From this it follows readily that the consistency of Z_C' is provable in Z_C. (Cf. Theorem IV, p. 260, of [2].)

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AN EXTENSION THEOREM FOR SOLUTIONS OF $d\omega=\Omega$

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Let U and V be open sets in E_n such that $\overline{V}\subset U$ and U is connected and homologically trivial, i.e., all homology groups of U beyond the zero-dimensional case vanish. Let Ω be an exterior differential form of degree p on E_n with infinitely differentiable coefficients whose exterior derivative vanishes: $d\Omega=0$. The well known converse to the Lemma of Poincaré asserts that $\Omega=d\omega$ where ω is an infinitely differentiable $p-1$ form on E_n. Let us suppose however that we are merely given a $p-1$ form α on U such that $d\alpha=\Omega$ on U. The question immediately arises as to whether it is possible to prolong α to all of E_n. The example $U=\{(x, y)|x>0\}$, $\alpha=(xdy-ydx)/(x^2+y^2)$, $\Omega=0$ shows us that the answer is negative. Nevertheless, *there exists a* $p-1$ *form* β *on* E_n *such that* $\beta=\alpha$ *on* V *and* $d\beta=\Omega$ *on* E_n.

To prove this, we shall take for granted the existence of an infinitely differentiable function f on E_n such that $f=1$ on V and $f=0$ outside of a closed subset of U. We have the form ω on E_n such that $d\omega=\Omega$ on E_n and the form α on U such that $d\alpha=\Omega$ on U. Thus $d(\alpha-\omega)=0$ on U and so it follows from the hypotheses on U and what is essentially de Rham's second theorem that $\alpha-\omega=d\lambda$ on U,

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where λ is an infinitely differentiable $p-2$ form on U.[1] One sets $\mu=f\lambda$ on U, $\mu=0$ outside U so that μ is a $p-2$ form on E_n such that $d\mu=d(f\lambda)=d\lambda=\alpha-\omega$ on V. The form $\beta=\omega+d\mu$ solves the problem, for $d\beta=d\omega=\Omega$ on E_n and $\beta=\omega+(\alpha-\omega)=\alpha$ on V.

If any doubt remains about the case $p=1$, it is quickly settled when one notes that $\alpha-\omega$ is a function and so $d(\alpha-\omega)=0$ on U implies that $\alpha-\omega=c$, a constant, on U. Now $\beta=\omega+c$ is the solution.

This result and method of proof can be extended to multiply connected regions; we give a single instance:

Let Ω be a p-form on E_n such that $d\Omega=0$ and suppose that α is a $(p-1)$-form on $r>0$ $(r^2=x_1^2+\cdots+x_n^2)$ such that $d\alpha=\Omega$ on $r>0$. Finally, suppose that

$$\lim_{\epsilon\to 0}\int_{r=\epsilon}\alpha = 0$$

in the case $p=n$. Then given any $\epsilon>0$, there exists a $(p-1)$-form β on E_n such that $d\beta=\Omega$ on E_n and $\beta=\alpha$ on $r>\epsilon$.

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[1] This form of de Rham's theorem is given in the paper of A. Weil, *Sur les théorèmes de de Rham*, Comment. Math. Helv. vol. 26 (1952). It is pointed out on p. 138 that in case U is convex, then an elementary proof is possible. Such a proof is given for the case in which U is a cell on p. 94 of the second edition of W. V. D. Hodge, *Harmonic integrals*.