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An exact solution for Burger's equation

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SUMMARY

An exact solution for Burger's equation useful for testing numerical methods. Copyright © 2006 John Wiley & Sons, Ltd.

KEY WORDS: Burger's equation; exact solution

When experimenting with numerical methods for the solution of a non-linear equation, it is helpful to have a particular case where there is an exact solution with which to test the results.

Burger's equation is

$$u_t + uu_x = \nu u_{xx} \tag{1}$$

With, say, $0 \leqslant x \leqslant 1$, $t \geqslant 0$, and

$$u(x,0) = f(x), \quad f(0) = f(1) = 0 \tag{2}$$

$$u(0,t) = u(1,t) = 0 \tag{3}$$

The Cole–Hopf transformation [1, 2] gives the solution of Equation (1) in the form:

$$u = -2\nu \frac{\theta_x}{\theta} \tag{4}$$

where θ is a solution of the diffusion equation:

$$\theta_t = \nu \theta_{xx} \tag{5}$$

We choose a solution of Equation (5) which satisfies $\theta_x = 0$ at $x = 0, 1$, e.g.

$$\theta = a + \exp(-\pi^2 \nu t)\cos(\pi x) \tag{6}$$

where a is an arbitrary constant.

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Thus

$$u = \frac{2\nu\pi \exp(-\pi^2 \nu t)\sin(\pi x)}{a + \exp(-\pi^2 \nu t)\cos(\pi x)}, \quad \text{with } a > 1 \tag{7}$$

is an exact solution of Burger's equation (1) with

$$u(x,0) = f(x) = \frac{2\nu\pi \sin(\pi x)}{a + \cos(\pi x)}, \quad a > 1 \tag{8}$$

This can be used to test numerical methods before using them on the more usual starting condition $f(x) = \sin(\pi x)$.

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