	**342**
34	

EXERCISE 4.3 Find the products using the above rules.

Time 60 seconds

1. 102 x 104	**2. 78 x 79**
3. 109 x 87	**4. 60 x 58**
5. 65 x 72	**6. 102 x 89**
7. 87 x 91	**8. 45 x 48**
9. 21 x 25	**10. 54 x 53**
11. 31 x 28	**12. 12 x 16**

ALGEBRAIC EXPLANATION OF THE RULE

$$(x+m)\times(x+n)=x^2+mx+nx+mn=x(x+m+n)+mn$$

It is actually so easy to perform multiplication,
x+m+n is what we do in the second step.
m.n is achieved by multiplying the complements.

There is no trick, only a brilliant use of an algebraic expression.

In the next chapter we shall consider cases of higher order multiplication in varying degrees of complexity.

CHAPTER 5 Higher Order Multiplication

Now we shall proceed to understand higher order multiplication. The principle used is same. The base order is 1000 or multiple/sub multiple of 1000 and even higher. Also an altogether new technique for multiplication of numbers that are not close to each other will be explained.

Base is 1000		991	-9
		x 986	-14
	991-14=977	977	126
so answer is		977126	

Base is 7 x 100		706	6
		x 691	-9
	691+6=697		-54
This step is important	697x 7= 4879		487900 (as base is 100)
	Answer is		487846

Some quick examples, please observe carefully. The details are purposefully hidden so that you can understand yourself.

base 100 x 5

467	-33
489	-11
456	3/63
456 x 5	
=2280	**228363**

base 1000 / 2

509	9
497	-3
506	-27
506/2	**253000**
=253	**-27**
	=252973

base 100 x 7

147	-553
689	-11
136	60/83
136 x 7=	**95200**
952	**+6083**
	=101283

Still simpler !

base 100 x 5

499	-1
18	-482
17	482
17 x 5=	**8982**
85	

base 100 x 6	
489	-111
612	12
501	-1332
501 x 6 =	**300600**
3006	**-1332**
	=299268

base 100 x 9	
890	-10
918	18
908	-180
908 x 9=	**817200**
8172	**-180**
	=817020

base 100 x 5	
499	-1
18	-482
17	482
17 x 5=	**8982**
85	

base 100 x 3	
303	3
312	12
315	36
315 x 3 =	**94536**
945	

It is seen that the products above are all close to each other, except for a few cases where the second number is small and just a little bit of lateral thinking enabled us to simplify the calculation.

EXERCISE 5.1 Find the products

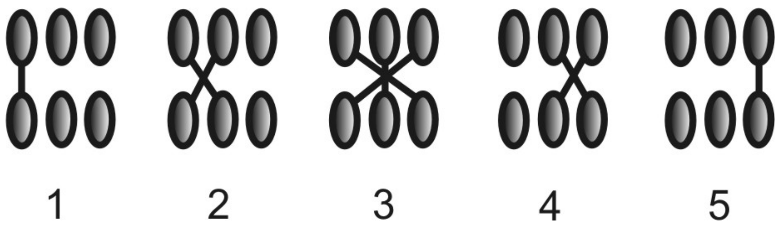
Time 90 seconds

1. 912 x 913
2. 667 x 707
3. 544 x 567
4. 812 x 813
5. 293 x 307
6. 907 x 880
7. 220 x 211
8. 898 x 12
9. 390 x 406
10. 309 x 289

Although the conventional technique of multiplication is acceptable and reliable but the following technique when mastered can turn the reader into a master in calculation without using a line more than just the answer. But this requires considerable practice and perfection as one error could lead to disastrous results and ***a lot of embarrassment***.

Let us begin with the smallest example. This method is only a reorder of the conventional technique, but is extremely useful in reducing the time.

Vertically and Crosswise multiplication



The above diagram depicts the procedure for vertically and crosswise multiplication.

- The two numbers should be written one above the other. Start from the right side of the number, which is the 5th step in the diagram.
- Multiply both the right side members and write down the unit digit and carry over.
- Now multiply crosswise as the 4th step and add the products, write down the unit digit and carry over the ten's digit.
- Now look at the 3rd step in the figure, crosswise multiply the extreme end digits and then add the vertical product of the middle digits.
- 1st and 2nd steps are self-evident.
- This is the one line product; remember to add the respective carryovers to the left side neighbouring digit.

The following example would clearly illustrate this

	3 5 7	
	x 7 3 2	
1st step 2 x 7 = 14	**21 4 0 1 4**	
2nd step 2 x 5 + 3 x 7 = 31	4...7...3...1.....	carry overs
3rd step 3 x 2 + 7 x 7 + 5 x 3 = 70		
4th step 3 x 3 + 5 x 7 = 44	**2 6 1 3 2 4**	Final result after adding carryovers
5th step 3 x 7 = 21		

Let us take another example

	2 9 6	
	x 4 5 7	
1^{st} step 6 x 7 = 42	**8 6 3 3 2**	
2^{nd} step 9 x 7 + 5 x 6 = 93	4...8...9...4.....	carry overs
3^{rd} step 2 x 7 + 6 x 4 + 9 x 5 = 83		
4^{th} step 2 x 5 + 9 x 4 = 46	**1 3 5 2 7 2**	Final result after adding carryovers
5^{th} step 2 x 4 = 8		

EXERCISE 5.2 Find the products in one line. Do all the additions mentally

Time 180 seconds

1. **678 x 346**
2. **294 x 498**
3. **145 x 768**
4. **380 x 189**
5. **674 x 350**
6. **987 x 876**
7. **452 x 469**
8. **961 x 798**
9. **190 x 781**
10. **129 x 459**

CHAPTER 6 Simple Division

To understand the technique of a division, we shall first look at the following examples of division by nine. Before we proceed let us remind us of the terms,

Dividend this is the number which is to be divided
Divisor this is a number which would divide.
Quotient the whole number whose product with divisor is closest to dividend.
Remainder the remaining part of the dividend after division.

divisor) ***dividend*** (quotient

remainder

dividend = (divisor x quotient) + remainder

AN INTERESTING PROPERTY

From the Ex 1 below⇒ 12 = (9 x 1) + 3

9)1/2	9)2/1	9)7/0	9)6/2
../1	../2	../7	../6
1/3	2/3	7/7	6/8
Ex 1	**Ex 2**	**Ex 3**	**Ex 4**

The first step is to divide the dividend in part with a / (slash). An interesting property emerges in the dividend.

The left part becomes the quotient and sum of the two numbers becomes the remainder.

Consider Ex 1; 12 divided by 9, place a slash between 1 and 2. Now write 1 below 2, add them to get 3. Write it below the line and then copy 1 below the line with a slash in between. Thus 1 is the QUOTIENT and 3 is the REMAINDER

Ex 4; 62 ÷ 9, write 6 below 2 , add 2 + 6 = 8 , thus 6 is quotient and 8 remainder

Using this basic property the division rules shall be deduced. Let us also see if the same property works for higher order numbers. The point to be kept in mind here is the left part of the slashed dividend has to be divided by 9 and then written below the original dividend.

```
9)20 / 5
 ..2 / 2
 -------
  22 / 7
```

In this case split the dividend into two parts, since a two digit number 20 is the left part, using the previous method, write down quotient and remainder of 20 below the number with a usual slash as depicted. Add up the right side and left side to get the final result.

One more example

```
9)14 / 7
 ..1 / 5
 --------
  15 / 12
  16 / 3
```

In this case split the dividend into two parts, since a two digit number 14 is the left part, using the previous method, write down quotient and remainder of 14 below the number with a usual slash as depicted. Add up the right side and left side to get the final result.

But here as you must be seeing 12 can certainly not be a remainder, divide 12 by 9 and add 1 to 15 and 3 is the remainder. So final answer is 16 quotient and 3 remainder.

The general rule is if the remainder is same or greater than the divisor, redivide the remainder by 9, carry the quotient over to the quotient column/side and retain the result which is final remainder.

RULES FOR DIVISION BY NEAR TO BASE METHOD

1. Decide on the base, for two digit numbers it is 100, three digit numbers 1000.
2. Find the complement of the divisor from this base i.e. for 89, 11 is the complement from 100. Similarly for 996, 4 is the complement from 1000.
3. Place a slash between the dividend depending on the base, thumb rule is slash should be placed as per the number of zeroes in the base, i.e. if base is 1000, place slash three digits from right.
4. Multiply the complement with quotient and follow the same rules.

```
8)4 / 7
 .. / 8
 -------
  4 / 15
  5 / 7
```

2 is the complement of 8

Thus 4x2=8 is written below 7

But 15 is higher than 8, the divisor, so redividing it by 8 we get 5 as quotient and 7 as remainder.

A better way to remove confusion during division is to write the complement below the divisor. Let us look at another example.

73) 1/15 27 ../27 —————— 1/42	To divide 115 by 73 Write 27 the complement of 73 from 100 Place a slash, two digit from right Multiply 27 x 1 and add to 15.

Let us look at some more examples.

888) 1/149
112   ../112
——————
     1/261
	7998) 1/3618
	2002 ../2002
	——————
	     1/5620
		993) 2/159
		7 .../014
		——————
		     2/173

EXERCISE 6.1 Using the rules explained above, solve the following

1. 678÷9
2. 254÷9
3. 119÷89
4. 2245÷997
5. 14567÷8998
6. 5467÷887
7. 32345÷8888
8. 4532÷991
9. 394567÷89997
10. 447÷86

All the above examples would have made the basic method clear. Now we shall seek to understand division when the dividend is much larger than divisor and ***there is more than one digit on the left side of the slash.***

Divide 106 by 7	7) 10 /6
Complement of 7 , 3 is written below 7	3
The first digit of the dividend is brought down to the answer	7) 10 /6 3 ____ 1 /
The first digit of the quotient,1 is multiplied by complement, 3 and answer is placed below the next dividend digit	7) 10 /6 3 3 ____ 1 /
The second column is added up 0+3=3, this is the second quotient digit.	7) 10 /6 3 3 ____ 13 /

The Final step is to multiply this new quotient digit by the complement, i.e. 3x3=9, which is placed after the slash. This column is added up to get the remainder.	7) 10 /6 3 3 /9 ――― 13 /15
But 15 obviously cannot be the remainder, when divided by 7 it gives 2 and remainder 1, add this quotient. Thus the final result is 15 as quotient and 1 as remainder.	7) 10 /6 3 3 /9 ――― 15 /1

Let us take another example

Divide 10113 by 88	88) 101 / 13
Complement of 88 , 12 is written below	12
The first digit of the dividend is brought down to the answer	88) 101 / 13 ――― 1 /
The first digit of the quotient,1 is multiplied by complement, 12 and answer is placed below the next dividend digit	88) 101 / 13 12 ――― 1 /
The second column is added up 0+1=1, this is the second quotient digit.	88) 101 / 13 12 ――― 11 /
This second quotient digit is again multiplied by complement, 12 and written below one digit being after the slash.	88) 101 / 13 12 1 / 2 ――― 114 /
The third quotient digit is now multiplied by complement and written below the next two digits of the dividend. Add up all the right side numbers for remainder REMEMBER ***the partial result is placed by shifting towards the right side by one digit after each step.***	88) 101 / 13 12 1 / 2 / 48 ――― **114 / 81**

Let us look at another simple example, try to figure out what has done

8997) 20 / 1151 1003 ..2 / 006 ..../ 2006 22 / 3217	• 1003 is the complement • Slash is placed 4 digits from right as 10000 is the base • 1003 x 2 = 2006 is written first • 0+2 is the second digit of quotient • Again 1003 x 2 = 2006 is written shifting the one digit towards right • Adding up for remainder

There is one very important thing to understand and the following example would clearly illustrate this. The reader may get confused on what to do when the addition ***results in more than one digit in quotient and when it is more than one digit in remainder.***

4363÷9 4 is the 1st digit of quotient 4x1=4 , 4+3=7 is the 2nd digit 7x1=7 , 6+7=13 is the 3rd digit. IMPORTANT Here the conventional rule applies for quotient. Carry over 1 to next column and 2nd digit becomes 7+1=8 But careful for the remainder 13x1=13 is placed after the slash and remainder become 3+13=16 and not 3+13=43 ***Here the rule is different, 16 should be redivided by the divisor 9 to get 7 as remainder and 1 is added to quotient digits 483 to get 484 as result***	9) 4 3 6 / 3 4 / 7 / /13 4 7 ***13*** / ***16*** 4 8 3/ 7 1 **4 8 4/ 7** **Final result**

EXERCISE 6.2 Using the rules explained above, solve the following

1. 23456÷89
2. 121237÷997
3. 125467÷887
4. 314578÷987
5. 121679÷898
6. 2104569÷9892

One very obvious question would be what about numbers above the base like 11, 109, 1007 etc. For such numbers method is same except that the complement is actually surplus and is called ***surplus.*** For 11, surplus is 1 as it is 1 above 10 the base.

After all this while, you must be wondering that these rules are brilliant but only for very specific cases where the divisor is close to some base. There is another rule, which can substantially reduce time for division by any number irrespective of the base. In competitive examinations and normal school examinations, the level of difficulty in division is not very high and commonly it is these small two digit or three digit divisors that pose a problem. The following technique in combination with above techniques provides a very simple and safe system for division. You only have to add or subtract and not be involved with big multiplication, which is another headache.

This new technique will be explained in the next chapter.

CHAPTER 7 More Division

The general method for division taught conventionally is called the straight division method. It covers all possibilities. There is one superb way of easing calculations with some mental arithmetic and no big multiplication etc.

Let us start with a very simple example of 467÷32

First step is to set out is the first digit of the divisor and write it on top of the remaining number, in this case 2 called the flag digit is written on top of 3. The remainder stroke will be placed one digit from right, as the number of FLAG digits is also one.	**2** **3 | 4 6 / 7**
Now only 3 shall be used for dividing. **4÷3 gives us quotient 1 and remainder 1.** Place the quotient below the line as the first digit of main quotient. Place the remainder between 4 and 6 in the next line.	**2** **3 | 4 6 / 7** **1** **1**
<u>IMPORTANT</u> Using the flag digit 2, subtract from 16 the product of this flag digit 2 and the previous quotient 1. **16 - {2(flag digit) x 1 (previous quotient)} =14** **This is now the new dividend. Divide 14 by 3 to get quotient 4 and remainder 2.** Place them as before 4 below the line and 2 between 6 and 7.	**2** **3 | 4 6 / 7** **1 2** **1 4**
We are now left with the remainder portion. Here we do not divide but merely find the remainder by subtracting the product of previous quotient digit and flag digit. **27 - {2(flag digit) x 4 (previous quotient)} =19.** **This is the final remainder.**	**2** **3 | 4 6 / 7** **1 2** **1 4 / 19**

It is heartening to find that without much to multiply and simple addition and subtraction with a set of rules we can divide any number by any number. There is a simple algebraic proof behind this, which is not very important to know as long as the procedure is understood.

SUMMARISING THE RULES FOR STRAIGHT DIVISION

- ✓ Set out the flag digit. The flag digit is obtained by removing the first digit of the divisor. For example, for 35 flag digit is 5 and in case of 367 flag digit is 67, after removing 3.
- ✓ The number of the flag digits indicates the number of digits to be placed after the remainder stroke.
- ✓ The first digit of the divisor, called modified divisor, does the dividing.
- ✓ The first digit of the quotient is obtained by dividing the first number of the dividend by the modified divisor.
- ✓ This quotient be placed below the line and the remainder just before the next number of the dividend.
- ✓ The new dividend is obtained by combining the remainder with the next digit, subtract from this digit the product of the flag digit and the previous quotient.
- ✓ Repeat this procedure till the / mark.
- ✓ The remainder is obtained by subtracting the product of the flag digit and the immediate quotient from the combined number.

Another example would depict the procedure clearly; we shall use a single line.
Find 98613 ÷ 63

- ✓ First find the flag digit which is 3
- ✓ Mark the slash, one digit from right side between 1 and 3

6^{3}	9	8	6	1	/	3
		3	5	5		3
	1	5	6	5	/	18
Steps	9÷6 q=1 r=3	38-(3x1) =35 35÷6 q=5 r=5	56-(3x5) =41 41÷6 q=6 r=5	51-(3x6) =33 33÷6 q=5 r=3		Remainder column 33-(3x5) =18

here, **q** depicts quotient and **r** is remainder.

Picking up another example. 178291÷83

- ✓ The flag digit is 3
- ✓ Slash to be placed before 1

8^{3}	1	7	8	2	9	/	1
			$_{1}$ 8	$_{4}$ 2	$_{7}$ 9	/	$_{3}$ 1
		2	1	4	8	/	7

Steps	17÷8 q=2 r=1	18-(3x2) = 12 12÷8 q=1 r=4	42-(3x1) =39 39÷8 q=4 r=7	79-(3x4) =67 67÷8 q=8 r=3	Remainder column 31-(3x8) =7

EXERCISE 7.1 Using the rules explained above, solve the following

1. 423÷21
2. 651÷31
3. 4911÷32
4. 1668÷54
5. 8466÷64
6. 59266÷53
7. 86369÷76
8. 38982÷73
9. 34632÷52
10. 59266÷83

ALTERED REMAINDER

Observe the following division using this method

6013÷76

7^{6}	6	0	1	/	3
			$_{4}$ 1	/	3
		8			

Steps	60÷7 q=8 r=4	41-(6x8) =41-48 which is -7

It is found that that in the next step the result is negative. This would be happening many times during divisions.

For cases like these the altered remainder is used, which is simply like this

60÷7 q=8, r=4
or q=7, r=11 here 7x7=49 and 60-49=11

This is called alteration of remainder.

The quotient is reduced by 1, and resulting remainder is picked for further division.

Now we shall proceed with the same problem using the altered remainder.

7^{6}	6	0	$_{11}$1	/	$_{6}$3
		7	9	/	9
Steps	**60÷7 q=7 r=11**		**111-(6x7) =111-42 = 69 69÷7 q=9 r=6**		**Remainder column 63-(6x9) =9**

Note : quotient is reduced from 8 to 7 in first step and new remainder 11 is carried over to form next number 111

Another example,

6^{6}	1	5	$_{3}$7	$_{7}$0	/	$_{10}$7
		2	3	7	/	65
Steps	**15÷6 q=2 r=3**		**37-(6x3) =25 25÷6, normally q=4, r=1 but r has to be modified q=3,r=7**	**70-(6x3) =52 52÷6, normally q=8, r=4 here also r needs to be modified q=7,r=10**		**Remainder column 107-(6x7) =65**

EXERCISE 7.2 Using the rules explained above, solve the following

1. 3412÷24	2. 8218÷33
3. 1408÷23	4. 4143÷18
5. 4919÷11	6. 5711÷54
7. 9678÷59	8. 6712÷53
9. 9836÷21	10. 601325÷76

DIVISION BY TRANSPOSE AND ADJUST

Using the Remainder Theorem And Horner's Process Of Synthetic Division, one very easy method for division of numbers **close to bases** has been devised.

Before we proceed let us review some concepts

Surplus and deficient.

Base 10

11 has	1 surplus
8 has	2 deficient

Base 100

109 has	9 surplus
89 has	11 deficient

For this method surplus is prefixed with – as it has to be subtracted.

The principle behind this is **GET TO BASE**.

To get 11 to base i.e. 10, we have to add -1 to 11; 11+(-1)=10

Similarly to get 98 to base i.e. 100, we have to add 2 to 98; 98+2=100

Let us start with a very simple example,

1467÷111

here the divisor 111 has 11 surplus, so this is written as -1 -1 below 111.

Since the base is 100 hence slash is marked before the two digits from right.

PROCEDURE

The method is same as explained in chapter 6, here also the complement/surplus is written below the divisor.

- The left most digit is written below the line
- The surplus is multiplied by this partial answer and placed below from the second digit from left onwards (please see the example below)
- The second part is added up to get the partial quotient and the surplus is multiplied with this result.
- The process is carried on till the last digit is covered.
- The normal rules for addition are used and the quotient and remainder are the numbers left and right of slash respectively.

The following examples will explain the method and some special cases.

1 1 1	**1**	**4**	**/ 6**	**7**
-1 -1		**-1**	**-1**	
			-3	**-3**
	1	**3**	**/ 2**	**4**

Thus 13 is the quotient and 24 the remainder.

Another example, 16999÷112

1 1 2	**1**	**6**	**9**	**/ 9**	**9**
-1 -2		**-1**	**-2**		
			-5	**-10**	
				-2	**-4**
	1	**5**	**2**	**/ -3**	**5**
			-1	**+11**	**2**
	1	**5**	**1**	**/ 8**	**7**

Here a strange case has emerged, negative remainder. There is a simple rule; it actually means we have ***over divided*** the dividend. Therefore reduce the quotient by 1 and add the divisor to the remainder.

Reduce 152 by 1 to have 151
Add 112 in the remainder column as shown, +11 with first digit and +2 with second digit.
Thus answer is q = 151 and r = 87.

Let us take another example,

1 1 3 2 / -1 -3 -2					
	1	2	/ 3	4	9
		-1	-3	-2	
			-1	-3	-2
1st	1	1	/ -1	-1	7
2nd		-1	+11	+3	+2
3rd	1	0	/ 10	2	9

Here again remainder had two negative digits, reduce 11 by 1 to get 10.
Add +11, +3and +2 to remainder columns respectively.

Thus q =10 , r =1029

Isn't this wonderful. Although this technique applies to specific cases but in most of the competitive examinations, the question setter ensures that the division remains as simple as possible. And for most of the cases these techniques would really help in reducing time for calculation.

EXERCISE 7.3 Using the rules explained above, solve the following

1. 1234÷112
2. 1241÷112
3. 1234÷160
4. 239479÷11203
5. 12789÷1104
6. 27868÷1211
7. 21188÷1003
8. 25987÷123
9. 15542÷110
10. 189765÷1221

DIVISION BY TRANSPOSE IF NUMBERS ARE LESS THAN BASE

Let us consider the number 819, the deficiency is 181, which means we use + sign and division by transpose and adjust method can be done.

8 1 9 / 1 8 1				
	1	/ 4	6	7
		1	8	1
	1	/ 5	14	8
	1	/ 6	4	8

This case was simple but big number 8 is involved. this can be simplified by judicious use of surplus and deficient. 819 is 1000 - 181 or (200-20+1), thus 181 can be written as 2 -2 1

8 1 9		1	/ 4	6	7
2 -2 1			2	-2	1
		1	/ 6	4	8

Let us see another example

49999÷9891

in this case 9891 is 10000-109 and 109 can be expressed as (100+10-1) or 1 1 -1, easing the calculation tremendously. That is a bit of lateral thinking.

9 8 9 1	4	/ 7	9	9	9
0 1 0 9		0	4	4	-4
0 1 1 -1					
1^{st}	4	/ 7	13	13	5
2^{nd}	4	/ 8	4	3	5

IMPORTANT: The number of operators is same as number of zeroes in the base.

For example in the previous example if you start working with only 1 1 –1 ignoring the 0 the whole result would be wrong. Be careful in such cases.

Notice that in 3rd and 4th column the additions result in larger than 10 numbers. The rule is ***add the carry over to the preceding column number starting from right side***. But remember each column is a unit, ten, hundred and so on, so the addition has to follow the same rule.

For example if 13 of 3rd column is to added by carry over 1 from 4th column, it will be like this

1 3
  1

IMPORTANT: Consider each column separately do not add in the conventional manner.

Suppose some addition leads to a negative number, then the result should be subtracted from preceding column considering its place value i.e adding a zero to it. For example if in 3rd and 4th column results in remainder row are 3 and -4 respectively, then the final column would result in 30-4 = 26 and not -1, because 3rd column's place value is 10 more.

To illustrate this let us consider another example,

Here 8829 = 10000 - 1171 and 1171 can be expressed as (1000+200-30+1) or 1 2 -3 1

8 8 2 9	2	/ 5	4	5	3
1 1 7 1		2	4	-6	2
1 2 -3 1					
	2	/ 7	8	-1	5
	Here -1 has to added to 8 x 10 = 80, This remainder row is actually				7000 800 -10 +5
	2	/ 7	7	9	5

Answer is q = 2 and r = 7795

EXERCISE 7.4 Using the rules explained above, solve the following

1. 13044÷988
2. 7101÷878
3. 31883÷879
4. 21002÷799
5. 10091÷883
6. 2683÷672
7. 39979÷9820
8. 20121÷818
9. 134567÷8893
10. 24434÷987

The reader must be wondering on how to choose a particular method. It can be very confusing to first learn all the techniques and then remember when to use what. One very useful way to know which technique to use when is,

Check whether the divisor is close to a base. The technique explained in Chapter 6 is actually the same as Division by transpose and adjust. The only difference is the method adopted. Sometimes the method in Chapter 6 is easier as it is direct. But for higher order numbers the technique explained in this chapter is better.

If the number is not close to the base use the direct division method, which can be applied to any set of numbers.

Let us do an exercise of choosing the method to be adopted.

EXERCISE 7.5 By visual inspection choose the method for division.

1. 13044÷988
2. 7101÷78
3. 39883÷29
4. 21002÷457
5. 401÷88
6. 7683÷78
7. 39979÷1003
8. 60121÷1008
9. 134567÷19
10. 54434÷563

Happy dividing. The best way to decrease the time for calculating is not to try to increase speed. Practice more such problems by making your own problems and checking the answers by reverse method of multiplying the divisor with quotient and adding the remainder to get the dividend.

DIVISOR x QUOTIENT + REMAINDER = DIVIDEND

CHAPTER 8 Quick Squaring

The multiplication techniques explained in Chapter 4 are employed for quick squaring with a slight modification. In most of the competitive examinations the squaring problems are usually not very complex, but calculating the same does take time, which can be reduced by 50% using the techniques, which will be explained now.

All the procedures lead to an increase in calculation speed once adequate practice has been done. The reader may now wonder on how to remember the techniques. Case to case examination will very smoothly orient your mind and the choice would be a reflex action, rather than a very conscious effort.

FOR NUMBERS NEAR TO A BASE

For numbers near to base

1. First find the complement of number from Base, for 97, complement is 3, 100-3, this can be marked as -3, for 1009, complement is 9 from base 1000.
2. Now "add" this complement to the number, i.e. add -3 to 97, to get 94 or add 9 to 1009 to get 1018.
3. Square the complement and place to the right side of the added result. Here it is important to note the base value will decide the number of digits on right side, e.g. in case of 97, complement 3 is squared to get 9 but 9 shall be written as 09 since we are using base 100 which has 2 zeroes.

The following examples will illustrate this technique,

Squaring of 97		
97	**-3**	100-97=3, complement is 3
97+(-3)	**9**	Add -3 to 97 Square -3, add 0 before 9 to make it 2 digit since we are dealing with base 100
94	**09**	
	9409	Thus 9409 is the answer, very straight

Another example,

Squaring of 1009		
1009	**9**	1000-1009= -9 complement is 9
1009+9	**81**	Add 9 to 1009 square 9 add 0 in front of 81 to make it 3 digit
1018	**081**	
	1018081	is the answer

Stepwise explanation was given so that you could understand the procedure, a few examples are solved in one line for illustration (try them once yourself).

98^2 = 98-2/04=9604

86^2 = 86-14/196=72+1/96=7396 (because we are operating in base 100)

989^2 = 989-11/121=978121

104^2 = 104+4/16=10816

1015^2 = 1015+15/225=1030225

79^2 = 79-21/441=58+4/41=6241

965^2 = 965-35/1225=930+1/225=931225(because we are operating in base 1000)

113^2 = 113+13/169=126+1/69=12769(because we are operating in base 100)

9991^2 = 9991-9/0081=99820081

99989^2 = 99989-11/00121=9997800121

EXERCISE 8.1 Find the squares of following numbers

1. 97
2. 108
3. 115
4. 998
5. 899
6. 989
7. 9987
8. 9979
9. 10032
10. 1023
11. 1012
12. 99997

FOR NUMBERS NOT NEAR TO A BASE

For numbers which are not near to base, the base can be chosen as multiple or sub multiple of a decimal base.

For 47, 50=100/2 or 50=10 x 5 can be the base.
For 509, 500=100 x 5 or 100/2 can be the base.
Let us now see how this works.

Suppose you were to find 49^2. Taking base as 50 which is 100/2

Squaring of 49		Take 50 = 100/2 base
49	**-1**	50+(-1)=49
49+(-1)	**01**	Add -1 to 49 Square 1 , write as 01
48 **48/2**	**01**	**IMPORTANT: divide the first part by 2 to get final answer**
	2401	

It is also possible to solve this by using 50=10 x 5

Squaring of 49		Take 50 = 10x5 base
49	**-1**	50+(-1)=49
49+(-1)	**1**	Add -1 to 49 square 1 write as 1
48 **48x5**	**1**	**IMPORTANT: multiply the first part by 5 to get final answer**
	2401	Same as above

By now it must be clear to you that the first part of the sub result has to divided or multiplied depending on base decided as multiple or sub multiple of nearest decimal base.

A few more examples,

32^2 = 32+2 / 4=34x3/4=1024 (choosing 10x3 as base)

19^2 = 19-1 / 1=18x2/1=361 (choosing 10x2 as base)

58^2 = 58-2 / 4=56x6/4=3364 (choosing 10x6 as base)

387^2 = 387-13 / 169=374x4+1/69=149769 (choosing 100x4 as base)

498^2 = 498-2 / 004=496÷2/004=248004 (choosing 1000÷2 as base)

253^2 = 253+3 / 009=256÷4/009=64/009=64009 (choosing 1000÷4 as base)

2021^2 = 2021+21 / 441=2042x2/441=4084441(choosing 1000x2 as base)

4996^2 = 4996-4 / 016=4992x5/016=24960016 (choosing 1000x5 as base)

4997^2 = 4997-3 / 0009=4994÷2/0009=24970009 (choosing 10000÷2 as base)

6992^2 = 6992-8 / 064=6984x7/064=48888064 (choosing 1000x7 as base)

EXERCISE 8.2 Find the squares of following numbers

1. 68	2. 208	3. 315
4. 298	5. 699	6. 789
7. 7987	8. 6987	9. 20032
10. 8023	11. 6012	12. 89997

SQUARING USING DUPLEX

There is yet another method for squaring using the concept of Duplex. In this method the basic formula,

$$(a+b)^2=a^2+2ab+b^2$$

must be remembered.

The Duplex is calculated using two basic rules,

1. For a single central digit, square the digit, a^2
2. For even number of digits or a and b equidistant from ends double the cross product, **2ab**

Don't worry if you haven't understood, the following examples will illustrate this,

2 is a single digit, $D=2^2=4$

43 is double digit, D=2 x (4 x 3)=24

313 is three digit, D= 1^2 + 2 x (3 x 3), as 1 is the central digit and 3 and 3 are equidistant from the ends. D= 19

5314 is four digit, there is no central digit, thus two numbers in center and at the ends are used thus, D= 2 x (5 x 4) + 2 x (3 x 1)=46

13451 is five digit, 4 is central digit thus D= 4^2 + 2 x (1 x 1) + 2 x (3 x 5)=48

Coming back to the method of squaring.

Note: The Square of a number containing n digits, will have 2n or 2n-1 digits.

So, first mark extra dots or space one less than the number of digits for the number to be squared, e.g. for 456 mark 2 dots, for 45678 mark 4 dots.

The Duplex, D has to be calculated for each part of the number to be squared starting from the right side. That is if the number to be squared is 12345, then D has to be calculated using the method explained above for 5, then 45, then 345, then 2345, then 12345 and then 012345 so on till the number of dots which are placed with assumed value of 0.

Getting confused, it is actually simple start from right side, pick up the digits one by one get their D and write them down with the units place on main line and the other digits a line below as depicted.

223^2 , add 2 dots ..223

D for 3 = 9 D for 23 = 2x2x3 = 12 D for 223 = 4 + 2x2x3 = 16

D for 0223 (considering dot as 0) = 2x0x3 + 2x2x2=8

D for 00223 = 2^2 + 2x0x3 + 2x0x2 = 4

All this may seem very cumbersome but it is not. All these calculations can be done mentally and there is no need to write all this. This is being done only to explain the method.

The method is to write the D's from right side like this, keeping the units digit on the main line and carryovers as subscript.

$4\ 8\ {}_{1}6\ {}_{1}2\ 9$,

now add the "carryovers" to the immediate left side digit,

Thus 223^2 = 49729

Let us now solve a few examples in one line,

$335^2 = ..335 \Rightarrow 9\ \ {}_{1}8\ \ {}_{3}9\ \ {}_{3}0\ \ {}_{2}5 \Rightarrow$ adding up we get 112225, so easy

$678^2 = ..678 \Rightarrow 36\ \ {}_{8}4\ \ {}_{14}5\ \ {}_{11}2\ \ {}_{6}4 \Rightarrow 459684$

$2234^2 = \ldots 2234 \Rightarrow 4\ \ 8\ \ {}_{1}6\ \ {}_{2}8\ \ {}_{2}5\ \ {}_{2}4\ \ {}_{1}6 \Rightarrow 4990756$

To understand the method better, solve the above yourself, and check at each stage. ***Do not haste.*** No time is saved if you consciously try to save time in calculations. Concentrate step wise on the problem. ***Remember this is the shortest possible method and no more shortcuts are possible.***

A few more squared numbers,

$347^2 = ..347 \Rightarrow 9\ \ {}_{2}4\ \ {}_{5}8\ \ {}_{5}6\ \ {}_{4}9 \Rightarrow 120409$

$413^2 = ..413 \Rightarrow 16\ \ 8\ \ {}_{2}5\ \ 6\ \ 9 \Rightarrow 170569$

$773^2 = ..773 \Rightarrow 49\ \ {}_{9}8\ \ {}_{9}1\ \ {}_{4}2\ \ 9 \Rightarrow 597529$

$2136^2 = \ldots 2136 \Rightarrow 4\ \ 4\ \ {}_{1}3\ \ {}_{3}0\ \ {}_{2}1\ \ {}_{3}6\ \ {}_{3}6 \Rightarrow 4562496$

$395^2 = ..395 \Rightarrow 9\ \ {}_{5}4\ \ {}_{11}1\ \ {}_{9}0\ \ {}_{2}5 \Rightarrow 156025$

$849^2 = ..849 \Rightarrow 64\ \ {}_{6}4\ \ {}_{16}0\ \ {}_{7}2\ \ {}_{8}1 \Rightarrow 720801$

$3247^2 = \ldots 3247 \Rightarrow 9\ \ {}_{1}2\ \ {}_{2}8\ \ {}_{5}8\ \ {}_{4}4\ \ {}_{5}6\ \ {}_{4}9 \Rightarrow 10543009$

$53491^2 = \ldots .53491 \Rightarrow 25\ \ {}_{3}0\ \ {}_{4}9\ \ {}_{11}4\ \ {}_{8}0\ \ {}_{7}8\ \ {}_{8}9\ \ {}_{1}8\ 1 \Rightarrow 2861287081$

$4632^2 = \ldots 4632 \Rightarrow 16\ \ {}_{4}8\ \ {}_{6}0\ \ {}_{5}2\ \ {}_{3}3\ \ {}_{1}2\ \ 4 \Rightarrow 21455424$

$7693^2 = \ldots 7693 \Rightarrow 49\ \ {}_{8}4\ \ {}_{16}2\ \ {}_{15}0\ \ {}_{11}7\ \ {}_{5}4\ \ 9 \Rightarrow 59182249$

In all the above cases

D the Duplex has been calculated at each stage

Digits other than the unit's place have been subscripted

And then added to left side main digit.

Please do all the above squaring yourself once before proceeding further. After this you would become an expert at squaring and it would really be appreciated how easy it has become, and you are a ***ONE LINE Expert***.

EXERCISE 8.3 Find the squares of following numbers using D method

1. 68	2. 208	3. 315
4. 298	5. 653	6. 789
7. 7141	8. 6507	9. 21313
10. 8617	11. 5538	12. 81347

SOME SPECIAL SQUARES

For Numbers ending with 5

There is a special case when the number ends in 5.

Put square of 5 as last two digits. Add 1 to the tens digit and multiply with original.

Take 45, place 25 and then 4 x (4+1) = 4 x 5 = 20 , thus we have 2025

125⇒ 12 x 13 ⇒ 156 ⇒ 125^2 = 15625

75⇒ 7x8 ⇒56 ⇒75^2 = 5625

For Numbers starting with 5

Square the last digit and place the squared quantity as last two digits.

The first two digits are 25 + the last digit

Taking 56 as example⇒ 6^2=36, place 36 as last two digits

25+6= 31, 3 & 1 are first two digits, thus 56^2=3136

53^2= 25+3⇒ 2809 (09 is square of 3)

59^2= 25+9⇒ 3481 (81 is square of 9)

CHAPTER 9 Square Roots

There are certain rules about square roots of numbers, which are listed below

- The number of digits in square root of the number will be same as the number of two digit groups in the number. This actually means if the number contains n digits, then the square root will contain **n/2 or (n+1)/2** digits.
- The exact square cannot end in 2,3,7 or 8.
- 1,5,6 and 0 as the last digits of the number repeat themselves in the square of the number.
- 1 and 9, 2 and 8, 3 and 7, 4 and 6, 5 and 5 have the same ending.

A number cannot be an exact square if

- if it ends in 2,3,7 or 8
- if there are odd number of zeroes at the end
- if its last digit is 6 but ***second last digit is even.***
- if the last digit is ***not 6*** but second last digit is ***odd***.
- the last two digits of even number are not divisible by 4.

To start with, first make 2 digit groups of the number this will give the number of digits in square root.

- Now take the right side digit group to find the first digit of the square root. **The number whose square is closest and lower to this digit group is the first digit**. For example if the digit group is 35 then 25 is lower closest to 35, hence 5 is the first digit.
- Find the difference between the first digit group and square of first digit of square root. This difference should be prefixed to the next digit as was done during the division process in a line below. This now becomes the new quotient.
- ***Double this first digit and place as dividend***.
- Start the division process and find **q** and **r**. Place **q** as the new digit and prefix **r** to the next digit of the number.

- Be careful in the next step, the new quotient should be reduced by **D,the duplex** of the right side of the partial square root. Now divide and place **q** as new digit in square root and **r** as the next digit.
- In the next step take the duplex of the last two digits, this process is continued till the last digit of the expected square root is reached. After this the D is calculated in decreasing order and after reaching the last digit of the number D of the last digit of the expected square root is used.
- In fact **this difference should be 0 for a perfect square after last digit of the square root**. It is proof of the correct answer.
- You may also need to alter the remainders as was done in straight division.

All this may be very confusing, but observe the following examples carefully.

Remember the Duplex D, the same shall be used here (refer chapter 8)

2	2 :	$_1$3	$_3$4 :	$_3$0	$_0$9
	1	5	3 :	0	0
Steps	**1 is the 1^{st} digit, thus 2 becomes the dividend $2-1^2=1$**	**using altered remainder 13÷2, q=5, r=3**	**D of 5 is 25, so 34-25=9, 9 is the new dividend, 9÷2, here again using altered remainder q=3, r=3**	**D of 53 is 30, 30-30=0**	**D of 3 is 9, 9-9=0**

This can be very confusing, but remember the square root of a number will have n/2 or (n+1)/2 digits, i.e. for a 5 digit number it will have 3 digits, and also for a 6 digit number 3 digits.

Let us take another exercise of finding square root of 20457529, here n= 8, thus number of digits in square root will be 4.

	2	0 :	4	5 :	7	5 :	2	9
8			4	4	4	3	1	0
		4	5	2	3 :	0	0	0
		4 is 1st digit, so 8 is dividend $20-4^2$ =4	44÷8 q=5 r=4	45-D(5) =45-25 =20 20÷8 q=2 r=4	47-D(52) =47-20 =27 27÷8 q=3 r=3	35-D(523) =35-34 =1 1÷8 q=0 r=1	12-D(23) =12-12 =0 q=0 r=0	9-D(3) =9-9=0 q=0 r=0

By now the use of D must be clear, ***start with single, then double, triple so on and again recede form higher to lower digits such that the last division is done with single digit D.***

Some more square roots (without details of calculation) are elucidated, do it yourself and then compare the calculations. It will serve to remove all doubts.

	3	0 :	1	4 :	0	1
10			5	11	8	8
		5	4	9 :	0	0

	7 :	1	8 :	2	4
4		3	7	10	6
	2	6	8 :	0	0

Note: Altered remainders are used here

	3	3 :	6	4
10			8	6
		5	8 :	0

It may be observed that all partial results will be zero after the square root is obtained; this is verification of the result.

			7		4	:	3		0	:	4		4		
16						10		7		2		0			
					8		6		2	:	0		0		

			5	:	5		8	:	8		4	:	9		6
4				1		3		5		6		4		1	
			2		3		6		4	:	0		0		0

	6		7	:	6		8	:	3		5	:	2		9
16				3		4		12		3		3		4	
			8		2		2		7	:	0		0		0

	5		1	:	0		6	:	5		3	:	1		6
14				2		6		9		3		5		3	
			7		1		4		6	:	0		0		0

EXERCISE 9.1 Find the square roots of following

1. 4624
2. 43264
3. 99225
4. 88804
5. 488601
6. 622521
7. 63792169
8. 48818169
9. 4129024
10. 64368529
11. 36144144
12. 80946009

CHAPTER 10 Quick Cubes

Let us remember the formula

$$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$$

If observed carefully it can be seen that each term is in geometrical ratio with common ratio ***b/a.***

Using this premise, rules for cubing are obtained for 2 digit numbers

1. Write the cube of first digit
2. The next three terms should be in geometric ratio between second digit and first digit of number to be cubed.
3. Now under second and third terms, double these terms and add them together.
4. Using normal rules for addition, the answer obtained is the cube.

The example would serve to illustrate the procedure completely.

12^3	**1**	**2**	**4**	**8**	Numbers written in geometric proportion, which is 2 in this case
double 2nd and 3rd terms		**4**	**8**		
carry over 1	**1**	**6**	**12**	**8**	
	1	**7**	**2**	**8**	

A few more examples,

52^3	**125**	**50**	**20**	**8**
		100	**40**	
	125	**150**	**60**	**8**
	140	**6**	**0**	**8**

27^3	**8**	**28**	**98**	**343**
		56	**196**	
	8	**84**	**294**	**343**
	19	**6**	**8**	**3**

34^3	**27**	**36**	**48**	**64**
		72	**96**	
	27	**108**	**144**	**64**
	39	**3**	**0**	**4**

Usually cubes of more than 2 digits are not really asked for in competitive examinations, hence getting adept at this would serve all purposes.

EXERCISE 10.1 Find the cubes of following

14	23	35
37	76	38
65	57	67
47	43	81

NUMBERS CLOSE TO BASE

For numbers close to base, whether surplus or deficient, a very simple and nice way is explained ahead.

The steps are:

1. The surplus or deficient number is doubled and added to original number. In case of deficient it is actually adding of negative number or subtraction. This is the left most part of answer

2. This new surplus or deficient is multiplied by original surplus or deficient, and put down encompassing same number of digits as that of zeroes in base as second portion.
3. Now place cube of original surplus or deficient as last portion of answer.

Cube of 102

Base 100 surplus 2	Double 2 i.e. 4 add to 102	**102+4=106**
6 is the new surplus	Multiply 6 with 2 and put down as 2nd portion	**106/12**
Cube the original surplus 2	place as 3rd portion 08 since base 100 has 2 zeroes	**106/12/08**
	Answer	**1061208**

Taking example of deficient, **cube of 997**

Base 1000 deficiency -3	Double -3 i.e. -6 add to 997	**997+(-6)=991**
-9 is the new deficiency	Multiply -9 with -3 and put down as 2nd portion	**991/027**
Cube the original surplus -3	place as 3rd portion -027 since base 1000 has 3 zeroes	**991/027/-027**
Conventional calculation 027000 -027	Answer	**991026973**

A few more solved examples, solve them yourself also.

$104^3 = 112/48/64$

$112^3 = 136/_{4}32/_{17}28 = 1404928$

$996^3 = 988/048/-064 = 998047936$

$96^3 = 88/48/-64 = 884736$

$9991^3 = 9973/0243/-0729 = 997302429271$

EXERCISE 10.2 Find the cubes of following

1. 98	2. 105	3. 113
4. 995	5. 1004	6. 1012
7. 9989	8. 10007	9. 10011
10. 9992	11. 10021	12. 998

CHAPTER 11 Cube Roots

CUBE ROOTS UP TO 6 DIGITS

Divide the number in two parts, put a separator after the first three digits e.g. 262,144.

The left part 262 is between 216 = 6^3 and 343 = 7^3. This means 262144 lies somewhere between 216000 = 60^3 and 343000 = 70^3. Hence the answer is surely between 60 and 70.

Something very special about cubes of single digits is that the last digit is unique. To understand let us see the following table, the second row is cube of the first row number respectively.

1	2	3	4	5	6	7	8	9
1	8	27	64	125	216	343	512	729

So all you have to know, rather remember very definitely is these cubes and the associated last digit which is unique.

For 1,4,5,6 and 9 it is the same digit.

For 3,7 and 2,8 last digits are reverse of the set.

Not a tough task at all after a few minutes of practice.

Applying this to our cube 262144 in just 15 seconds the answer 64

Lets try finding cube root of 571787.

83 » as 571 is between 512 and 729 i.e. first digit of cube root is 8.
Since 7 is the last digit associated with cube of 3, therefore we have 3.

This is actually mental, calculation without need of pencil and paper.

GENERAL CUBE ROOTS

Certain rules about cubes:

- **the last digit of the cube root is always distinct as explained above.**
- **The number of digits in a cube root is same as the number of three digit groups in the original cube.**
- **The first digit of the cube root is from the first group in the cube.**
- **After finding the first digit and the last digit, the work of extracting the cube root begins.**

Let F be the first digit, L the last digit and n be the number of digits in the cube root. Remember we are dealing with exact cubes.

Let us consider a few exact cubes

Exact cube	F	L	n
39,304	3	4	2
2,299,968	1	2	3
278,445,077	6	3	3
35,578,826,569	3	9	4

The decimal expansion of any number is

a + 10b + 100c +1000d + 10000e and so on, using the various parts of the cube of this algebraic expression, a procedure is formulated.

Let the digits of the cube be denoted by ***a,b,c,d,e*** so on from last to first respectively, i.e. if the number is 2345 then ***a=5, b=4, c=3, d=2.***

1. Subtract $\mathbf{a^3}$ from the unit's place, this eliminates the last digit.
2. Now subtract $\mathbf{3a^2b}$, eliminating the last but one digit.
3. Subtract $\mathbf{3a^2c + 3ab^2}$, this eliminates the third last digit.
4. Now from the thousands place deduct $\mathbf{b^2 + 6abc}$, this is continued till required.

All this may seem very complex, but a few examples will show how convenient it is,

Cube root of 74,618,461	
Here cube root will be 3 digit.	
The first digit is 4 as 4^3=64 and 5^3=125 and 74 is between 64 and 125.	**74 618 461**
Last digit, a = 1. Subtracting 1^3 from last digit	**1**
$3a^2b$ = $3x1^2xb$, this should be ending in 6 as it is the second last digit of the cube, **thus b=2**	**74 618 46**
So all the three digits are known and cube root is	**421**

Another Example

Cube root of 178,453,547	
Here cube root will be 3 digit.	
The first digit is 5 as 5^3=125 and 6^3=216 and 178 is between 125 and 216.	**178 453 547**
Last digit, a = 3. Subtracting 3^3 i.e. 27 from last digit	**27**
$3a^2b$ = $3x3^2xb$, this should end in 2 as it is the second last digit of the cube, thus **b = 6** as 27 x 6 = 162, ending in 2	**178 453 52**
So all the three digits are known and cube root is	**563**

Let us now consider a cube of 4 digit number

Cube root of 180,170,657,792	
Here cube root will be 4 digit.	
The first digit is 5 as 5^3=125 and 6^3=216 and 180 is between 125 and 216.	**180 170 657 792**

Last digit, a = 8. Subtracting 8^3 i.e. 512 from last digit	**512**
$3a^2b = 3x8^2xb$, this should end in 8 as 8 is the second last digit of the cube, thus **b = 4** as 192x4 = 768, ending in 8	**180 170 657 28**
Subtracting 768 from the remaining number	**7 68**
$3a^2c+3ab^2$ should end in 6, $3x8^2xc + 3x8x4^2 = 192c+384$, thus **c=6** since this expression will end in 6	**180 170 49 6**
Now all the four digits are known	**5648**

A few quick examples, try them yourself and compare the results,

143,877,824	
First digit 5, last digit 4	143 877 824
$4^3=64$	-64
$3x4^2xb$ should end in 6, hence b=2 or 8, but integer sum leads to 524 and not 584	143 877 76
so cube root = 524	

674,526,133	
First digit 8, last digit 7	674 526 133
$7^3=343$	-343
$3x7^2xb$ should end in 9, hence b=7	674 525 79
So cube root = 877	

921,167,317	
First digit 9, last digit 3	921 167 317
$3^3=27$	-27
$3x3^2xb$ should end in 9, hence b=7	921 167 29
So cube root = 973	

45,384,685,263	
First digit 3, last digit 7	45 384 685 263
$7^3=343$	-343
$3x7^2xb$ should end in 2, hence b=6	…….84 92
subtracting 147x6	8 82
$3a^2c+3ab^2$ should end in 1 147c+756 ends 1 with c= 5	76 1
Cube root = 3567	

Note: ***There may be cases where in step 2 for ten's digits more than one number may suffice the requirement, this slight ambiguity can be removed by verifying the first digit after continuing the next step.*** For example, 108.b ending in 2, here both 4 and 9 can be the answers. So some intelligent guessing is a must, which is a key to quick calculations. And most importantly use integer sum for verifying the result.

EXERCISE 11.1 Find the cube roots of following

1. 941192
2. 474552
3. 14348907
4. 40353607
5. 91733851
6. 961504803
7. 180362125
8. 480048687

CHAPTER 12 Recurring Decimals

Let us review concept of fraction. A fraction number is one, which has a numerator and denominator.

$$\text{Fraction} = \frac{\text{Numerator}}{\text{Denominator}}$$

Some very common problems are associated with finding the decimal equivalent of fractions. It is quite a cumbersome process if the conventional method is used. Recurring decimals have a set of numbers after the decimal, which repeat themselves. For example 1/3 = 0.333333…… , 1/9 = 0.11111……..

FOR DENOMINATORS ENDING IN 9

There are certain observations regarding numbers ending in 9,

1. The recurring decimal will always end in 1.
2. The part of denominator except the 9 is increased by 1 to make the ***operator*** i.e. in case of 19, the ***operator*** is 1+1=2, for 29 it is 2+1=3.
3. Put down 1 as last digit and start multiplying it by the ***operator*** from right towards left. If the intermediate result is two digit, it is normally carried to the next product and added.
4. When this product is ***equal to difference between denominator and numerator***, half the result is achieved.
5. The remaining half is found by writing down complements from 9, in simple terms the addition of terms should be 9.

Let us start with 19,
operator is 1+1=2

Putting 1 as last digit and continually multiplying by 2 from right to left

$$\ldots\ldots 9\ {}_{1}4\ 7\ {}_{1}3\ {}_{1}6\ 8\ 4\ 2\ 1$$

STEPS 1x2=2 2x2=4 4x2=8 8x2=16, keep 6 carry over 1

6x2+1=13, keep 3 carry over 1 3x2+1=7 7x2=14, keep 4 carry over 1

4x2+1=9 9x2=18 this is equal to 19-1, thus stop here. Half the result is found.

9 4 7 3 6 8 4 2 1
0 5 2 6 3 1 5 7 8 **complements from 9**

Thus **1/19 = . 0 5 2 6 3 1 5 7 8 9 4 7 3 6 8 4 2 1** ……….(repeating this set).

Now we shall calculate 1/29

2+1=3, thus operator is 3

………$_{1}9 \ _{1}6 \ _{1}5 \ 5 \ _{2}1 \ 7 \ _{1}2 \ 4 \ _{1}1 \ _{2}3 \ _{2}7 \ 9 \ 3 \ 1$

This ends at 9 because 3x9+1=28 which is equal to 29-1.

Finding complements from 9, the first half is found

9 6 5 5 1 7 2 4 1 3 7 9 3 1
0 3 4 4 8 2 7 5 8 6 2 0 6 8

Thus **1/29= . 0 3 4 4 8 2 7 5 8 6 2 0 6 8 9 6 5 5 1 7 2 4 1 3 7 9 3 1 … …**

FOR DENOMINATORS ENDING IN OTHER THAN 9

Now we shall involve fractions with denominators like 1, 3 or 7.

1. The last digit of the recurring decimal is such that the product of last digit of denominator and this last digit of decimal is 9. Thus denominators ending in 7,3 and 1 will yield decimals ending with 7,3 and 9 as 7x7=49, 3x3=9 and 1x9=9.
2. Convert the fraction into one with 9 ending denominator. i.e. 1/7 can be written as 7/49.
3. This resulting denominator will serve to find the ***operator***. 1 added to first digit, in case of 49, 1+4=5.
4. The same method as explained above is then applied.

The simplest case of 1/7 is taken

1/7 = 7/49

Last digit will be 7, and 5 will be the *operator*

Starting with 7 at the right end. $_{2}$**8** $_{3}$**5** **7**

Now 8x5+2=42 is equal to 49-7, using complements rule **1/7= 0.142 857**

Let us take another example of 1/23

1/23 = 3/69, thus 7 is the ***operator***

Putting 3 as last digit $\mathbf{{}_{3}9{}_{4}5{}_{3}6{}_{1}5{}_{1}2{}_{5}1{}_{2}7{}_{6}39{}_{2}13}$

This ends as 7x9+3=66 = 69-3, using complements,

1/23 = 0 . 0 4 3 4 7 8 2 6 0 8 6 9 5 6 5 2 1 7 3 9 1 3 … … …

EXERCISE 12.1 Find the recurring decimals

1. 1/13	2. 2/13	3. 1/17
4. 4/7	5. 2/19	6. 1/69
7. 1/89	8. 3/23	9. 2/49

CHAPTER 13 Divisibility Rule For Any Number

A very important aspect of Arithmetic in School is the rule for divisibility. There are standard rules for numbers like 2,3,4,5,6,8,9,11 etc. But for numbers like 7,12 and so on no rules are found in textbooks.

Schooling teaches a few basic rules which are summarized thus,

2	If the last digit divisible by two, then number is too
3	If the sum of the digits of the number is divisible by three, then number is too
4	If the last two digits are divisible by four, then the number is too
5	If the last digit is 5 or 0, then it is divisible by 5
6	If is divisible by 2 and by 3, then number is divisible by 6
8	If the last three digits are divisible by 8, then number is too
9	If the sum of the digits of the number is divisible by nine, then it is too
10	If the last digit is 0, then number is divisible by 10

11 also has a rule which is slightly complicated. Add up all the even placed digits and odd placed digits. Subtract these two sums and if the result is 0 or divisible by 11, then the number is divisible.

But for other numbers like 7,13,17,19 etc., there are no straight rules. In this chapter we will discuss the divisibility rule for any number, by any number the author means any number. The method is adopted from the Vedic mathematics Sutra called ***Ekādhikena Purveņa*** and sub sutra ***Veşţanam.***

It is the method of osculation using the *Veşţana,* finding the *Ekādhika* of the divisor. We shall not go into the details for calculating the *Ekādhika* of the divisor, but explain the procedure for checking divisibility.

Let us understand what osculation means,

Suppose you were to osculate ***(which means to form connecting links)*** 21 with 5, then multiply the last digit with 5 i.e. 5 x 1 =5, add this to previous digit 2 and thus get 7. This is the simplest of osculations.

A few more illustrative examples. **4321 with 7**

432 + 7x1 = 439,

43+7x9=106,

10+7x6=52,

5+7x2=19,

1+7x9=64 and so on to get 34,31,10,1

NOTE: The purpose of osculation is to reach a number, which is easily identifiable as divisible or not by the divisor.

Firstly the ***Ekādhika*** *which shall be called the* ***osculator*** for the numbers.

For 9,19,29,39, … they are 1,2,3,4,……respectively

For 3,13,23,33, … they are 1,4,7,10,…..respectively

For 7,17,27,37, … they are 5,12,19,26...respectively

For 1,11,21,31, … they are 1,10,19,28...respectively

To find the osculator multiply the number with the least such number that the result has 9 as last digit. The osculator is simply 1 more than the number left after removing its last digit 9.

For example, 13, here multiplying it by 3 we get 39, 1 more to 3 is 4, thus 4 is the osculator.

Similarly, for 67, 67x7=469, thus 1 more than 46 i.e. 47 is the osculator.

OSCULATION BY THE *Ekādhika*

Let us check the divisibility of 9994 by 19. We have to osculate 9994 with 2, which is the osculator for 19 (1 more than 1, first digit of 19).

The ***Ekādhika (osculator)*** for 19 is 2, so multiply the last digit by 2 and add the product 8 to previous digit 9.	**9994** 999+4x2=1007
Multiply last digit of intermediate result, 7 with 2 and add	100+7x2=114
Now multiply 4 with 2 and add with left digits 11 to get 19, thus it is proven that 9994 is divisible by 19	11+4x2=19

Do you realize how simple it is, the only thing to remember is the osculator or the *Ekādhika*.

Let us find out whether 7755 is divisible by 33 using the straight osculation method.

The *Ekādhika* for 33 is 10 (33x3=99 and 1 more than 9 is 10). Thus for 7755

775+50=825⇒82+50=132⇒13+20=33

A few more solved examples, try them yourself also

- 3453 by 53, osculator is 16 {53x3=159}

  345+48⇒393⇒39+48=87, thus not divisible

- 3445 by 53, osculator is 16

  344+80=424⇒42+64=106, 106=53x2, thus it is divisible

- 191023 by 29, osculator is 3

  19102+9=19111⇒1911+3=1914⇒191+12=203⇒20+9=29,divisible

- 95591 by 17, osculator is 12 {17x9=119}

  9559+12=9571⇒957+12=969⇒96+108=204=20+48=68, 68=17x4

- 101672 by 179, osculator is 18

  10167+36=10203⇒1020+54=1074⇒107+72=179, divisible

- 5860391 by 103, osculator is 31

  586039+31=586070⇒58607⇒5860+217=6077⇒607+217=824⇒82+124=206, divisible

EXERCISE 13.1 Check for divisibility using above method

1. 298559 by 17
2. 1937468 by 39
3. 643218 by 23
4. 2565836 by 49
5. 934321 by 21
6. 1131713 by 199

7. 468464 by 59 8. 1772541 by 69 9. 10112124 by 33

10. 1452132 by 57 11. 777843 by 49 12. 1335264 by 21

But there is a slight problem with numbers ending in 1 and 7 the process may become cumbersome, as the *Ekādhika, OSCULATOR* is a higher order number like 5,12,19,26 etc. For this another brilliant solution is the **negative osculation** method. Here the principle is same except that there is subtraction instead of addition and if the end result is a multiple of the number or zero *(which is true for most cases)* the divisibility is proved.

NEGATIVE OSCULATION METHOD

If P is the positive osculator and N is the negative osculator, then the rule is divisor, D = P+N.

For number 7, P=5, thus N=7-2=5

For number 21, P=19, thus N=21-19=2

The Negative osculators for the numbers, they are used mainly for numbers ***ending in 7 and 1.***

For 7,17,27,37, … they are 2,5,8,11…...respectively

For 11,21,31,41, .. they are 1,2,3,4…......respectively

For 9,19,29,39,….they are 8,17,26,35.. respectively

For 3,13,23,33,…..they are 2,9,16,23….respectively

NOTE: To remember the negative osculator multiply the number to get a product ending in 1, remove the 1 and the remaining number is the negative osculator.

The important thing to remember here is that unlike the positive osculation here osculate by ***subtracting*** intermediate results from remaining number.

Let us consider example of testing divisibility of 165763 with 41

The negative osculator is 4.

16576 - 4x3=16564
1656 - 4x4=1640
164 - 4x0=164
16- 4x4=0

Let us consider another example of 10171203 by 67, here multiply 67 by 3 which is 201, hence negative osculator is 20.

1017120-20x3=1017060
101706-20x0=101706
10170-20x6=10050
1005-20x0=1005
100-20x5=0
Thus it is divisible

A few direct examples, try yourself also.

- 358248 by 11, here negative osculator is 1 and positive osculator is 10

  35824-8=35816⇒3581-6=3575⇒357-5=352⇒35-2=33

- 1193766 by 21, here negative osculator is 2

  119376-12=119364⇒11936-8=11928⇒1192-16=1176⇒117-12=105⇒ 10-10=0

- 43928 by 17, here negative osculator is 5 as 17x3=51, removing 1

  4392-40=4352⇒435-10=425⇒42-25=17

- 943093 by 37, here negative osculator is 11 as 37x3=111, removing 1

  94309-33=94276⇒9427-66=9361⇒936-11=925⇒92-55=37

- 1310598 by 51, here negative osculator is 5

  131059-40=131019⇒13101-45=13056⇒1305-30=1275⇒127-25=102 ⇒10-10=0

- 387863 by 67, here negative osculator is 20 as 67x3=201

  38786-60=38726⇒3872-120=3752⇒375-40=335⇒ 33-100=-67 (divisible)

EXERCISE 13.2 Check for divisibility using above method

1. 138369 by 21
2. 204166 by 31
3. 1044885 by 41
4. 1013743 by 47
5. 1368519 by 37
6. 416789 by 37
7. 941379 by 67
8. 412357 by 81
9. 713473 by 17
10. 956182 by 17
11. 1518831 by 27
12. 426303 by 81

CHAPTER 14 Quick Averaging

Finding the average of a set of numbers is a very common problem in competitive examinations as well as school level tests. It often involves long calculations and trick division. If the weighted average is to be found out then multiplication also comes in.

This chapter would deal with simple average problems commonly encountered, using some techniques explained earlier in this book.

The average of a set of numbers is the number, which lies in the middle of the set, or all numbers have values centered on this average.

5,2,7,8,9,11 are 6 numbers. Now to find their average
First add them up 5+2+7+8+9+11=42 , divide this sum by 6 to get 7.

This was fairly simple but what about the numbers,

145, 136, 143, 145, 138, 137, 135, 145 set of 8 numbers.

Here it is observed that the numbers are fairly close to each other and it is easier if 140 is chosen as the base and then the average of deviations from 140 has to be added to 140.

$$\text{Average} = 140 + \frac{5 - 4 + 3 + 5 - 2 - 3 - 5 + 5}{8} = \mathbf{140} + \mathbf{4}/\mathbf{8} = \mathbf{140.5}$$

See how simple and easy it becomes. In most of the cases this technique would work.

Let us check a few more examples

67, 78, 81, 76, 69, 70, 81, 75, 69, 65 a set of 10 numbers.

The first number that would strike as base is 70, but if another thought is given 75 is a better choice.

$$\text{Average} = 75 + \frac{-8 + 3 + 6 + 1 - 6 - 5 + 6 + 0 - 6 - 10}{10} = \mathbf{75} + (-\mathbf{19})/\mathbf{10} = \mathbf{73.1}$$

The same method can be extended to decimal averages,

Find average of **9.9, 9.8, 10.1, 11.2, 12.0, 9.7, 10.5, 11.4**

Here 10 is the most convenient base.

$$\text{Average} = 10 + \frac{-0.1 - 0.2 + 0.1 + 1.2 + 2.0 - 0.3 + 0.5 + 1.4}{8} = \mathbf{10 + 4.6/8 = 10.575}$$

EXERCISE 14.1 Find the average of the numbers using a convenient base

1. 78, 79, 84, 75, 86, 90
2. 205, 210, 215, 216, 217, 218, 220, 203
3. 5.25, 5.75, 5.45, 6.1, 5.85
4. 1768, 1890, 1798, 1759, 1803, 1802, 1750, 1765,1780,1810

Suppose you were asked to find the average weight of a class of 100 students with following data (***weight is rounded of to nearest 5***)

Weight	Number of students
40	10
45	11
50	37
55	23
60	19

A common mistake is to take the average of weight column, here comes in concept of ***weighted average.*** Firstly find the ***weight*** of each row item by multiplying number of students by weight.

Weight	Number of students	Total weight
40	10	400
45	11	495
50	37	1850
55	23	1265
60	19	1140
	Grand total	**5150**

Therefore average 5150÷100= 51.5

These kinds of problems are very common in Data Interpretation section; the weighted average needs to be calculated for table data as well as graphical data. There is no direct shortcut, but addition and multiplication techniques explained earlier can reduce the time taken.

CHAPTER 15 Application of Techniques to Algebraic equations

FACTORISATION OF QUADRATIC EXPRESSIONS

Let us consider the quadratic expression

$$2x^2 + 2y^2 + z^2 + 4xy + 3xz + 3yz \ldots\ldots\ldots(1)$$

This is the homogeneous class quadratic. For factorization of such expressions a very simple and easy method has been devised.

1. Put z=0 to eliminate z, retain x and y which is a simple quadratic. Factorize this expression.
2. Now put y=0 and factorize the simple quadratic in x and z.
3. Now combine these two expressions by filling the gaps.

The ensuing steps will serve the purpose of explaining the method.

Put z=0 in (1),

$$2x^2 + 2y^2 + 4xy \Rightarrow (2x + 2y)(x + y)$$

Now putting y=0 in (1)

$$2x^2 + z^2 + 3xz \Rightarrow (2x + z)(x + z)$$

Combining these two expressions we have,

$(2x + 2y + z)(x + y + z)$ taking common coefficients.

A few more examples,

i. $2x^2 - y^2 + 3z^2 - xy + 5xz - yz$ put z=0 and y=0 to get expressions $(2x + y)(x - y)$ and $(2x + 3z)(x + z)$, combining we get $(2x + y + 3z)(x - y + z)$

ii. $2x^2 - 2y^2 + 6z^2 - 3xy + 7xz - 4yz$ put z=0 and y=0 to get expressions $(2x + y)(x - 2y)$ and $(2x + 3z)(x + 2z)$, combining we get $(2x + y + 3z)(x - 2y + z)$

iii. $2x^2 - 5y^2 - 6z^2 + 3xy + 4xz - 11yz$ put z=0 and y=0 to get expressions $(2x+5y)(x-y)$ and $(2x+6z)(x-z)$, combining we get $(2x+5y+6z)(x-y-z)$

EXERCISE 15.1 Factorize the following quadratic expressions

1) $3x^2 + y^2 - 2z^2 - 4xy - xz - yz$
2) $12x^2 - y^2 - 4z^2 + xy + 2xz + 3yz$
3) $2x^2 + 2y^2 + 6z^2 + 4xy - 8xz - 8yz$

FACTORISATION OF CUBIC EXPRESSIONS

First note a very interesting and important property of factorization and factors.

The sum of coefficients in the expression(product) is equal to the product of the sum of the coefficients of each factor.

Illustrating with the help of an example,

(x+3)(x+2)= x^2+5x+6

(1+3)(1+2)=1+5+6 =12

The same holds good for all kinds of expression like cubics, biquadratics etc.

Let us consider a cubic equation,

(x+2)(x+3)(2x+1)=$2x^3$+$11x^2$+17x+6

(1+2)(1+3)(2+1)=2+11+17+6=36

Let us call this sum **Sc (sum of coefficients).**

A cubic expression is expanded thus,

$$(x+a)(x+b)(x+c) = x^3 + x^2(a+b+c) + x(ab+ac+bc) + abc \quad \text{......(2)}$$

This brings out some rules,

1. The coefficient of x^2 is sum of the three zero power coefficients.
2. The product of these coefficients is equal to the last term of the expression.
3. If sum of coefficients of even powers is equal to sum of coefficients of odd power then (x+1) is a factor.
4. If Sc is 0, then one of the factors is (x-1).

Let us take the example of

$x^3+6x^2+11x+6$

Here the last term is 6 whose factors are 1,1,6 or 1,2,3. But the coefficient of x^2 is 6, hence their sum should be 6. Thus 1,2,3 is the only possibility.

So factorization is **(x+1) (x+2) (x+3)**

Taking another example which is more complex,

Here it must be kept in mind that this is a method to speed up factorization using some logic which could be different with different expressions, the basic equation (2) must be memorized and the key is to manipulate the coefficients.

$x^3+12x^2+44x+48$

here **a+b+c=12** and **abc=48**, this means a,b,c should be below 12,

Factors of 48 below 12 are 1,2,3,4,6,8 of which only 2,4,6 can satisfy both conditions of sum and product thus factorization is **(x+2)(x+4)(x+6)**

This method is applicable only for expressions whose coefficient of x^3 is unity.

Let us now take an example with negative coefficients,

$x^3+7x^2+7x-15$, here Sc=0, thus it is clear one of the factors is (x-1)

Now a+b+c=7 and abc=-15 since a=-1, hence the only other possibility is 3 and 5

Thus the factors are **(x-1), (x+3) and (x+5).**

EXERCISE 15.2 Factorize the following cubic expressions

1. $x^3+5x^2-2x-24$
2. $x^3+9x^2+20x+12$
3. $x^3+4x^2-19x+14$

SIMULTANEOUS SIMPLE EQUATIONS

There are a fair amount of problems where simultaneous equations have to be solved. They are usually not very complex but its does take time to solve. Here is a very useful technique to solve equations of the type,

$$ax + by = m$$
$$cx + dy = n$$

Just remember this simple formula

$$x = \frac{bn - dm}{bc - ad} \text{ and } y = \frac{cm - an}{bc - ad}$$

the denominator is common and a cyclical order is used, notice for y, the lower equation is used first i.e*. **cm - an*** .

Let us consider an example,

$$2x + 3y = -2$$
$$5x + 4y = 2$$

here

$$x = \frac{3.2 - (-2.4)}{3.5 - 2.4} = \frac{6 - (-8)}{15 - 8} = \frac{14}{7} = 2 \text{ and } y = \frac{-2.5 - 2.2}{3.5 - 2.4} = \frac{-10 - 4}{15 - 8} = \frac{-14}{7} = -2$$

Some quick examples,

$$4x - 2y = 7$$
$$x + 3y = 4$$

here

$$x = \frac{-2.4 - 7.3}{-2.1 - 4.3} = \frac{13}{14} \text{ and } y = \frac{7.1 - 4.4}{-2.1 - 4.3} = \frac{9}{14}$$

Taking another example,

$5x + 2y = 23$

$-3x + 7y = 19$

here

$x = \dfrac{2.19 - 23.7}{5.7 - (-2.3)} = \dfrac{123}{41} = 3$ and $y = \dfrac{-23.3 - 5.19}{5.7 - (-2.3)} = \dfrac{164}{41} = 4$

A few more solved examples

1.

$5x + 3y = 2$

$9x + 2y = 8$

here

$x = \dfrac{3.8 - 2.2}{9.3 - (5.2)} = \dfrac{20}{17}$ and $y = \dfrac{3.8 - 2.2}{9.3 - (5.2)} = \dfrac{-22}{17}$

2.

$x + 9y = -10$

$9x + y = -3$

here

$x = \dfrac{-27 - (-10)}{81 - 1} = \dfrac{-17}{80}$ and $y = \dfrac{-90 - (-27)}{81 - 1} = \dfrac{-63}{80}$

3.

$4x + 2y = 13$

$6x + 5y = 21$

here

$x = \dfrac{42 - 65}{12 - 20} = \dfrac{23}{8}$ and $y = \dfrac{78 - 84}{12 - 20} = \dfrac{6}{8}$

Solutions to Exercises in Chapters

Exercise 1.3

1.

			2	8	9	7
			4	3’	5’	6’
			7’	8	4	3
			5	4’	3	4
running total		0	7	1	10	9
ticks		0	1	2	1	1
Grand total		**2**	**0**	**5**	**3**	**0**

2.

			3	7	6	8
			5	4’	6’	7’
			9’	0	8	7’
			8’	7	9’	6
running total		0	3	7	7	6
ticks		0	2	1	2	2
Grand total		**2**	**7**	**1**	**1**	**8**

3.

		3	4	5	2	6
		1	8’	9’	7	6’
		9’	8	7	6’	8
		4	5’	3’	6	7’
		1	3	9’	8’	7’
running total	0	7	6	0	7	1
ticks	0	1	2	3	2	3
Grand total	**2**	**1**	**1**	**6**	**2**	**4**

4.

			2	3	4	1
			5	4	6	7
			9’	0	8’	8’
			7’	0	0	9’
			3	2	1	9’
running total		0	4	9	8	1
ticks		0	2	0	1	3
Grand total		**2**	**7**	**1**	**2**	**4**

5.

		1	4	6	7	8
		4	5	3	2	5’
		5	8’	3’	2’	6
		9’	0	0	8	7’
		7’	6’	5	0	9’
running total	0	4	1	6	8	2
ticks	0	2	2	1	1	3
Grand total	**2**	**8**	**4**	**9**	**2**	**5**

Exercise 1.4

1. 361547980 7
2. 376539246 9
3. 546372 9
4. 900876 3
5. 9994563 9
6. 918273645 9
7. 225162 9

Exercise 3.1

1. 507732
2. 346758
3. 3413418
4. 5405184

Exercise 3.2

1. 242634
2. 679651
3. 3988341
4. 2148188

Exercise 3.3

1. 173310
2. 485465
3. 2848815
4. 1534420

Exercise 3.4

1. 381282
2. 1068023
3. 6267393
4. 3375724

Exercise 3.5

1. 415944
2. 1165116
3. 6837156
4. 3682608

Exercise 3.6

1. 311103
2. 8781564
3. 748375
4. 2887199935
5. 4811844
6. 4761988
7. 238518
8. 3994668
9. 6306104
10. 904908072

Exercise 4.1

1. 156
2. 5159
3. 3618
4. 168
5. 3551
6. 7200
7. 242
8. 1044
9. 5040
10. 2670

Exercise 4.2

1. 8722
2. 8536
3. 8811
4. 8075
5. 9603
6. 8742

Exercise 4.3

1.	10608	2.	6162
3.	9483	4.	3480
5.	4680	6.	9078
7.	7917	8.	2160
9.	525	10.	2862
11.	868	12.	192

Exercise 5.1

1.	832656	2.	471569
3.	308448	4.	660516
5.	89951	6.	798160
7.	46420	8.	10776
9.	158340	10.	89301

Exercise 5.2

1.	234588	2.	146412
3.	111360	4.	71820
5.	235900	6.	864612
7.	211988	8.	766878
9.	148390	10.	59211

Exercise 6.1

1.	Q=75 R=3	2.	Q=28 R=2
3.	Q=1 R=30	4.	Q=2 R=251
5.	Q=1 R=5569	6.	Q=6 R=145
7.	Q=3 R=5681	8.	Q=4 R=568
9.	Q=4 R=34579	10.	Q=5 R=17

Detailed solutions Exercise 6.1

Divide 67 by 9, Q=7, R=4
(one more way to do this)

9) 67 / 8
..7 / 4
6(14) / 12
74 / 12
75 / 3

9) 25 / 4
..2 / 7
27 / 11
28 / 2

89 / 11) 1 / 19
.. / 11
1 / 30

997 / 003) 2 / 245
.. / 006
2 / 251

8998 / 1002) 1 / 4567
.. / 1002
1 / 5569

887 / 113) 5 / 467
.. / 565
5 / 1032
6 / 145

8888 / 1112) 3 / 2345
.. / 3336
3 / 5681

991 / 009) 4 / 532
.. / 036
4 / 568

89997 / 10003) 3 / 94567
.. / 30009
3 / 124576
4 / 34579

86 / 14) 4 / 47
.. / 56
4 / 103
5 / 17

Exercise 6.2

1. Q=263 R=49
2. Q=121 R=600
3. Q=141 R=400
4. Q=318 R=712
5. Q=135 R=449
6. Q=212 R=7465

Detailed solutions Exercise 6.2

$$\begin{array}{l} \frac{89}{11}\Big) 234/56 \\ ..22 \\ ....5/5 \\ ....../(12)1 \\ \hline 25.11/ \\ 261../237 \\ +2/-(89\times 2) \\ 263/49 \end{array}$$

$$\begin{array}{l} \frac{997}{003}\Big) 121/237 \\ ..00/3 \\ ....0/06 \\ ....../003 \\ \hline 121/600 \end{array}$$

$$\begin{array}{l} \frac{887}{113}\Big) 125/467 \\ ..11/3 \\ ....3/39 \\ ....../(10)17 \\ \hline 139/2174 \\ ....2/-(887\times 2) \\ 141/400 \end{array}$$

$$\begin{array}{l} \frac{987}{013}\Big) 314/578 \\ ..03/9 \\ ....0/13 \\ ....../091 \\ \hline 317/1699 \\ ..+1/-(987) \\ 318/712 \end{array}$$

$$\begin{array}{l} \frac{898}{102}\Big) 121/679 \\ ..10/2 \\ ....3/06 \\ ....../408 \\ \hline 134/1347 \\ .+1/-(898) \\ 135/449 \end{array}$$

$$\begin{array}{l} \frac{9892}{0108}\Big) 210/4569 \\ ..02/16 \\ ....0/108 \\ ....../0216 \\ \hline 212/7465 \end{array}$$

Exercise 7.1

1. Q=20 R=3
2. Q=21 R=0
3. Q=153 R=15
4. Q=30 R=48
5. Q=132 R=18
6. Q=1118 R=12
7. Q=1136 R=33
8. Q=534 R=0
9. Q=666 R=0
10. Q=714 R=4

Detailed Solutions to Exercise 7.1

1

1				
2	4	2	/	3
	0		0	
	2	0	/	3

2

1				
3	6	5	/	1
	0		0	
	2	1	/	0

3

2					
3	4	9	1	/	1
	1	2		2	
	1	5	3	/	15

4

4					
5	1	6	6	/	8
		1		4	
		3	0	/	48

5

4					
6	8	4	6	/	6
	2	2		2	
	1	3	2	/	18

6

3						
5	5	9	2	6	/	6
	0	1	4		3	
	1	1	1	8	/	12

7

6						
7	8	6	3	6	/	9
	1	3	6		6	
	1	1	3	6	/	33

8

3						
7	3	8	9	8	/	2
		3	3		1	
		5	3	4	/	0

9

2						
5	3	4	6	3	/	2
		4	4		1	
		6	6	6	/	0

10

3						
8	5	9	2	6	/	6
		3	3		1	
		7	1	4	/	4

Exercise 7.2

1. Q=142 R=4
2. Q=249 R=1
3. Q=61 R=5
4. Q=230 R=3
5. Q=447 R=2
6. Q=105 R=41
7. Q=164 R=2
8. Q=126 R=34
9. Q=468 R=8
10. Q=7912 R=13

Detailed Solutions to Exercise 7.2

1

4					
2	3	4	1	/	2
	1	2		1	
	1	4	2	/	4

2

3					
3	8	2	1	/	8
	2	4		2	
	2	4	9	/	1

3

3					
2	1	4	0	/	8
		2		0	
		6	1	/	5

4

8					
1	4	1	4	/	3
	2	2		0	
	2	3	0	/	3

5

1					
1	4	9	1	/	9
	0	1		0	
	4	4	7	/	2

6

4					
5	5	7	1	/	1
	0	3		6	
	1	0	5	/	41

7

9					
5	9	6	7	/	8
	4	7		3	
	1	6	4	/	2

8

3					
5	6	7	1	/	2
	1	4		5	
	1	2	6	/	34

9

1					
2	9	8	3	/	6
	1	2		1	
	4	6	8	/	8

10

6							
7	6	0	1	3	2	/	5
		11	6	2		2	
		7	9	1	2	/	13

Exercise 7.3

1. Q=11 R=2
2. Q=11 R=9
3. Q=7 R=114
4. Q=21 R=4216
5. Q=11 R=645
6. Q=23 R=15
7. Q=21 R=125
8. Q=211 R=34
9. Q=141 R=32
10. Q=155 R=510

Detailed solutions to Exercise 7.3

1

1 1 2	1	2	/ 3	4
-1 -2		-1	-2	
			-1	-2
	1	1	/ 0	2

2

1 1 2	1	2	/ 4	1
-1 -2		-1	-2	
			-1	-2
	1	1	/ 1	-1
	1	1	/ 0	9

3

1 6 0	1	2	/ 3	4
-6 0		-6	0	
			24	0
1st	1	-4	/ 27	4
		+1	-16	0
	(10-4+1)	7	/ 11	4

4

1 1 2 0 3	2	3	/ 9	4	7	9
-1 -2 0 3		-2	-4	0	-6	
			/ -1	-2	0	-3
	2	1	/ 4	2	1	6

5

1 1 0 4	1	2	/ 7	8	9
-1 0 -4		-1	0	-4	
			-1	0	-4
	1	1	/ 6	4	5

6

1 2 1 1	2	7	/ 8	6	8
-2 -1 -1		-4	-2	-2	
			-6	-3	-3
	2	3	/ 0	1	5

7

1 0 0 3	2	1	/ 1	8	8
0 0 -3		0	0	-6	
			0	0	-3
	2	1	/ 1	2	5

8

1 2 3	2	5	9	/ 8	7
-2 -3		-4	-6		
			-2	-3	
				-2	-3
	2	1	1	/ 3	4

9

1 1 0	1	5	5	/ 4	2
-1 0		-1	0		
			-4	0	
				-1	0
	1	4	1	/ 3	2

10

1 2 2 1	1	8	9	/ 7	6	5
-2 -2 -1		-2	-2	-1		
			-12	-12	-6	
				10	10	5
	1	6	-5	/ 4	10	10
	1	5	5	/ 5	1	0

Note: 4x100+10x10+10x1=400+100+10=510

Exercise 7.4

1. Q=13 R=200
2. Q=8 R=77
3. Q=36 R=239
4. Q=26 R=228
5. Q=11 R=378
6. Q=3 R=667
7. Q=4 R=699
8. Q=24 R=489
9. Q=15 R=1172
10. Q=24 R=746

Detailed solutions to Exercise 7.4

1

9 8 8	1	3	/ 0	4	4
0 1 2		0	1	2	
			0	3	6
	1	3	/ 1	9	10
	1	3	/ 2	0	0

2

8 7 8	7	/ 1	0	1
1 2 2		7	14	14
	7	/ 8	14	15
	7+1=8		800+140+15=955 955-878=77	

3

8 7 9	3	1	/ 8	8	3
1 2 1		3	6	3	
			4	8	4
	3	4	/ 18	19	7
		34+2=36		1800+190+7=1997 1997-(879x2)=239	

4

7 9 9	2	1	/ 0	0	2
2 0 1		4	0	2	
			10	0	5
	2	5	/ 10	2	7
		25+1=26		1027-799=228	

5

8 8 3	1	0	/ 0	9	1
1 2 -3		1	2	-3	
			1	2	-3
	1	1	/ 3	8	-2
				300+80-2=378	

6

6 7 2	2	/ 6	8	3
3 2 8		6	6	-4
3 3 -2				
	2	/ 12	14	-1
	2+1=3		1200+140-1=1339 1339-672=667	

Note: the operators have been modified to ease calculations

7

9 8 2 0	3	/ 9	9	7	9
0 1 8 0		0	6	-6	0
0 2 -2 0					
	3	/ 9	15	1	9
	3+1=4			10519-9820=699	

8

8 1 8	2	0	/ 1	2	1
1 8 2		4	-4	4	
2 -2 2			8	-8	8
	2	4	/ 5	-2	9
					489

9

8 8 9 3	1	3	/ 4	5	6	7
1 1 0 7		1	1	1	-3	
1 1 1 -3			4	4	4	-12
	1	4	/ 9	10	7	-5
		14+1=15		9000+1000+70-5=10065 10065-8893=1172		

10

9 8 7	2	4	/ 4	3	4
0 1 3		0	2	6	
			0	4	12
	2	4	/ 6	13	16
				600+130+16=746	

Exercise 8.1

1. 97-3/09=9409
2. 108+8/64=11664
3. 115+15/${}_{2}$25=13225
4. 998-2/004=996004
5. 899-101/${}_{10}$201=798000+10201=808201 (be careful here)
6. 989-11/121=978121
7. 9987-13/0169=99740169
8. 9979-21/0441=99580441
9. 10032+32/1024=100641024
10. 1023+23/529=1046529
11. 1012+12/144=1024144
12. 99997-3/00009=9999400009

Exercise 8.2

1. Take base 70=10x7; 68-2;66x7/4=4624
2. Take base 200=100x2;208+8;216x2/64=43264
3. Take base 315=100x3;315+15;330x3/${}_{2}$25=99225
4. Take base 300=100x3;298-2;296;296x3/04=88804
5. Take base 800=100x8;789-11;778;778x8/${}_{1}$21=622521
6. Take base 700=100x7;699-1;698;698x7/01=488601
7. Take base 8000=1000x8;7987-13;7974;7974x8/169=63792169

8. Take base 7000=1000x7;6987-13;6974;6974x7/169=48818169
9. Take base 20000=10000x2;20032+32;20064;20064x2/01024=4012801024
10. Take base 8000=1000x8;8023+23;8046;8046x8/529=64368529
11. Take base 6000=1000x6;6012+12;6024;6024x6/144=36144144
12. Take base 90000=10000x9;89997-3;89994x9/0009=8099460009

Exercise 8.3

1. $68^2 = .68 \Rightarrow 36\ \ {}_{9}6\ \ {}_{6}4 \Rightarrow 4624$
2. $208^2 = ..208 \Rightarrow 4\ \ 0\ \ {}_{3}2\ \ 0\ \ {}_{6}4 \Rightarrow 43264$
3. $315^2 = ..315 \Rightarrow 9\ \ 6\ \ {}_{3}1\ \ {}_{1}0\ \ {}_{2}5 \Rightarrow 99225$
4. $298^2 = ..298 \Rightarrow 4\ \ {}_{3}6\ \ {}_{11}3\ \ {}_{14}4\ \ {}_{6}4 \Rightarrow 88804$
5. $653^2 = ..653 \Rightarrow 36\ \ {}_{6}0\ \ {}_{6}1\ \ {}_{3}0\ \ 9 \Rightarrow 426409$
6. $789^2 = ..789 \Rightarrow 49\ \ {}_{11}2\ \ {}_{19}0\ \ {}_{14}4\ \ {}_{8}1 \Rightarrow 622521$
7. $7141^2 = ...7141 \Rightarrow 49\ \ {}_{1}4\ \ {}_{5}7\ \ {}_{2}2\ \ {}_{1}8\ \ 8\ \ 1 \Rightarrow 50993881$
8. $6507^2 = ...6507 \Rightarrow 36\ \ {}_{6}0\ \ {}_{2}5\ \ {}_{8}4\ \ {}_{7}0\ \ 0\ \ {}_{4}9 \Rightarrow 42341049$
9. $21313^2 = ...21313 \Rightarrow 4\ \ 4\ \ {}_{1}3\ \ {}_{1}0\ \ {}_{2}3\ \ {}_{1}2\ \ {}_{1}9\ \ 6\ \ 9 \Rightarrow 454243969$
10. $8617^2 = ...8617 \Rightarrow 64\ \ {}_{9}6\ \ {}_{5}2\ \ {}_{12}4\ \ {}_{8}5\ \ {}_{1}4\ \ {}_{4}9 \Rightarrow 74252689$
11. $5538^2 = ...5538 \Rightarrow 25\ \ {}_{5}0\ \ {}_{5}5\ \ {}_{11}0\ \ {}_{8}9\ \ {}_{4}8\ \ {}_{6}4 \Rightarrow 30669444$
12. $81347^2 = ...81347 \Rightarrow 64\ \ {}_{1}6\ \ {}_{4}9\ \ {}_{7}0\ \ {}_{12}9\ \ {}_{3}8\ \ {}_{5}8\ \ {}_{5}6\ {}_{4}9 \Rightarrow 6617334409$

Exercise 9.1

1.	68	2.	208	3.	315
4.	298	5.	699	6.	789
7.	7987	8.	6987	9.	2032
10.	8023	11.	6012	12.	8997

Detailed solutions to Exercise 9.1

1

	4		6	:	2		4
12				10		6	
			6		8	:	0

2

	4	:	3		2	:	6		4
4		0		3		0		6	
	2		0		8	:	0		0

3

	9	:	9		2	:	2		5
6		0		3		1		2	
	3		1		5	:	0		0

4

	8	:	8		8	:	0		4
4		4		12		15		6	
	2		9		8	:	0		0

5

	4	8	:	8		6	:	0		1
12			12		20		17		8	
		6		9		9	:	0		0

6

	6	2	:	2	5	:	2	1
14			13	20		15	8	
		7		8	9	:	0	0

7

	6	3	:	7	9	:	2	1	:	6	9
14			14	21		26	20		11	4	
		7		9	8		7 :	0		0	0

8

	4	8	:	8	1	:	8	1	:	6	9
12			12	20		24	20		11	4	
		6		9	8		7 :	0		0	0

9

	4	:	1	2	:	9	0	:	2	4
4		0	1		0	1		1	0	
	2		0	3		2 :	0		0	0

10

	6	4	:	3	6	:	8	5	:	2	9
16			0	3		4	0		1	0	
		8		0	2		3 :	0		0	0

11

	3	6	:	1	4	:	4	1	:	4	4
12			0	1		2	0		0	0	
		6		0	1		2 :	0		0	0

12											
	8	0	:	9	4	:	6	0	:	0	9
16			16	25		29		22	13		4
		8		9	9		7	:	0	0	0

Exercise 10.1

1.	2744	2.	12167	3.	42875
4.	50653	5.	438976	6.	54872
7.	274625	8.	185193	9.	300763
10.	103823	11.	79507	12.	531441

Exercise 10.2

1. 98^3 = 94/12/-08 = 941192
2. 105^3 = 115/75/$_1$25 = 1157625
3. 113^3 = 139/$_5$07/$_{21}$97 = 1442897
4. 995^3 = 985/075/-125=985074875
5. 1004^3 = 1012/048/064=1012048064
6. 1012^3 = 1036/432/$_1$728=1036433728
7. 9989^3 = 9967/0363/-1331=996703628669
8. 10007^3 = 10021/0147/0343=1002101470343
9. 10011^3 = 10033/0363/1331 = 1003303631331
10. 9992^3 = 9976/0192/-0512 = 997601919488
11. 10021^3 = 10063/1323/9261 = 100631329261
12. 998^3 = 994/012/-008 = 994011992

Exercise 11.1

1. 941,192 here 941 is more than 9^3, thus first digit is 9, and last digit is 8 as it ends in 2. Cube root is 98.

2. 474,552 here 474 is more than 7^3, thus first digit is 7, and last digit is 8 as it ends in 2. Cube root is 78.

3. 14,348,907
First digit c is 2, last digit a is 3, eliminating last digit 907-27=880
Now $3a^2b$ should end in 8, so 3x 3^2 b = 27b ends in 8 if b=4
Thus cube root is 243

4. 40,353,607
c=3, a=3, eliminating last digit 607-27=580
Now $3a^2b$ should end in 8, so 3x 3^2 b = 27b ends in 8 if b=4
Thus cube root is 343

5. 91,733,851
c=4, a=1, eliminating last digit 5 is left
Now $3a^2b$ should end in 5, so 3 b ends in 5 if b=5
Thus cube root is 451

6. 961,504,803
c=9, a=7, eliminating last digit 803-343=460
Now $3a^2b$ should end in 6, so 3x49 b= 147b ends in 6 if b=8
Thus cube root is 987

7. 180,362,125
c=5, a=5, eliminating last digit 125-125=000
Now $3a^2b$ should end in 0, so 3x25 b= 75b ends in 6 if b=0,2,4,6 or 8
Here lies a confusion.
Integer sum of 180362125 is 1
Thus cube root is 5_5, where _ could be 0 or 6 as integer sum of 505^3 = 1^3 =1 and of 565^3 = 7^3 =343= 1.
With an intelligent guess(which is a must in quick maths) answer is 565

8. 480,048,687
c=7, a=3, eliminating last digit 687-27=660
Now $3a^2b$ should end in 6, so 3x9 b= 27b ends in 6 if b=8
Thus cube root is 783

Exercise 12.1

1. 0.076923
2. 0.153846
3. 0.0588235294117647
4. 0.571428
5. 0.105263157894736842
6. 0.0144927536231884057971
7. 0.01123595505617977528089887640449438202247191
8. 0.1304347826086956521739
9. 0.040816326530612244897551O2

Exercise 13.1

1. 298559 by 17 not divisible

   The ekadhika for 17 is 12

   29855+108=29963⇒2996+36=3032⇒303+24=327⇒32+84=116⇒11+72=83⇒ 8+36=44 is not divisible by 17

2. 1937468 by 39 not divisible

   The ekadhika for 39 is 4

   193746+32=193778⇒19377+32=19409⇒1940+36=1976⇒197+24=221⇒22+ 4=26 is not divisible by 39

3. 643218 by 23 divisible

   The ekadhika for 23 is 7

   64321+56=64377⇒6437+49=6486⇒648+42=690⇒69 is divisible by 23

4. 2565836 by 49 divisible

   The ekadhika for 49 is 5

   256583+30=256613⇒25661+15=25676⇒2567+30=2597⇒259+35=294⇒29+ 20=49 is divisible by 49

5. 934321 by 21 not divisible

   The ekadhika for 21 is 19

   93432+19=93451⇒9345+19=9364⇒936+76=1012⇒101+38=139 is not divisible by 21

6. 1131713 by 199 divisible

   The ekadhika for 199 is 20

   113171+60=113231⇒11323+20=11343⇒1134+60=1194⇒119+80=199 is divisible by 199

7. 468464 by 59 not divisible

   The ekadhika for 59 is 6

   46846+24=46870⇒4687+0=4687⇒468+42=510⇒51 is not divisible by 59

8. 1772541 by 69 divisible

   The ekadhika for 69 is 7

   177254+7=177261⇒17726+7=17733⇒1773+21=1794⇒179+28=207⇒20+49 =69 is divisible by 69

9. 10112124 by 33 divisible

   The ekadhika for 33 is 10

   1011212+40=1011252⇒101125+20=101145⇒10114+50=10164⇒1016+40⇒ 1056⇒105+60=165⇒16+50=66 is divisible by 33

10. 1452132 by 57 divisible

    The ekadhika for 57 is 40

    145213+80=145293⇒14529+120=14649⇒1464+360=1824⇒182+160=342⇒ 34+80=114 is divisible by 57

11. 777843 by 49 not divisible

    The ekadhika for 49 is 5

    77784+15=77799⇒7779+45=7824⇒782+20=802⇒80+10=90 is not divisible by 49

12. 1335264 by 21 divisible

    The ekadhika for 21 is 19

    133526+76=133602⇒13360+38=13398⇒1339+152=1491⇒149+19=168 is divisible by 21

Exercise 13.2

1. 138369 by 21 divisible

   The negative osculator for 21 is 2

   13836-18=13818⇒1381-16=1365⇒136-10=126⇒12-12=0 is divisible by 21

2. 204166 by 31 divisible

   The negative osculator for 31 is 3

   20416-18=20398⇒2039-24=2015⇒201-15=186⇒18-18=0 is divisible by 31

3. 1044885 by 41 divisible

   The negative osculator for 41 is 4

   104488-20=104468⇒10446-32=10414⇒1041-16=1025⇒102-20=82⇒8-8=0 is divisible by 41

4. 1013743 by 47 divisible

   The negative osculator for 47 is 14

   101374-42=101332⇒10133-28=10105⇒1010-70=940⇒94 is divisible by 47

5. 1368519 by 37 divisible

   The negative osculator for 37 is 11

   136851-99=136752⇒13675-22=13653⇒1365-33=1332⇒133-22=111⇒11-11=0 is divisible by 37

6. 416789 by 37 not divisible

   The negative osculator for 37 is 11

   41678-99=41579⇒4157-99=4058⇒405-88=317 is not divisible by 37

7. 941379 by 67 not divisible

   The negative osculator for 67 is 20

   94137-180=93957⇒9395-140=9255⇒925-100=825⇒82-100=-18 is not divisible by 67

8. 412357 by 81 not divisible

   The negative osculator for 81 is 8

   41235-56=41179⇒4117-72=4045⇒404-80=364⇒36-32=4 is not divisible by 81

9. 713473 by 17 divisible

   The negative osculator for 17 is 5

   71347-15=71332⇒7133-10=7123⇒712-15=697⇒69-35=34 is divisible by 17

10. 956182 by 17 divisible

    The negative osculator for 17 is 10

    95618-10=95608⇒9560-40=9520⇒952⇒95-10=85 is divisible by 17

11. 1518831 by 27 divisible

    The negative osculator for 27 is 8

    151883-8=151875⇒15187-40=15147⇒1514-56=1458⇒145-64=81⇒8-8=0 is divisible by 27

12. 426303 by 81 divisible

    The negative osculator for 81 is 8

    42630-24=42606⇒4260-48=4212⇒421-16=405⇒40-40=0 is divisible by 81

Exercise 14.1

1. Choose base as 80, Av = 80 +(-2-1+4-5+6+10)/6 = 82
2. Choose base as 210, Av =210 +(-5+0+5+6+7+8+10-7)/8 = 213
3. Choose base as 5.5, Av = 5.5 +(-0.25+0.25-0.05+0.6+0.35)/5 = 5.68
4. Choose base as 1800, Av = 1800 +(-32+90-2-41+3+2-50-35-20+10)/10=1792.5

Exercise 15.1

1. (x-y-z)(3x-y+2z) 2. (4x-y+2z)(3x+y-z) 3. (x+y-z)(2x+2y-6z)

Exercise 15.2

1. (x+4)(x+3)(x-2) 2. (x+7)(x-2)(x-1) 3. (x+6)(x+2)(x-1)