An assessment of Evans' unified field theory I

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Abstract

Evans developed a classical unified field theory of gravitation and electromagnetism on the background of a spacetime obeying a Riemann-Cartan geometry. This geometry can be characterized by an orthonormal coframe ϑ^α and a (metric compatible) Lorentz connection $\Gamma^{\alpha\beta}$. These two potentials yield the field strengths torsion T^α and curvature $R^{\alpha\beta}$. Evans tried to infuse electromagnetic properties into this geometrical framework by putting the coframe ϑ^α to be proportional to four extended electromagnetic potentials $\mathcal{A}^\alpha$; these are assumed to encompass the conventional Maxwellian potential A in a suitable limit. The viable Einstein-Cartan(-Sciama-Kibble) theory of gravity was adopted by Evans to describe the gravitational sector of his theory. Including also the results of an accompanying paper by Obukhov and the author, we show that Evans' ansatz for electromagnetism is untenable beyond repair both from a geometrical as well as from a physical point of view. As a consequence, his unified theory is obsolete.

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1 Introduction

One of the problems in evaluating Evans' unified field theory is that its content is spread over hundreds of pages in articles and books of Evans and his associates. There is no single paper in which the fundamentals of Evans' theory are formulated in a concise and complete way. Nevertheless, we can take Evans' papers [23, 24], which subsume also work done earlier, as a starting point. Now and then, when additional information is required, we will use other publications of Evans and collaborators, too [19, 25–28]. We will try to put the fundamental equations of Evans' theory in a way as condensed as possible; in fact, we will come up with the nine equations from (78) to (86) that characterize Evans' theory. Incidentally, we came across Evans' unified field theory in the context of a refereeing process. And in the present paper, we will formulate our assessment in considerable detail.

We use, as Evans does in [24], the calculus of exterior differential forms. A translation for Evans' notation into ours is given in Table 1 (at the end of the Introduction).

The evaluation of Evans' theory is made more demanding since his articles contain many mathematical mistakes and inconsistencies, as has been amply shown by Bruhn [2–10] and Rodrigues et al. [18, 58]. Let me just illustrate this point with two new examples. I take Evans' "Einstein equation" in [24], App.4, Eq.(11), namely $R^a_b = kT^a_b$. According to Evans' definition [24], Eq.(16), the left hand side represents the curvature 2-form in a Riemann-Cartan geometry (i.e., $R^{\alpha\beta} = -R^{\beta\alpha}$) and the right hand side is proportional to the components of the canonical energy-momentum tensor T^a_b. Clearly, this equation is incorrect since a 2-form $R^a_b = R^a_{\mu\nu b}\, dx^\mu \wedge dx^\nu/2$ with its 36 components cannot be equated to the 16 component of a second rank tensor. If we generously interpreted R^a_b as Ricci tensor, even though Evans denotes the Ricci tensor always as $R_{\mu\nu}$, the equation would be wrong, too. A second example, we can find nearby: In [24], App.4, Eq.(10), Evans claims that the energy-momentum of his generalized Einstein equation obeys $D \wedge T^a_b = 0$. It is well-known, however, that in a spacetime with torsion there can be no zero on the right-hand-side, rather torsion and curvature dependent terms must enter, see [35], Eq.(3.12). Similar examples can be found easily.

One may argue, as I will do in future, that a scientist educated as chemist may have a great idea in physics even if the mathematical details of his articles are not quite sound. Accordingly, I sometimes followed not only that subclass of Evans' formulas that deemed correct to me, but also his prose in oder to understand Evans' underlying "philosophy".

It is clear from [24] that the 4-dimensional spacetime in which Evans' theory takes place obeys a Riemann-Cartan geometry (RC-geometry) [35] or, in the words of Evans, a “Cartan-geometry”. We decided to take what Evans calls an antisymmetric part of the metric $\vartheta^{\alpha\beta} := \vartheta^\alpha \wedge \vartheta^\beta$ (here ϑ^α is the coframe of the RC-spacetime) not seriously as a part of the metric, see Bruhn [7] for a detailed investigation. The quantity $\vartheta^{\alpha\beta} = -\vartheta^{\beta\alpha}$ is an antisymmetric tensor-valued 2-form with 36 independent components and it is a respectable quantity, but it certainly cannot feature as a metric. Since Evans very often claims to use a RC-geometry, we take his word for it. Then, an asymmetric metric is ruled out. One cannot have both, RC-geometry and an asymmetric metric. Both are mutually exclusive.

In a RC-geometry, a linear connection Γ and a metric g with Minkowskian signature $(+ - - -)$ are prescribed, furthermore metric compatibility is required. This guarantees that *lengths and angles* are integrable in RC-geometry. Evans arrives at a RC-geometry by means of what he calls the “tetrad postulate”, see [24], Eqs.(32) and (33), and Rodrigues et al. [58].

In Sec.2 we will display the geometric properties of a RC-geometry in the 4-dimensional spacetime in quite some detail. In particular, we define torsion and curvature and decompose torsion irreducibly under the local Lorentz group. We introduce the contortion and the Ricci 1-forms and the curvature scalar. Moreover, the two Bianchi identities are displayed and two irreducible pieces projected out leading to the Cartan and Einstein 3-forms. The Ricci identity will be mentioned shortly.

In Sec.3 we will take Evans’ ansatz relating the coframe ϑ^α to a generalized electromagnetic potential $\mathcal{A}^\alpha$ according to $\mathcal{A}^\alpha = a_0\, \vartheta^\alpha$, where a_0 is a scalar factor of the dimension *magnetic flux /length* $\stackrel{\text{SI}}{=} Wb/m = V s/m$, see Evans and Eckardt [28], p.2. We will point out that in this way one finds four Lorentz-vector valued 1-forms or, in other words, extended electromagnetic $SO(1,3)$-covariant potentials $\mathcal{A}^\alpha$ with 16 components, in contrast to what Evans finds, namely $O(3)$-covariant potentials. Then the extended electromagnetic field strength $\mathcal{F}^\alpha$ is defined and the generalized Maxwell equations displayed and discussed.

In Sec.4 we show that Evans just adopted the viable Einstein-Cartan theory (EC-theory) of gravity for his purpose literally. His generalized Einsteinian field equation is the same as the first field equation of EC-theory. As a consequence of the angular momentum law, Evans also used the second field equation of EC-theory, even though he used it only in words in identifying torsion with the spin of matter. Then we display the energy-momentum and angular momentum laws. It is pointed out that the so-called Evans wave equation for the coframe ϑ^α is a *redundant* structure since the dynamics of ϑ^α is already controlled by the generalized

Notion	Evans	here
coframe	$q^a = q^a_\mu dx^\mu$	$\vartheta^\alpha = e_i{}^\alpha dx^i$
connection	$\omega^a_b = \omega^a_{\mu b} dx^\mu$	$\Gamma_\alpha{}^\beta = \Gamma_{i\alpha}{}^\beta dx^i$
torsion	$T^a = \frac{1}{2}\, T^a_{\mu\nu} dx^\mu \wedge dx^\nu$	$T^\alpha = \frac{1}{2}\, T_{ij}{}^\alpha dx^i \wedge dx^j$
curvature	$R^a_b = \frac{1}{2}\, R^a_{b\mu\nu} dx^\mu \wedge dx^\nu$	$R_\alpha{}^\beta = \frac{1}{2}\, R_{ij\alpha}{}^\beta dx^i \wedge dx^j$
Ricci tensor/1-form	$R_{\mu\nu}$	$\mathrm{Ric}_\alpha = e_\beta \rfloor R_\alpha{}^\beta = \mathrm{Ric}_{\beta\alpha} \vartheta^\beta$
Evans' elmg. potential	A^a	$\mathcal{A}^\alpha$
Evans' elmg. constant	$\mathcal{A}^{(0)}$	a_0
Evans' elmg. field strength	F^a	$\mathcal{F}^\alpha$
Evans' hom. current	j^ν	$\mathcal{J}^\alpha_{\mathrm{hom}}$
Evans' inh. current	J^ν	$\mathcal{J}^\alpha_{\mathrm{inh}}$
can. energy-mom. density	T^a_b	$\Sigma_\alpha = \mathfrak{T}_\alpha{}^\beta \eta_\beta$
spin ang. mom. density	?	$\tau_{\alpha\beta} = \mathfrak{S}_{\alpha\beta}{}^\gamma \eta_\gamma$
Hodge duality	$\widetilde{\Psi}^a$	${}^\star\Psi^\alpha$

Table 1: Translation of Evans' notation into ours. Note that $\eta_\alpha = {}^\star\vartheta_\alpha$. In Evans' work, T^a_b is also sometimes used as symmetric energy-momentum tensor.

Einstein equation together with the Cartan equation.

In Sec.5 we collect the fundamental equations of Evans' theory in the nine equations from (78) to (86). We will explain what exactly we call Evans' unified field theory. In an accompanying paper by Obukhov and the author [41], we propose a new variational principle for Evans' theory and derive the corresponding field equations of Evans' theory. It turns out that for all physical cases we can derive the vanishing of torsion and thus the collapse of Evans' theory to Einstein's ordinary field equation. We discuss our findings, including the results of [41], and summarize our objections against Evans' unified theory.

A few historical remarks may be in order. Cartan himself noticed in a letter to Einstein, see [17], page 7, that one irreducible piece of the torsion T "has precisely all the mathematical characteristics of the electromagnetic potential"; it is apparently the vector piece $T_{\rm vec}$ that he determined earlier, between 1923 and 1925, in [14]. Thus he discussed $T_{\rm vec} \sim A$, where A is the potential of Maxwellian electrodynamics. Note that this assumption is totally different from Evans' ansatz $\vartheta^\alpha \sim \mathcal{A}^\alpha$. Moreover, Cartan did *not* develop a corresponding electromagnetic theory. In fact, in the same papers [14], he linked, within a consistent theoretical framework, torsion to the spin of matter. He laid the groundwork to what we call nowadays the EC-theory of gravity [1, 35, 68]. This excludes the mentioned identification of a piece of the torsion with the electromagnetic potential.

Later Eyraud [29] and Infeld [45] and, more recently, Horie [44] tried to link torsion to the electromagnetic field. But these attempts did lead to nowhere. For more details, one may consult Tonnelat [66] and Goenner [31].

2 Geometry: Riemann-Cartan geometry of spacetime

2.1 Defining RC-geometry

We assume a 4-dimensional differential manifold. At each point, the basis of the tangent space are the four linearly independent *vectors* $\mathbf{e}_\alpha = e^i{}_\alpha \partial_i$, here $\alpha, \beta, ... = 0, 1, 2, 3$, the (anholonomic) tetrad indices, number the vectors and $i, j, k, ... = 0, 1, 2, 3$, the (holonomic) coordinate indices, denote the components of the respective vectors. The basis of the cotangent space is span by the four linearly independent *covectors* or 1-forms $\boldsymbol{\vartheta}^\beta = e_j{}^\beta \, dx^j$. The bases of vectors and covectors are dual to each other. Consequently, we have $e^i{}_\alpha \, e_i{}^\beta = \delta^\beta_\alpha$ and $e^i{}_\beta \, e_j{}^\beta = \delta^i_j$. We call collectively the e_α's and the ϑ^β's also *tetrads*. We follow the conventions[1] specified in [37].

On our manifold we impose a *connection* 1-form $\Gamma_\alpha{}^\beta = \Gamma_{i\alpha}{}^\beta dx^i$ that allows us to define the parallel transport of quantities. Accordingly, we have a *covariant exterior derivative* operator $D := d + \Gamma_\alpha{}^\beta \, f^\alpha{}_\beta$, where $f^\alpha{}_\beta$ represents the behavior of the quantity under linear transformations. Additionally, we impose a symmetric

[1] We build from the coframe ϑ^α by exterior multiplication and by applying the Hodge star the following expressions: $\vartheta^{\alpha\beta} := \vartheta^\alpha \wedge \vartheta^\beta$, $\vartheta^{\alpha\beta\gamma} := \vartheta^{\alpha\beta} \wedge \vartheta^\gamma$, etc.; $\eta := {}^\star 1$ (volume 4-form) , $\eta^\alpha := {}^\star\vartheta^\alpha$, $\eta^{\alpha\beta} := {}^\star\vartheta^{\alpha\beta}$, etc., see [37, 38]. We denote antisymmetrization by brackets $[ij] = (ij - ji)/2$ and symmetrization by parentheses: $(ij) = (ij + ji)/2$. Analogously for more indices, as, e.g., for $[ijk]$, where we have $[ijk] = (ijk - jik + jki - + \cdots)/3!$, see Schouten [60, 61].

metric $g = g_{ij}\, dx^i \otimes dx^j$, with $g_{ij} = g_{ji}$. Referred to a tetrad, we have $g_{\alpha\beta} = e^i{}_\alpha e^j{}_\beta\, g_{ij}$.

Metric and connection are postulated to be metric compatible, that is, the nonmetricity 1-form $Q_{\alpha\beta} := -Dg_{\alpha\beta}$ is postulated to vanish: $Q_{\alpha\beta} = 0$. This guarantees that lengths and angles are constant under parallel transport. In accordance with this fact, it is convenient to choose the tetrads to be *orthonormal* once and for all. Then, $g_{\alpha\beta} = \mathrm{diag}(+1,-1,-1,-1) =: o_{\alpha\beta}$, where $o_{\alpha\beta}$ is the Minkowski metric. If we raise the α-index of the connection, then $\Gamma^{\alpha\beta} = -\Gamma^{\beta\alpha}$. This is known as the Lorentz (or spin) connection, since the indices α and β transform locally under the Lorentz group $SO(1,3)$. Hence, the variables ϑ^α and $\Gamma^{\alpha\beta}$, i.e., coframe and Lorentz connection, specify the geometry completely.

In the subsequent section, we need to discuss the transformation properties of the coframe ϑ^α. Under a coordinate transformation, it behaves like a 1-form, in components, $e_{i'}{}^\alpha = \frac{\partial x^j}{\partial x^{i'}}\, e_j{}^\alpha$. Under local SO(1,3) Lorentz rotations $\Lambda_\beta{}^{\alpha'}$, it transforms as a Lorentz vector $\vartheta^{\alpha'} = \Lambda_\beta{}^{\alpha'}\, \vartheta^\beta$. The spatial rotation group $O(3)$ is a subgroup of $SO(1,3)$. But the Lorentz group includes also the boosts. In other words, whereas a spatial $O(3)$-rotation of ϑ^α is an allowed procedure, the theory is only locally Lorentz covariant — and thus takes place in a RC-geometry — if the coframe ϑ^α transforms under the complete $SO(1,3)$. If a theory admitted only an $O(3)$ transformation, then it would violate Lorentz invariance.

The geometry defined so far is called a RC-geometry. It is also clear from the statements of *Evans* that he uses *exactly the same geometry*. Thus, we have a secure platform for our evaluation. In four dimensions, RC-geometry was first used in the viable Einstein-Cartan(-Sciama-Kibble) theory of gravity, for short EC-theory [34, 35, 49, 62, 63, 67, 68].

2.2 Torsion and curvature

From our variable ϑ^α and $\Gamma_a{}^\beta$, we can extract two Lorentz tensors, the *torsion* and the *curvature* 2-forms, respectively:

$$T^\alpha := D\vartheta^\alpha = d\vartheta^\alpha + \Gamma_\beta{}^\alpha \wedge \vartheta^\beta\,, \tag{1}$$
$$R_\alpha{}^\beta := d\Gamma_\alpha{}^\beta - \Gamma_\alpha{}^\gamma \wedge \Gamma_\gamma{}^\beta\,. \tag{2}$$

In RC-geometry, we have, because of the metric compatibility, $\Gamma^{\alpha\beta} = -\Gamma^{\beta\alpha}$, and thus, $R^{\alpha\beta} = -R^{\beta\alpha}$. In four dimensions, torsion and curvature have 24 and 36 independent components, respectively.

If one applies a Cartan displacement (rolling without gliding) around an infinitesimal loop in the manifold, then *torsion* is related to the *translational* misfit and the curvature to the rotational misfit; a discussion can be found in [15], see also Sharpe [65], our book [38], Sec.C.1.6, and the recent article of Wise [70]. Alternatively, one may build up an infinitesimal parallelogram, then the *translational closure failure is proportional to the torsion*. This is important: Geometrically, from the point of view of RC-geometry, torsion has nothing to do with spin, but rather with translations. It is for this reason that torsion can be understood as the field strength of a translational gauge theory, see Gronwald [32]. Consequently, when Evans treats torsion and spin (Spin of matter? Spin of gravity? Spin of electromagnetism?) synonymously, as he does in all of his articles on his theory,[2] then this can only be understood as an additional *dynamical assumption* that is independent from the RC-geometry of the underlying spacetime. We will see further down in detail that this is, indeed, the case.

The torsion 2-form T^α can be contracted by $e_\alpha\rfloor$ to a covector $e_\alpha\rfloor T^\alpha$ and multiplied by ϑ_α to yield a 3-form $\vartheta_a \wedge T^\alpha$ or, using the Hodge star, to a covector with twist ${}^\star\,(\vartheta_a \wedge T^\alpha)$. These expressions correspond to the vector and the axial vector pieces of the torsion. More formally, we can decompose the torsion tensor irreducibly under the local Lorentz into three pieces:

$$T^\alpha = {}^{(1)}T^\alpha + {}^{(2)}T^\alpha + {}^{(3)}T^\alpha\,. \tag{3}$$

The second and the third pieces correspond to the mentioned *vector* and *axial vector* pieces, respectively,

$$ {}^{(2)}T^\alpha \;:=\; \frac{1}{3}\,\vartheta^\alpha \wedge (e_\beta\rfloor T^\beta)\,, \tag{4}$$

$$ {}^{(3)}T^\alpha \;:=\; \frac{1}{3}\,e^\alpha\rfloor(\vartheta_\beta \wedge T^\beta)\,, \tag{5}$$

whereas the first piece can be computed by using (3).

For a comparison with Riemannian geoemtry, it is often convenient to decompose the connection 1-form into a Riemannian part, denoted by a tilde, and a tensorial post-Riemannian part according to

$$\Gamma_\alpha{}^\beta = \widetilde{\Gamma}_\alpha{}^\beta - K_\alpha{}^\beta\,. \tag{6}$$

[2] "There are two fundamental differential forms...that together describe any spacetime, the torsion or spin form and Riemann or curvature form." See Evans [24], p.434. Just by the choice of Evans' language, torsion is always identified with spin. We are not told what sort of spin we have to think of. In Sec.4 we will see that it has to be the total spin of all matter and the electromagnetic field, with exception of gravity.

In RC-geometry, the $K_\alpha{}^\beta$ can be derived by evaluating $Dg_{\alpha\beta} = 0$. We find the *contortion* 1-form as [37]

$$K_{\alpha\beta} := 2e_{[\alpha}\rfloor T_{\beta]} - \frac{1}{2}\, e_\alpha\rfloor e_\beta\rfloor (T_\gamma \wedge \vartheta^\gamma) = -K_{\beta\alpha}\,. \tag{7}$$

Resolved with respect to the torsion, we have $T^\alpha = K^\alpha{}_\beta \wedge \vartheta^\beta$.

The curvature 2-form yields, by contraction, the Ricci 1-form

$$\mathrm{Ric}_\alpha := e_\beta\rfloor R_\alpha{}^\beta = \mathrm{Ric}_{\beta\alpha}\,\vartheta^\beta = R_{\gamma\beta\alpha}{}^\gamma \vartheta^\beta. \tag{8}$$

In a RC-geometry, the components of the Ricci 1-form are asymmetric in general: $\mathrm{Ric}_{\alpha\beta} \neq \mathrm{Ric}_{\beta\alpha}$. By transvection with the metric, we find the curvature scalar

$$R := g^{\alpha\beta}\,\mathrm{Ric}_{\alpha\beta} = g^{\alpha\beta} e_\alpha\rfloor \mathrm{Ric}_\beta = e_\alpha\rfloor e_\beta\rfloor R^{\alpha\beta} = -e_\alpha\rfloor \left[e_\beta\rfloor {}^\star\!\left({}^\star R^{\alpha\beta}\right)\right]\,. \tag{9}$$

After some algebra, see [38], p.338, we find for the curvature scalar, with $\eta_{\alpha\beta} = {}^\star\,(\vartheta_\alpha \wedge \vartheta_\beta)$, the following:

$$R = e_\alpha\rfloor e_\beta\rfloor R^{\alpha\beta} = {}^\star\!\left(\eta_{\alpha\beta} \wedge R^{\alpha\beta}\right)\,. \tag{10}$$

The expression under the star can be taken as a Lagrangian 4-form of the gravitational field.

The curvature 2-form can be decomposed into 6 different pieces, see [37]. Among them, we find the symmetric tracefree Ricci tensor, the curvature scalar, and the antisymmetric piece of the Ricci tensor.

2.3 Bianchi identities

If we differentiate (1) and (2), we find the two Bianchi identities for torsion and curvature, respectively:

$$DT^\alpha = R_\beta{}^\alpha \wedge \vartheta^\beta\,, \tag{11}$$
$$DR_\alpha{}^\beta = 0\,. \tag{12}$$

Incidentally, Evans agrees that he and we use the same RC-geometry.[3]

[3]M.W. Evans states in his internet blog http://www.atomicprecision.com/blog/2006/12/08/endorsement-of-ece-by-the-profession/ the following: “The two Cartan structure equations, two Bianchi identities and tetrad postulate used by Carroll, Hehl and myself are the same.” See, however, a note of Bruhn [8] on some mistake in the corresponding considerations of Evans.

Both Bianchi identities can be decomposed irreducibly under the local Lorentz group into 3 and 4 irreducible pieces, respectively; for details, see [36, 37, 53]. We remind ourselves of the 1-form $\eta_{\alpha\beta\gamma} = {}^\star(\vartheta_\alpha \wedge \vartheta_\beta \wedge \vartheta_\gamma)$, where the star denotes the Hodge operator. Then, by exterior multiplication of the Bianchi identities with this 1-form, we can extract from (11) an irreducible piece with 6 independent components,

$$DT^\gamma \wedge \eta_{\gamma\alpha\beta} = R_\delta{}^\gamma \wedge \vartheta^\delta \wedge \eta_{\gamma\alpha\beta}\,, \tag{13}$$

and from (12) one with 4 independent components,

$$DR^{\beta\gamma} \wedge \eta_{\beta\gamma\alpha} = 0\,. \tag{14}$$

We define the *Cartan* and the *Einstein* 3-forms,

$$C_{\alpha\beta} \;:=\; \frac{1}{2}\,\eta_{\alpha\beta\gamma} \wedge T^\gamma\,, \tag{15}$$

$$G_\alpha \;:=\; \frac{1}{2}\,\eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma}\,, \tag{16}$$

respectively. Now we shift in (13,14) by partial integration the 1-form $\eta_{\alpha\beta\gamma}$ under D. After some algebra, we find,

$$DC_{\alpha\beta} \;=\; -\eta_{[\alpha} \wedge \mathrm{Ric}_{\beta]}\,, \tag{17}$$

$$DG_\alpha \;=\; \frac{1}{2}\,\eta_{\alpha\beta\gamma\delta}\,R^{\beta\gamma} \wedge T^\delta\,. \tag{18}$$

Thus, the Cartan 3-form $C_{\alpha\beta}$ and the Einstein 3-form G_α are important quantities since they appear in the two contracted Bianchi identities (17) and (18) under the differentiation symbol.

Using (16) and the formula $\vartheta_{[\alpha} \wedge G_{\beta]} = \eta_{[\alpha} \wedge \mathrm{Ric}_{\beta]}$, Eq.(17) can be rewritten in terms of G_α. Thus, finally we have for the two contracted Bianchi identities, see also [54, 55],

$$DC_{\alpha\beta} + \vartheta_{[\alpha} \wedge G_{\beta]} \;=\; 0\,, \tag{19}$$

$$DG_\alpha \;=\; \frac{1}{2}\,\eta_{\alpha\beta\gamma\delta}\,R^{\beta\gamma} \wedge T^\delta\,. \tag{20}$$

We will come back to these $6+4$ independent equations below. Note that (20) is only valid in four dimensions. In *three* dimensions — then G_α is a 2-form — the term on the right-hand-side of (20) vanishes,[4] see [36], that is, $DG_\alpha = 0$ (for $\alpha = 1, 2, 3$).

[4] É. Cartan [12, 13] worked very intuitively. One of his goals in analyzing Einstein's theory was

2.4 Ricci identity

If we differentiate covariantly the vector-valued p-form Ψ^α, we find, because of the Poincaré lemma $dd = 0$, the Ricci identity

$$DD\Psi^\alpha = R_\beta{}^\alpha \wedge \Psi^\beta \,. \tag{21}$$

This is particularly true for the coframe,

$$DD\vartheta^\alpha = R_\beta{}^\alpha \wedge \vartheta^\beta \,. \tag{22}$$

Evans is calling the Ricci identity (22) simply "Evans' lemma", see [25], p.464. However, he made a calculational mistake and found for the right-hand-side of (22) effectively the expression $R\,{}^\star\vartheta^\alpha$, instead of the correct $R_\beta{}^\alpha \wedge \vartheta^\beta$.

3 Electromagnetism: Evans' ansatz for extended electromagnetism

3.1 Evans' ansatz

Up to now, everything is quite conventional. The RC-geometry, which Evans is using, has been introduced earlier in gauge theories of gravity and is well understood, see [33]. However, in the electromagnetic sector, Evans has a highly unconventional ad hoc ansatz. He assumes the existence of an extended electromagnetic potential $\mathcal{A}^\alpha$ that is proportional to the coframe ϑ^α,

$$\mathcal{A}^\alpha = a_0\,\vartheta^\alpha \qquad \text{or} \qquad \mathcal{A}_i{}^\alpha = a_0\, e_i{}^a \,, \tag{23}$$

see Evans [24], Eq.(12). Here a_0 denotes a scalar constant of dimension $[a_0] =$ *magnetic flux/length;* it has supposedly to be fixed by experiment.

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to get a geometrical understanding of the Einstein *tensor*, that is, to get hold of the Einstein tensor without using analytical calculations. He achieved that for three dimensions, where $DG_\alpha = 0$. Obviously Cartan's intuition worked with three dimensions. There is evidence for this, namely, he constructed a special 3-dimensional model of a RC-space [12, 13], the Cartan spiral staircase; for a discussion, see Garcia et al. [30], Sec.V. Apparently over-stretching his intuition, Cartan also assumed $DG_\alpha = 0$ for four dimensions and run into difficulties with his gravitational theory. We go into such details here, since Evans [25], p. 464, commits the same mistake as Cartan did and assumes $DG_\alpha = 0$ for four dimensions, whereas, in fact, (20) is correct. The comparison between the four-dimensional EC-theory and a three-dimensional continuum theory of lattice defects has been reviewed by Ruggiero & Tartaglia [59].

Due to the omnipresence of the coframe ϑ^α, an extended electromagnetic potential $\mathcal{A}^\alpha$ is created by (23) everywhere. Thus, one may call such an ansatz pan-electromagnetic. The constant a_0 must be thought of as a universal constant. Otherwise a geometric theory, which supposedly describes a universal interaction, looses its raison d'être. The dimension of a_0 doesn't point to its universality. Remember that universal constants usually have the dimensions of $q^{n_1}\,\mathfrak{h}^{n_2}$, where q denotes the dimension of a charge and $\mathfrak{h}$ that of an action, see Post [57] and [40]. Constants built according to this rule, are 4-dimensional scalars, since q and $\mathfrak{h}$ carry exactly this property. Observationally it turns out that n_1 and n_2 are integers. Examples for such dimensionful 4-scalars are

$$q \rightarrow \text{electric charge}\,, \quad \frac{\mathfrak{h}}{q} \rightarrow \text{magnetic flux}\,, \quad \frac{\mathfrak{h}}{q^2} \rightarrow \text{electric resistance}\ldots \tag{24}$$

Thus, $n_1, n_2 = 0, \pm 1, \pm 2, \ldots$. Accordingly, the impedance of free space $\Omega_0 = \sqrt{\mu_0/\varepsilon_0}$, for example, is a 4-dimensional scalar and a universal constant, whereas ε_0 and μ_0 for themselves are no 4-scalars. And for a_0, we have $[a_0] = \mathfrak{h}/(q \times \textit{length})$. This doesn't smell particularly universal. The constant a_0 is not expected to qualify as a 4-scalar, since it defies the scheme (24).

Evans has the following to say[5] ([24], p.435): "Here $A^{(0)}$ denotes a $\hat{C}$ negative scalar originating in the magnetic fluxon $\hbar/e$, a primordial and universal constant of physics." From [22], p.2 we learn that we have "...a scalar factor $A^{(0)}$, essentially a primordial voltage." In fact, the dimension of a_0 is neither that of a magnetic flux nor that of a voltage. Thus, the meaning of this constant is not clear to us.

For convenience we can parametrize a_0 with the help of the magnetic flux quantum $h/(2e)$. Here h is the Planck constant and e the elementary charge. Then,

$$a_0 = \frac{h}{2e\ell_{\mathrm{E}}}\,. \tag{25}$$

Thus, the length ℓ_{E}, the E stands for Evans, is the new unknown constant. According to Evans, it should be negative.

The extended electromagnetic potential $\mathcal{A}^\alpha$ is represented four 1-forms,

$$\mathcal{A}^0 = \mathcal{A}_i{}^0\, dx^i\,, \;\; \mathcal{A}^1 = \mathcal{A}_i{}^1\, dx^i\,, \;\; \mathcal{A}^2 = \mathcal{A}_i{}^2\, dx^i\,, \;\; \mathcal{A}^3 = \mathcal{A}_i{}^3\, dx^i\,. \tag{26}$$

Thus, it has 16 independent components, quite a generalization as compared to the Maxwellian potential $A = A_i\, dx^i$ with only 4 independent components. Evans

[5] We denote Evans' constant $A^{(0)}$ by a_0, see our Table.

doesn't give a Lorentz covariant prescription of how to extract from $\mathcal{A}^\alpha$ the Maxwellian potential A. According to (23), $\mathcal{A}^\alpha$ transforms under a local *Lorentz transformation* $\Lambda_\beta{}^{\alpha'}$ like the coframe:

$$\mathcal{A}^{\alpha'} = \Lambda_\beta{}^{\alpha'} \mathcal{A}^\beta \,. \tag{27}$$

Suppose we try to identify the Maxwellian A with $\mathcal{A}^0$. Then, under a local Lorentz transformation, this identification is mixed up:

$$\mathcal{A}^{0'} = \Lambda_\beta{}^{0'} \mathcal{A}^\beta = \Lambda_0{}^{0'} \mathcal{A}^0 + \Lambda_1{}^{0'} \mathcal{A}^1 + \Lambda_2{}^{0'} \mathcal{A}^2 + \Lambda_3{}^{0'} \mathcal{A}^3 \,. \tag{28}$$

In the new frame, indicated by a prime, $\mathcal{A}^{0'}$ cannot be identified with A since it contains three non-Maxwellian admixtures. However, for the physical description the new frame is *equivalent* to the old one. In other words, *the identification of* $\mathcal{A}^0$ *as Maxwellian potential is not Lorentz covariant and has to be abandoned.* This is an inevitable consequence of the fact that $\mathcal{A}^\alpha$ transforms as a vector under the Lorentz group $SO(3,1)$, as it does, according to Evans' ansatz (23). Similar considerations apply to $\mathcal{A}^1$, $\mathcal{A}^2$, and $\mathcal{A}^3$.

One could try to kill the α-index in $\mathcal{A}^\alpha$ by some contractions procedure, such as $\vartheta_\alpha \wedge \mathcal{A}^\alpha$ or $e_\alpha \rfloor \mathcal{A}^\alpha$; however, the former yields a 2-form, the latter a 0-form. Also the Hodge star doesn't help, since ${}^\star\vartheta_\alpha \wedge \mathcal{A}^\alpha$, e.g., represents a 4-form. Since Maxwell's theory in a RC-spacetime is locally Lorentz covariant, the extraction of Maxwell's potential 1-form A from $\mathcal{A}^\alpha$ doesn't seem to be possible.

Evans also considers 3-dimensional spatial rotations $\rho_\beta{}^{\alpha'}$. The corresponding rotation group $O(3)$, is a subgroup of the Lorentz group $SO(1,3)$. Hence we can study the behavior of $\mathcal{A}^\alpha$ under these rotations:

$$\mathcal{A}^{\alpha'} = \rho_\beta{}^{\alpha'} \mathcal{A}^\beta \,. \tag{29}$$

This equation is contained in (27), which, additionally, encompasses *boosts* in three linearly independent directions. Clearly, the $O(3)$ is not the covariance group of $\mathcal{A}^\alpha$. It is just a subgroup of the $SO(1,3)$. An $O(3)$ covariant electromagnetic potential cannot be derived from the ansatz (23) in a Lorentz covariant way — in contrast to what Evans claims [24].

Thus, instead of the desired $O(3)$-covariant extended electrodynamics, Evans in fact, due to his ansatz (23), constructed willy nilly a $SO(1,3)$-covariant extended electrodynamics. Still, he insists that the $O(3)$-substructure has a meaning of its own; however, certainly not in a Lorentz covariant sense.

If we differentiate Evans' ansatz, we find for the extended electromagnetic field strength

$$\mathcal{F}^\alpha := D\mathcal{A}^\alpha = d\mathcal{A}^\alpha + \Gamma_\beta{}^\alpha \wedge \mathcal{A}^\beta \tag{30}$$

the relation[6]

$$\mathcal{F}^\alpha = a_0 \, T^\alpha \, . \tag{31}$$

Now we have 6×4 components of the extended electromagnetic field strength. As we have shown in Sec.2, the torsion T^α is a quantity related to translations and, accordingly, to energy-momentum. On the left-hand-side, we have an extended electromagnetic quantity that is eventually related to hypothetical extended electric currents. Also $\mathcal{F}^\alpha$, like the potential $\mathcal{A}^\alpha$, transforms as a vector under Lorentz transformations:

$$\mathcal{F}^{\alpha'} = \Lambda_\beta{}^{\alpha'} \mathcal{F}^\beta \, . \tag{32}$$

Before we turn to the extended electromagnetic field equations of Evans, let us first remind ourselves of the fundamental structure of Maxwell's theory. With the field strength 2-form $F = B + E \wedge dt$ and the excitation 2-form $H = \mathcal{D} - \mathcal{H} \wedge dt$, the homogeneous and the inhomogeneous Maxwell equations read, respectively,

$$dF = 0 \, , \qquad dH = J \, . \tag{33}$$

Here $J = \rho - j \wedge dt$ is the electric current 3-form. The homogeneous equation $dF = 0$ corresponds to *magnetic flux* conservation and the inhomogeneous one $dH = J$ is a consequence of *electric charge* conservation $dJ = 0$. Both equations correspond to separate physical facts and are thus independent from each other, see [38], where a corresponding axiomatic framework was set up.

In order to complete the theory, we have to specify, in addition to the Maxwell equations, a *constitutive law*. In *vacuum*, that is, in free space without space charges, the field strength and the excitation are related by

$$H = \frac{1}{\Omega_0} \, {}^\star F \, , \tag{34}$$

where $\Omega_0 = \sqrt{\mu_0/\varepsilon_0}$ is the impedance of free space. Now the Maxwell equations for vacuum can be put into the form[7]

$$dF = 0 \, , \qquad d\, {}^\star F = \Omega_0 \, J \, . \tag{35}$$

[6]As we mentioned in the Introduction, Cartan observed that the vector piece of the torsion $e_\alpha \rfloor T^\alpha$ has the same transformation behavior as Maxwell's potential A. In contrast, in (31) the field strength $\mathcal{F}^\alpha$ is involved and not the potential $\mathcal{A}^\alpha$.

[7] For no obvious reason, Evans writes $d\,{}^\star F = \mu_0 \, J$ instead. Apparently the ε_0 got lost.

Note that in no sense the inhomogeneous equation is the "dual" of the homogeneous one, or vice versa, provided $J \neq 0$.

3.2 Lorentz force density

In analogy to Maxwell's theory, we should have as Lorentz force density

$$f_\alpha = (e_\alpha \rfloor \mathcal{F}^\beta) \wedge \mathcal{J}_\beta \quad \text{(?)}, \tag{36}$$

with a Lorentz covariant electric current $\mathcal{J}_\alpha$, which we will discuss below. However, we didn't find a corresponding definition in Evans' work. Hence we marked this formula by a question mark.

3.3 "Homogeneous" field equation of extended electromagnetism

The exterior covariant derivative of the extended field strength (30) reads

$$D\mathcal{F}^\alpha = R_\beta{}^\alpha \wedge \mathcal{A}^\beta \qquad \text{or} \qquad d\mathcal{F}^\alpha = R_\beta{}^\alpha \wedge \mathcal{A}^\beta - \Gamma_\beta{}^\alpha \wedge \mathcal{F}^\beta \,. \tag{37}$$

This *Ricci identity for* $\mathcal{A}^\alpha$ poses in Evans' unified field theory as the extension of the homogeneous Maxwell equations. Eq.$(37)_1$ is the analog of the Maxwellian $dF = 0$.

If we follow Evans and substitute Evans' ansatz (23) into the right-hand-side of $(37)_2$, we have

$$d\mathcal{F}^\alpha = \Omega_0\, \mathcal{J}^\alpha_{\rm hom} \,, \tag{38}$$

with what Evans [20] calls the homogeneous current

$$\mathcal{J}^\alpha_{\rm hom} := \frac{a_0}{\Omega_0} \left(R_\beta{}^\alpha \wedge \vartheta^\beta - \Gamma_\beta{}^\alpha \wedge T^\beta \right) \,. \tag{39}$$

Eq.$(37)_1$ coincides with Evans [24], Eq.(20). However, [23], Eq.(29), which is also claimed to represent the homogeneous equation, seems simply wrong. We are not sure why Evans substitutes his ansatz only into the right-hand-side of $(37)_2$ and not completely into the whole equation, but this is just the way he did it in order to find his field equation.

It is strange that the "current" (39) depends on the torsion and thus on the extended electromagnetic field strength itself: $\mathcal{F}^\beta = a_0\, T^\beta$. Moreover, this *current is not Lorentz covariant* since its right-hand-side depends on the connection explicitly. The pseudo-conservation law

$$d\mathcal{J}^\alpha_{\rm hom} = 0 \,, \tag{40}$$

which follows from (38), is not Lorentz covariant either. Whereas the whole equation (38) is Lorentz covariant, as we recognize from $(37)_1$, its left-hand-side and its right-hand-side for themselves are *not* Lorentz covariant.

We differentiate $(37)_1$ covariantly and recall the second Bianchi identity (12):

$$DD\mathcal{F}^\alpha = R_\beta{}^\alpha \wedge \mathcal{F}^\beta \,. \tag{41}$$

For reasons unknown to us, Evans [24], p.442, calls this equation "the generally covariant wave equation". If $R_\beta{}^\alpha \wedge \mathcal{F}^\beta = 0$ — this corresponds to 4 conditions — he speaks of the condition for independent fields (no mutual interaction of gravitation and electromagnetism).

If (i) the curvature vanishes, $R_\beta{}^\alpha = 0$, and (ii) the frames are suitably chosen, $\Gamma_\beta{}^\alpha \stackrel{*}{=} 0$, then the field equation (37) of Evans' theory is really homogeneous: $d\mathcal{F}^\alpha \stackrel{*}{=} 0$. Otherwise we have to live with inhomogeneous terms. However, Evans claims the following ([24], p.440): *Experimentally* it is found that $(37)_2$ "must split into the particular solution"

$$d\mathcal{F}^\alpha = 0 \,, \tag{42}$$

$$\Gamma_\beta{}^\alpha \wedge \mathcal{F}^\beta = R_\beta{}^\alpha \wedge \mathcal{A}^\beta \,. \tag{43}$$

Clearly, Eqs.(42) and (43) represent an additional assumption. But note, neither (42) nor (43) is covariant under local Lorentz transformations of the frame and, accordingly, they are of very dubious value.

Since torsion is proportional to the extended electromagnetic field strength, the first Bianchi identity (11) and its contractions (17) and (19) are alternative versions of (37), provided one substitutes (31). Eq.(19) then reads

$$D\left(\eta_{\alpha\beta\gamma} \wedge \mathcal{F}^\gamma\right) + 2a_0\, \vartheta_{[\alpha} \wedge G_{\beta]} = 0 \,. \tag{44}$$

Now the Evans ansatz is exploited and, in order to get the extension of the inhomogeneous Maxwell equation, Evans had to invest a new idea.

3.4 Inhomogeneous field equation of extended electromagnetism

According to our evaluation, Evans' recipe amounts simply to take the homogeneous equation $(37)_1$ and to apply the substitution rule

$$\mathcal{F}^\alpha \rightarrow {}^\star\mathcal{F}^\alpha \quad \text{and} \quad R_\alpha{}^\beta \rightarrow {}^\star R_\alpha{}^\beta \tag{45}$$

to it. Then one finds

$$D\,{}^\star\mathcal{F}^\alpha = {}^\star R_\beta{}^\alpha \wedge \mathcal{A}^\beta \qquad \text{or} \qquad d\,{}^\star\mathcal{F}^\alpha = {}^\star R_\beta{}^\alpha \wedge \mathcal{A}^\beta - \Gamma_\beta{}^\alpha \wedge {}^\star\mathcal{F}^\beta\,. \tag{46}$$

Since $\mathcal{F}^\alpha$ as well as $R_\alpha{}^\beta$ are both 2-forms, the recipe is consistent. Eq.$(46)_1$ is the analog of the sourceless inhomogeneous Maxwell equation $d\,{}^\star F = 0$.

We substitute the ansatz (23) only into the right-hand-side of (46) and find

$$d\,{}^\star\mathcal{F}^\alpha = \Omega_0\,\mathcal{J}^\alpha_{\rm inh}\,, \tag{47}$$

with the inhomogeneous current

$$\mathcal{J}^\alpha_{\rm inh} := \frac{a_0}{\Omega_0}\left({}^\star R_\beta{}^\alpha \wedge \vartheta^\beta - \Gamma_\beta{}^\alpha \wedge {}^\star T^\beta\right)\,. \tag{48}$$

Evans [20] claims that (47) can be derived from (38) by applying the Hodge star to (38). However, this is not possible. Inter alia, he supposes erroneously that ${}^\star d\mathcal{F}^\alpha = d\,{}^\star\mathcal{F}^a$, see also the slides of Eckardt [19]. The inhomogeneous equation represents a new assumption that can be made plausible by the substitution rule (45).

As with the homogeneous current, we have again a pseudo-conservation law

$$d\mathcal{J}^\alpha_{\rm inh} = 0\,, \tag{49}$$

which is not Lorentz covariant either. We apply a Lorentz transformation to the left-hand-side of (49),

$$\Lambda_\beta{}^{\alpha'}(d\mathcal{J}^\alpha_{\rm inh}) = d\mathcal{J}^{\alpha'}_{\rm inh} - (d\Lambda_\beta{}^{\alpha'})\mathcal{J}^\beta_{\rm inh}\,. \tag{50}$$

The last term destroys Lorentz covariance. If we substitute (49), we find

$$d\mathcal{J}^{\alpha'}_{\rm inh} - (d\Lambda_\beta{}^{\alpha'})\mathcal{J}^\beta_{\rm inh} = 0\,. \tag{51}$$

Clearly, the law (49) is not Lorentz covariant. Analogous equations are true for $\mathcal{J}^\beta_{\rm hom}$.

If we write the inhomogeneous field equation in analogy to the inhomogeneous Maxwell equation with source $d\,{}^\star F = J$, we have

$$D\,{}^\star\mathcal{F}^\alpha = \mathcal{J}^\alpha \qquad \text{with} \qquad \mathcal{J}^\alpha := a_0\,{}^\star R_\beta{}^\alpha \wedge \vartheta^\beta\,. \tag{52}$$

The Lorentz covariant current $\mathcal{J}^\alpha$ seems to be the only current that could enter the definition (36) of the Lorentz force density. The currents $\mathcal{J}^\alpha_{\rm hom}$ or $\mathcal{J}^\alpha_{\rm inh}$ don't seem to qualify because of their lack of being Lorentz covariant; see, however, the next section. We differentiate $\mathcal{J}^\alpha$ covariantly:

$$D\mathcal{J}^\alpha = a_0 \left[(D\,{}^\star R_\beta{}^\alpha) \wedge \vartheta^\beta + {}^\star R_\beta{}^\alpha \wedge {}^\star T^\alpha \right] . \tag{53}$$

It is not conserved. *Local electric charge conservation* of classical electrodynamics $dJ = 0$ (note that we have only an exterior derivative here) is substituted by the four extended charge non-conservation laws (53). Local electric charge conservation, a law that is experimentally established to a high degree of accuracy (see Particle Data Group [56], p.91, and also Lämmerzahl [52]), *is irretrievably lost* since the connection $\Gamma_\alpha{}^\beta$ as well as the torsion T^α and the curvature $R_\alpha{}^\beta$ get involved in (53). In Maxwell's theory no such thing happens for $dJ = 0$.

3.5 Lorentz force density revisited

We discussed the Lorentz force density earlier, see (36), since it represents the key formula for the operational definition of the electromagnetic field strength. This should be also true in Evans' framework. However, Evans supplied no corresponding formula and, accordingly, his field strength $\mathcal{F}^\alpha$ has no operational support. However, after defining the homogeneous and the inhomogeneous currents, the following observation[8] helps:

The homogeneous current $\mathcal{J}_\alpha^{\rm hom}$ of Evans is of a magnetic type, whereas $\mathcal{J}_\alpha^{\rm inh}$ is of an electric type. Now we recall that in Maxwell's theory, if an independent magnetic current 3-form K is allowed for, the Maxwell equations read

$$dH = J\,, \qquad dF = K\,, \tag{54}$$

compare (33). If the Lorentz force density is adapted to this new situation, then we find, see Kaiser [48] and [39],

$$f_\alpha = (e_\alpha \rfloor F) \wedge J - (e_\alpha \rfloor H) \wedge K\,. \tag{55}$$

Let us translate this into Evans' framework,

$$F \rightarrow \mathcal{F}^\alpha\,, \quad K \rightarrow \Omega_0 \mathcal{J}_\alpha^{\rm hom}\,, \quad H \rightarrow \frac{1}{\Omega_0}\,{}^\star\mathcal{F}^\alpha\,, \quad J \rightarrow \mathcal{J}_\alpha^{\rm inh}\,, \tag{56}$$

that is,

$$f_\alpha = (e_\alpha \rfloor \mathcal{F}^\beta) \wedge \mathcal{J}_\beta^{\rm inh} - (e_\alpha \rfloor {}^\star\mathcal{F}^\beta) \wedge \mathcal{J}_\beta^{\rm hom}\,. \tag{57}$$

[8] I owe this observation to Robert G. Flower (private communication). It is also mentioned in Eckardt's workshop slides [19], as I found out later.

We substitute the currents (48) and (39):

$$f_\alpha = \frac{a_0}{\Omega_0}\left[(e_\alpha\rfloor\mathcal{F}^\beta)\wedge({}^\star R_{\gamma\beta}\wedge\vartheta^\gamma-\Gamma_{\gamma\beta}\wedge{}^\star T^\gamma)\right.$$
$$\left.-(e_\alpha\rfloor{}^\star\mathcal{F}^\beta)\wedge(R_{\gamma\beta}\wedge\vartheta^\gamma-\Gamma_{\gamma\beta}\wedge T^\gamma)\right]\,. \tag{58}$$

The noncovariant, connection dependent terms on the right-hand-side of (58) drop out, provided we substitute the Evans ansatz $\mathcal{F}^\alpha = a_0\,T^\alpha$. We are left with

$$f_\alpha = \frac{a_0}{\Omega_0}[(e_\alpha\rfloor\mathcal{F}^\beta)\wedge\underbrace{{}^\star R_{\gamma\beta}\wedge\vartheta^\gamma}_{\text{el.type cur.}}-(e_\alpha\rfloor{}^\star\mathcal{F}^\beta)\wedge\underbrace{R_{\gamma\beta}\wedge\vartheta^\gamma}_{\text{mg.type cur.}}]\,. \tag{59}$$

This formula fills the bill. The currents are those on the right-hand-sides of the covariantly extended Maxwell equations $(46)_1$ and $(37)_1$, respectively.

Eq.(59) represents the Lorentz force formula in Evans' theory. At the same time (59) supports our earlier conclusions that $\mathcal{J}_\alpha^{\rm hom}$ and $\mathcal{J}_\alpha^{\rm inh}$, being non-covariant, have no legitimate place as physical observables in Evans' theory. The "real currents" can only be read off from the right-hand-sides of the covariant electromagnetic field equations $(46)_1$ and $(37)_1$.

4 Gravitation: Evans adopted Einstein-Cartan theory of gravity

4.1 First field equation of gravity

According to Evans, the Einstein equation of general relativity needs to be generalized such the on the left-hand-side we have an asymmetric Einstein tensor based on RC-geometry and on the right-hand-side an asymmetric canonical energy-momentum tensor, see [25], p.103, Eq.(5.31). Then his generalized Einstein equation, valid for a spacetime obeying a RC-geometry, reads (in exterior calculus)

$$G_\alpha = \kappa\,\Sigma_\alpha \qquad \text{(first field eq.)}, \tag{60}$$

where G_α is the Einstein 3-form (16) and $\kappa := 8\pi G/c^3$ (called k by Evans), with G as Newton's gravitational constant and c the velocity of light. According to Evans, we have to understand Σ_α as canonical energy-momentum that "has an antisymmetric component representing canonical angular energy[9]/angular momentum" (see Evans [24], p. 437). Thus, we take the antisymmetric piece of (60),

$$\vartheta_{[\alpha}\wedge G_{\beta]} = \kappa\,\vartheta_{[\alpha}\wedge\Sigma_{\beta]}\,. \tag{61}$$

[9]Whatever angular energy may mean in this context.

4.2 Second field equation of gravity

It is known from special relativistic field theory, see Corson [16], Eq.(19.23a), that angular momentum conservation, with the canonical spin angular momentum current of matter $\tau_{\alpha\beta}$ and the canonical energy-momentum current of matter Σ_α, can be expressed as[10]

$$D\tau_{\alpha\beta} + \vartheta_{[\alpha} \wedge \Sigma_{\beta]} = 0\,. \tag{62}$$

In this form the law is also valid in a RC-spacetime, see [37].

Let us now take a look at the contracted first Bianchi identity (19). Then (19) and (62), substituted into (61), yield

$$D\,(C_{\alpha\beta} - \kappa\,\tau_{\alpha\beta}) = 0\,. \tag{63}$$

In this derivation, we invested the asymmetric Einstein equation à la Evans (rather à la Sciama-Kibble, see below), the generally accepted angular momentum law, and the contracted first Bianchi identity. Consequently, up to a gradient term, we find

$$C_{\alpha\beta} = \kappa\,\tau_{\alpha\beta} \qquad\qquad \text{(second field eq.)}. \tag{64}$$

Now we recall Evans' insistence that spin and torsion are equivalent (rather proportional to each other, we should say). Provided we drop the gradient term mentioned, we arrive at (64) — and this, indeed, expresses the proportionality of spin and torsion. Therefore, we have shown that (64), which is sometimes called Cartan equation, represents *a hidden tacit assumption of Evans' theory*. This proportionality between spin and torsion, which is *not* a geometrical property of torsion, but rather the result of picking (60) as one field equation for gravity, is always advocated by Evans in slogans, but never stated in an explicit formula, as far as I am aware. Because of the angular momentum law (62), it is clear that the spin $\tau_{\alpha\beta}$ in (64) is the spin of all matter, including that of the electromagnetic field. Similarly, the energy-momentum Σ_α in (62) and (60) represents the energy-momentum of all matter, including that of the electromagnetic field.

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[10] Corson [16] formulates angular momentum conservation in tensor calculus in Cartesian coordinates as $\partial_k \mathfrak{S}_{ij}{}^k - 2\mathfrak{T}_{[ij]} = 0$. Here $i, j, ... = 0, 1, 2, 3$ are holonomic coordinate indices and $\mathfrak{S}_{ij}{}^k$ and $\mathfrak{T}_{ij}$, in Corson's notation, canonical spin angular momentum and canonical energy-momentum, respectively. If we define the 3-forms of spin and energy-momentum as $\tau_{\alpha\beta} = \mathfrak{S}_{\alpha\beta}{}^\gamma \eta_\gamma$ and $\Sigma_\alpha = \mathfrak{T}_{\alpha\beta}\,\eta^\beta$, respectively, and substitute the partial by a covariant exterior derivative, then Corson's relation can be translated into (62). Note that $\mathfrak{S}_{\alpha\beta}{}^\gamma$ and $\mathfrak{T}_{\alpha\beta}$ are ordinary tensors here, not, however, tensor densities. We use the Gothic $\mathfrak{T}$ for energy-momentum in order not to confuse it with the T of the torsion.

Evans' states repeatedly that, within his theory, electromagnetism is an effect of spin. Let us translate that prose into a quantitative relation. For this purpose we have to resolve the second field equation[11] (64) with respect to the torsion T^γ:

$$T^\alpha = \kappa\,\eta^{\beta\gamma\delta\varepsilon}\left[\delta^\alpha_\beta(e_\gamma\rfloor\tau_{\delta\varepsilon}) - \frac{1}{4}\,\vartheta^\alpha\wedge(e_\beta\rfloor e_\gamma\rfloor\tau_{\delta\varepsilon})\right]. \qquad (65)$$

Using Evan's ansatz (31), this transforms into a relation between the extended electromagnetic field $\mathcal{F}^\alpha$ and the spin $\tau_{\gamma\delta}$:

$$\mathcal{F}^\alpha = a_0\kappa\,\eta^{\beta\gamma\delta\varepsilon}\left[\delta^\alpha_\beta(e_\gamma\rfloor\tau_{\delta\varepsilon}) - \frac{1}{4}\,\vartheta^\alpha\wedge(e_\beta\rfloor e_\gamma\rfloor\tau_{\delta\varepsilon})\right]. \qquad (66)$$

As soon as we have a source with spin, whatever the source may be, then, as a consequence of Evans' ansatz (80), an extended electromagnetic field is created via (66).

We would like to stress that (64) and (60) are *the field equations of the Einstein-Cartan theory of gravity*[12] (1961). In other words, without stating this explicitly anywhere, Evans just adopted, knowingly or unknowingly, the two field equations of the Einstein-Cartan theory. This insight makes a lot of his considerations more transparent.

In the EC-theory, the gravitational field variables are coframe ϑ^α and Lorentz connection $\Gamma^{\alpha\beta} = -\Gamma^{\beta\alpha}$. The *first* field equation corresponds to the variation of a Hilbert type Lagrangian with respect to the *coframe* and the *second* field equation with respect to the *Lorentz connection*. Consequently, the dynamics of ϑ^α and $\Gamma^{\alpha\beta}$ is controlled by the two field equations (60) and (64).

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[11] We multiply (64), with $C_{\alpha\beta}$ substituted according to (15), from the left with $e_\delta\rfloor$,

$$\frac{1}{2}(e_\delta\rfloor\eta_{\alpha\beta\gamma})\wedge T^\gamma - \frac{1}{2}\,\eta_{\alpha\beta\gamma}\,e_\delta\rfloor\wedge T^\gamma = \kappa\,e_\delta\rfloor\tau_{\alpha\beta}\,.$$

We have $e_\delta\rfloor\eta_{\alpha\beta\gamma} = \eta_{\alpha\beta\gamma\delta}$, see [37, 38]. Moreover, in order to kill the free indices α,β,δ, we multiply with $\eta^{\alpha\beta\delta\mu}$ and note $\eta_{\alpha\beta\gamma} = \eta_{\alpha\beta\gamma\nu}\,\vartheta^\nu$:

$$-\frac{1}{2}\,\eta^{\alpha\beta\delta\mu}\eta_{\alpha\beta\delta\gamma}\wedge T^\gamma - \frac{1}{2}\,\eta^{\alpha\beta\delta\mu}\eta_{\alpha\beta\gamma\nu}\,\vartheta^\nu\wedge e_\delta\rfloor T^\gamma = -\kappa\,\eta^{\mu\beta\gamma\delta}e_\beta\rfloor\tau_{\gamma\delta}\,.$$

After some algebra with the products of the η's, we find

$$T^\alpha = -\vartheta^\alpha\wedge e_\beta\rfloor T^\beta + \kappa\,\eta^{\alpha\beta\gamma\delta}e_\beta\rfloor\tau_{\gamma\delta}\,.$$

We determine the trace $e_\alpha\rfloor T^\alpha$ of the last equation and re-substitute. This yields the desired result.

[12] Evans calls (60) generously the "Evans field equation of gravity", see [25], p.465.

4.3 Trace of the first field equation

The trace of the first field equation (60) plays a big role in Evans' publications. Hence we want to determine it exactly. We multiply (60) by ϑ^α. Then we get a scalar-valued 4-form with only one independent component:

$$\vartheta^\alpha \wedge G_\alpha = \frac{1}{2}\,\vartheta^\alpha \wedge \eta_{\alpha\beta\gamma} \wedge R^{\beta\gamma} = \kappa\,\vartheta^\alpha \wedge \Sigma_\alpha\,. \tag{67}$$

After some light algebra, we find the 4-form (recall $\Sigma_a = \mathfrak{T}_\alpha{}^\beta \eta_\beta$)

$$\eta_{\beta\gamma} \wedge R^{\beta\gamma} = \kappa\,\vartheta^\alpha \wedge \Sigma_\alpha = \kappa\,\mathfrak{T}\eta\,, \tag{68}$$

with the trace of the canonical energy-momentum tensor $\mathfrak{T} := \mathfrak{T}_\alpha{}^\alpha$. By taking its Hodge dual, remembering (10) and ${}^\star\eta = {}^{\star\star}1 = -1$, we can put it into the scalar form

$$R = -\kappa\,\mathfrak{T}\,. \tag{69}$$

This is the generalization of Einstein's trace of his field equation $\widetilde{R} = -\kappa\,\mathfrak{t}$ to the more general case of EC-theory. With a *tilde* we denote the *Riemannian* part of a certain geometrical quantity (not to be confused with Evans' Hodge duality symbol). In general relativity, the source of Einstein's equation is the symmetric Hilbert energy-momentum tensor $\mathfrak{t}_{\alpha\beta} = \mathfrak{t}_{\beta\alpha}$; its trace we denote by $\mathfrak{t} := \mathfrak{t}_\alpha{}^\alpha$. The corresponding 3-form is $\sigma_\alpha = \mathfrak{t}_\alpha{}^\beta \eta_\beta$.

In order to make a quantitative comparison with general relativity, we decompose, within a RC-spacetime, the canonical Noether energy-momentum $\Sigma_\alpha = \mathfrak{T}_{\alpha\beta}\,\eta^\beta$ into the symmetric Hilbert energy-momentum σ_α and spin dependent terms according to [37]

$$\Sigma_\alpha = \sigma_\alpha - e_\beta \rfloor (T^\beta \wedge \mu_\alpha) + D\mu_\alpha\,. \tag{70}$$

The spin energy potential μ_α, a 2-form, is related to the spin angular momentum 3-form as follows: $\tau^{\alpha\beta} = \vartheta^{[\alpha} \wedge \mu^{\beta]}$. Similarly, we decompose the curvature scalar R into its Riemannian part $\widetilde{R}$ and torsion dependent terms. The calculations are quite involved. We defer them to the Appendix. We end up with the final relation

$$\widetilde{R}^{\alpha\beta} \wedge \eta_{\alpha\beta} = \kappa\,\left(\vartheta^\alpha \wedge \sigma_\alpha + K^{\alpha\beta} \wedge \tau_{\alpha\beta}\right)\,. \tag{71}$$

Here $K^{\alpha\beta}$ is the contortion 1-form defined in (7). The scalar version of (71) reads

$$\widetilde{R} = -\kappa\left[\mathfrak{t} + {}^\star(\tau_{\alpha\beta} \wedge K^{\alpha\beta})\right]\,. \tag{72}$$

Thus we recognize that in the EC-theory the Riemannian piece $\widetilde{R}$ of the curvature scalar R obeys a relation like in general relativity, however, the Einsteinian source $\mathfrak{t}$ has to be supplemented by a *spin-contortion term*.

Our trace formula (72) of the first field equation, which is an exact consequence of the EC-theory, should be distinguished from Evans' corresponding hand-waving expression like, e.g., [21], Eq.(17). The T in Evans' formula changes its meaning within that paper several times; moreover, he uses the "Einstein Ansatz" $R = -k\,T$ (in our notation $\widetilde{R} = -\kappa\,\mathfrak{t}$) even though he is in a RC-spacetime, where (72) should have been used instead.

4.4 Energy-momentum and angular momentum laws

Within the EC-theory, the energy-momentum law reads, see [35, 37, 55],

$$D\Sigma_\alpha = (e_\alpha \rfloor T^\beta) \wedge \Sigma_\beta + (e_\alpha \rfloor R^{\beta\gamma}) \wedge \tau_{\beta\gamma}\,. \tag{73}$$

Evans assumes incorrectly (as did Cartan in his original papers) that there has to be a zero on the right-hand-side of (73), see Evans [25], p. 464. This basic mistake, which has far-reaching consequences, if (73) is compared with (20), apparently induced Cartan to abandon his gravitational theory in a RC-spacetime. We recognize from (73) that, instead of a zero, there rather emerge gravitational Lorentz type forces of the structure *mass* $\times$ *torsion* $+$ *spin* $\times$ *curvature*. Remember, in electrodynamics we have *charge* $\times$ *field strength*.

The angular momentum law, as we saw in (62), keeps its form as in flat spacetime, namely

$$D\tau_{\alpha\beta} + \vartheta_{[\alpha} \wedge \Sigma_{\beta]} = 0\,. \tag{74}$$

4.5 Evans' wave equation as a redundant structure

It is puzzling, besides the structure we discussed up to now, Evans provides additionally a wave equation for the coframe ϑ^α. He derives it, see [25], p.149, Eq.(8.8), from the gravitational Lagrangian (in his notation)

$$L_{\rm Ev} = -\frac{c^2}{k}\left[\frac{1}{2}\,(\partial_\mu q^a_\nu)(\partial^\mu q^\nu_a) + \frac{R}{2} q^a_\nu q^\nu_a\right]\,. \tag{75}$$

It is astonishing, Evans presupposes a RC-spacetime; nevertheless, he takes partial derivatives that are not diffeomorphism invariant. In order to translate (75) into a respectable Lagrangian, we (i) substitute the partial by covariant derivatives $\partial_\mu \rightarrow$

D_μ, (ii) interpret $D_\mu q^a_\nu$ as $2D_{[\mu}q^a_{\nu]}$, and (iii) insert a missing factor[13] $\frac{1}{4}$. Then we have (in our notation)

$$L_{\mathrm{Ev}'} = -\frac{1}{2\kappa}\left(D\vartheta^\alpha \wedge {}^\star D\vartheta_\alpha + {}^\star R\right) . \tag{76}$$

With the definition of torsion and with (10), we can rewrite it as

$$L_{\mathrm{Ev}'} = -\frac{1}{2\kappa}\left[T^\alpha \wedge {}^\star T_\alpha + {}^\star(\vartheta_\alpha \wedge \vartheta_\beta) \wedge R^{\alpha\beta}\right] . \tag{77}$$

It is now obvious, this is a purely gravitational Lagrangian; note also the appearance of the gravitational constant in it. Lagrangians of this type have been widely investigated in the framework of the Poincaré gauge theory of gravity, see [33, 54]. There, in contrast to EC-theory with its R-Lagrangian, propagating torsion occurs. However, for Evans' theory, (77) is an incorrect Lagrangian. Only if we dropped the quadratic torsion term, would we recover the generalized Einstein equation (60) that Evans used from the very beginning. Therefore the Lagrangian $L_{\mathrm{Ev}'}$ is false. But it is more, it is a redundant structure at the same time.

Our argument is independent of the details of our translation procedure from (75) to (76). Evans' Lagrangian (75) depends on the gravitational constant and the only field variables present are ϑ^α and $\Gamma^{\alpha\beta}$, i.e., it *is* a gravitational field Lagrangian. However, since Evans postulates the validity of the generalized Einstein equation (60) and of the Cartan equation (64), the dynamics of the variable ϑ^a is already taken care of by (60) and (64). There is no place for a further wave equation.

In the framework of Evans' theory, the subculture that developed around Evans' wave equation, is largely inconsistent with Evans' theory proper, the latter of which will be defined exactly in Sec.5.2. Apparently, Evans is misunderstanding his own theory.

5 Assessment

5.1 Summary of the fundamental structure of Evans' theory

Since the publications of Evans and associates are not very transparent to us, we distilled from all their numerous papers and books the "spirit" of Evans' theory.

[13] Evans equates his expression $q^a_\nu\, q^\nu_a$ always consistently to 1, whereas 4 is correct, namely $e_\alpha \rfloor \vartheta^\alpha = \delta^\alpha_\alpha = 4$. The trace of the unit matrix in 4 dimensions is 4.

Geometry: Spacetime obeys a RC-geometry that can be described by an orthonormal coframe ϑ^α, a metric $g_{\alpha\beta} = \mathrm{diag}(+1,-1,-1,-1)$, and a Lorentz connection $\Gamma^{\alpha\beta} = -\Gamma^{\beta\alpha}$. In terms of these quantities, we can define torsion and curvature by, respectively,

$$
\begin{aligned}
T^\alpha &:= D\vartheta^\alpha\,, &(78)\\
R_\alpha{}^\beta &:= d\Gamma_\alpha{}^\beta - \Gamma_\alpha{}^\gamma\wedge\Gamma_\gamma{}^\beta\,. &(79)
\end{aligned}
$$

The Bianchi identities (11,12) and their contractions (19,20) follow therefrom.

Electromagnetism: Evans' ansatz relates an extended electromagnetic potential to the coframe,

$$\mathcal{A}^\alpha = a_0\,\vartheta^\alpha\,. \tag{80}$$

The electromagnetic field strength is defined according to

$$\mathcal{F}^\alpha := D\mathcal{A}^\alpha\,. \tag{81}$$

The extended homogeneous and inhomogeneous Maxwell equations read in Lorentz covariant form

$$D\mathcal{F}^\alpha = R_\beta{}^\alpha\wedge\mathcal{A}^\beta \qquad \text{and} \qquad D{}^\star\mathcal{F}^\alpha = {}^\star R_\beta{}^\alpha\wedge\mathcal{A}^\beta\,, \tag{82}$$

respectively. Alternatively, with Lorentz non-covariant sources and with partial substitution of (80) and (81), they can be rewritten as

$$
\begin{aligned}
d\mathcal{F}^\alpha &= \Omega_0\,\mathcal{J}^\alpha_{\rm hom}\,, &\qquad \mathcal{J}^\alpha_{\rm hom} &:= \frac{a_0}{\Omega_0}\left(R_\beta{}^\alpha\wedge\vartheta^\beta - \Gamma_\beta{}^\alpha\wedge T^\beta\right)\,, &(83)\\
d\,{}^\star\mathcal{F}^\alpha &= \Omega_0\,\mathcal{J}^\alpha_{\rm inh}\,, &\qquad \mathcal{J}^\alpha_{\rm inh} &:= \frac{a_0}{\Omega_0}\left({}^\star R_\beta{}^\alpha\wedge\vartheta^\beta - \Gamma_\beta{}^\alpha\wedge{}^\star T^\beta\right)\,. &(84)
\end{aligned}
$$

Gravitation: Evans assumes the EC-theory of gravity. Thus, the field equations are those of Sciama [62, 63] and Kibble [49], which were discovered in 1961:

$$
\begin{aligned}
\frac{1}{2}\,\eta_{\alpha\beta\gamma}\wedge R^{\beta\gamma} &= \kappa\,\Sigma_\alpha = \kappa\left(\Sigma^{\rm mat}_\alpha + \Sigma^{\rm elmg}_\alpha\right)\,, &(85)\\
\frac{1}{2}\,\eta_{\alpha\beta\gamma}\wedge T^{\gamma} &= \kappa\,\tau_{\alpha\beta} = \kappa\left(\tau^{\rm mat}_{\alpha\beta} + \tau^{\rm elmg}_{\alpha\beta}\right)\,. &(86)
\end{aligned}
$$

Here $\eta_{\alpha\beta\gamma} = {}^\star(\vartheta_\alpha\wedge\vartheta_\beta\wedge\vartheta_\gamma)$. The total energy-momentum of matter plus electromagnetic field is denoted by Σ_α, the corresponding total spin by $\tau_{\alpha\beta}$.

5.2 Five cornerstones define Evans' unified field theory

In order to prevent misunderstandings, I'd like to define clearly what I understand as Evans' unified field theory. Such a statement, which overlaps with the last subsection, seems necessary since there are numerous inconsistencies and mistakes in Evans' work, see Bruhn [2–11] and Rodrigues et al. [18, 58], such that it is necessary to distiguish between the relevant and the irrelevant parts of Evans' articles. Let me formulate what I consider to be the five cornerstones of Evans' theory:

1. Physics takes place in a Riemann-Cartan spacetime, see (78) and (79).

2. The extended electromagnetic potential is proportional to the coframe, see (80), and the extended electromagnetic field strength to the torsion, see (81).

3. The extended Maxwell equations are given by (82).

4. The Einstein equation gets generalized such that on its left-hand-side we have the asymmetric Einstein tensor of a Riemann-Cartan spacetime and on its right-hand-side, multiplied with the gravitational constant, there acts as source the asymmetic canonical energy-momentum tensor of the extended electromagnetic field plus that of matter, see (85).

5. Torsion is proportional to spin.

One may wonder what Evans understood exactly as spin. However, since he specified the canonical energy-momentum tensor under cornerstone 4, we concluded that he opts likewise for the corresponding spin angular momentum tensor under cornerstone 5. This all the more, since Evans [24], p. 437, mentioned the *canonical* spin explicitly. Starting from cornerstone 4, we were able to show, using only the angular momentum law and a piece of the first Bianchi identity, that cornerstone 5 implies the second field equation (86).

There is not more than these five cornerstones. Our conclusions in this paper and the one accompaying it [41] are derived only from these 5 cornerstones by the use of the appropriate mathematics.

We disregarded the following two main points:

A) The antisymmetric part of the metric. Evans has some small talk about it mixed with partially incorrect formulas, see Bruhn [7]. Because of cornerstone 1, an asymmetric metric is excluded. Hence we didn't follow this train of thoughts of Evans any longer.

B) Evans derived a wave equation for the *coframe* in a not too transparent way, see [25], p.149, Eq.(8.5). All the results in the context of this wave equation we don't consider to belong to Evans' theory proper, as defined above. Since the generalized Einstein equation of cornerstone 4, together with cornerstone 5, rules already the dynamics of the coframe — after all, one can find the generalized Einstein equation by variation of the curvature scalar with respect to the coframe — there is no place for a further equation of motion for the coframe.

In the accompanying paper [41] we propose a variational principle for Evans' theory that reproduces the facts mentioned in cornerstones 1 to 5. In this context, there emerges an additional piece $D\,^\star T_\alpha$ on the right-hand-side of the generalized Einstein equation, which, because of $D\,^\star T_\alpha = D\,^\star D\vartheta_\alpha$, is, indeed, in the linearized version a wave operator applied to the coframe. And this structure is reminiscent of those in Evans' wave equation. However, our result was achieved by just taking the five cornerstones for granted and by constructing an appropriate Lagrangian. We didn't use any additional assumption, whereas Evans introduces his wave equation as an ad hoc structure without consistent motivation.

5.3 Points against Evans' theory

5.3.1 Electrodynamics has nothing to do with the geometry of spacetime

In gravity the experimentally well established equality of inertial and gravitational mass $m_{\rm in} = m_{\rm gr}$ is a fundamental feature. It is the basis of Einstein's equivalence principle and of a *geometric* interpretation of gravity in the framework of general relativity. The *universality* of this feature is decisive. Since there is no physical object without energy-momentum, the equivalence principle applies equally well to all of them, without any known exception.

Is there a similar physical effect known in electromagnetism? No, not to my knowledge. Rather, the decisive features of electromagnetism are electric charge and magnetic flux conservation (yielding the Maxwell equations [38]). And these conservation laws have nothing to do with spacetime symmetries, whereas energy-momentum, the source in Einstein's gravitational theory, is related, via Noether's theorem, to *translations* in spacetime. In the Maxwell-Dirac theory (Maxwell's theory with a Dirac electron as source), electric charge conservation emerges due to the $U(1)$ phase (gauge) invariance of the theory, that is, due to an *internal* symmetry (unrelated to external, i.e., spacetime symmetries). Moreover, charge conservation is universally valid. However, it has nothing to say about electrically and magnetically *neutral* matter, as, e.g., the neutrinos $\nu_{\rm e}, \nu_\mu, \nu_\tau$, the photon γ, the gauge boson Z, the neutral pion π^0, etc.

Evans provides *no new insight* into this question. His only argument is that any ansatz (like his $\mathcal{A}^\alpha = a_0\vartheta^\alpha$) must be permitted and only experiments can decide on its validity. However, Evans' ansatz $\mathcal{A}^\alpha = a_0\vartheta^\alpha$ *presupposes* that electromagnetism, like the coframe ϑ^α, is a universal phenomenon, which it isn't, since neutral matter is exempt from it. The lack of universality of electromagnetism makes its geometrization a futile undertaking.

This argument is sufficient for me to exclude Evans' theory right from the beginning. However, some people, like Evans himself, don't find it so convincing. Therefore we collect more evidence.

5.3.2 Uncharged particles with spin and charged particles without spin cause unsurmountable problems for Evans' theory

Take a neutrino, say the electron neutrino ν_e. It has no electric charge ($< 10^{-14}$ electron charges), no magnetic moment ($< 10^{-10}$ Bohr magnetons), and no charge radius squared [$< (-2.97$ to $4.14) \times 10^{-32}$ cm^2], see [56]. Hence the ν_e is electromagnetically neutral in every sense of the word. But is carries spin $1/2$. Consequently, according to Evans' doctrine, see (66), it should create an electromagnetic field, But halt, this cannot be true! A neutrino creating an electromagnetic field? Even Evans abhors such an idea. And his remedy? For a neutrino we have to put $a_0 = 0$, is Evans' stunning answer to a corresponding question, see Evans' blog.[14] A *unified* field theory of *geometric* type that switches off a coupling constant for a certain type of matter, doesn't it lose all credentials?

Complementary is the charged pion $\pi^\pm$. It carries electric charge but *no* spin. Evan concludes[15] that it cannot carry an electromagnetic field either!

Of course, according to Evans' ansatz $\mathcal{A}^\alpha = a_0\vartheta^\alpha$, electromagnetism is assumed to be an universal phenomenon. Since this assumption is incorrect, Evans' theory must run into difficulties for neutral and for spinless matter willy nilly.

[14]http://www.atomicprecision.com/blog/2007/02/19/elementary-particles-charge-and-spin-of-ece-theory-2/

[15]http://www.atomicprecision.com/blog/2007/02/19/elementary-particles-charge-and-spin-of-ece-theory/

5.3.3 There doesn't exist a scalar electric charge, electric charge conservation is violated

In Maxwell's theory the current J integrated over a (3-dimensional) spacelike hypersurface Ω_3 yields a 4-dimensional scalar charge $\int_{\Omega_3} J$. In Evans' theory no such structure is available since any current $\mathcal{J}^\alpha$, because it is vector-valued, doesn't qualify as an integrand. Accordingly, in Evans' theory, a global electric charge cannot be defined in a Lorentz covariant way.

By the same token, as was shown in (53), electric charge conservation is violated: $D\mathcal{J}^\alpha \neq 0$. Under such circumstances even the concept of a test charge is dubious. Charge conservation is a law of nature. Exceptions are not known, see the experimental results collected by the Particle Data Group [56]. Therefore Evans' theory grossly contradicts experiment.

To take Evans' $\mathcal{J}^\alpha_{\rm hom}$ or $\mathcal{J}^\alpha_{\rm inh}$ as a substitute for a decent conserved current is impossible, even when $d\mathcal{J}^\alpha_{\rm hom} = 0$ and $d\mathcal{J}^\alpha_{\rm inh} = 0$. They both, $\mathcal{J}^\alpha_{\rm hom}$ and $\mathcal{J}^\alpha_{\rm inh}$, depend explicitly on the connection and don't transform as Lorentz vectors. Their physical interpretation, as given by Evans, since not Lorentz covariant, is null and void. The expression $d\mathcal{J}^\alpha_{\rm hom} = D\mathcal{J}^\alpha_{\rm hom} - \Gamma_\beta{}^\alpha \wedge \mathcal{J}^\beta_{\rm hom}$ is not Lorentz covariant either and thus unsuitable for the formulation of a law of physics, in contrast to Evans' claims to the opposite. An analogous consideration applies to $\mathcal{J}^\alpha_{\rm inh}$.

5.3.4 There doesn't exist a well-defined Maxwellian limit, the superposition principle is violated

According to our considerations in Sec.3.1, we cannot extract from the $SO(1,3)$ electrodynamics proposed by Evans in a Lorentz covariant way an $O(3)$ sub-electrodynamics, the latter of which Evans claims to be a physical theory. Moreover, we have shown that the index α in $\mathcal{A}^\alpha$ cannot be compensated in a Lorentz covariant way such as to find the Maxwellian potential A in some limit. Thus, we have a potential $\mathcal{A}^\alpha$ with 16 independent components and we don't know what to do with them, provided we insist on Lorentz covariance.

Bruhn [3] has even shown explicitly that a plane wave in Evans' $O(3)$ electrodynamics, if subject to a Lorentz transformation, will not be any longer a plane wave. A proof cannot be more telling. In addition, Bruhn [10] pointed out in detail how Evans suppresses the undesired $\mathcal{A}^0$ component of his potential in order to arrive at his $O(3)$ structure, compare also Bruhn and Lakhtakia [11, 51].

Wielandt [69] demonstrated that the superposition principle, valid in Maxwell's theory, breaks down in Evans' $O(3)$ electrodynamics. In a non-linear theory this is inevitable. However, the superposition principle cannot even be recovered for small amplitudes and under suitable supplementary conditions. In this sense, Maxwell's theory as a limiting case seems to be excluded.

5.3.5 Evans' theory is not really unified

The energy-momentum and spin angular momentum 3-forms of matter $\Sigma_\alpha^{\rm mat}$ and $\tau_{\alpha\beta}^{\rm mat}$, entering the two field equations (85) and (86), have to be determined form other physical theories, like from Dirac's electron theory. Thus Evans' theory is not really unified.

On top of these five main counterarguments — remember that one conclusive counterargument is enough to kill a theory — we were able to formulate a variational principle for Evans' theory:

5.3.6 Evans' theory is trivial and collapses to general relativity in all physical cases

As Obukhov and the author have shown in an accompanying paper [41], Evans theory can be characterized by a dimensionless constant

$$\xi := \frac{a_0^2\kappa}{\Omega_0}\,, \tag{87}$$

a fact that was apparently overlooked by Evans. If Evans' ansatz for a unified field theory is to be taken seriously, then certainly one would expect a_0, and thus ξ, to be an universal constant that cannot be adjusted freely (see, however, Evans' treatment of the neutrino that was discussed above).

We proposed a variational principle [41] with a Lagrange multiplier term that enforces Evans' ansatz. This approach reproduces all features of Evans' theory. We find two field equations with 10 + 24 independent components, respectively. The second field equation, it is (64) with the spin of the $\mathcal{A}^\alpha$ field on its right-hand-side, is algebraically linear in torsion and can be solved. In all physical cases, the torsion vanishes completely and, because of $\mathcal{F}^\alpha = a_0\,T^\alpha$, Evans' extended electromagnetic field vanishes, too. Consequently, in all physical cases Evans' theory collapses to the Einstein vacuum field equation.

Probably Evans will argue that he doesn't like our variational principle and that our principle ammends the inhomogeneous electromagnetic field equation $(82)_2$ and the first gravitational field equation (85) with terms induced by the Lagrangian multiplier. And that these terms are not contained in his original theory. This is true. However, we have shown a consistent way (we believe, it is the only way) to include Evans' ansatz $\mathcal{A}^\alpha = a_0\,\vartheta^\alpha$ into the the electromagnetic and gravitational field equations of Evans' theory. If Evans rejects our variational principle, he will have a problem. If he substitutes his ansatz into the extended Maxwell equations (82), he will get field equations for ϑ^α and $\Gamma^{\alpha\beta}$, which are of second order in ϑ^α (basically wave type equations); if he substitutes his ansatz also into the gravitational field equations (85) and (86), which, after an elimination prodecure, are also of second order in ϑ^α, how will he guarantee that these two different sets of wave type equations are consistent with each other? Clearly, this cannot be guaranteed. However, our Lagrange multiplier method does guarantee consistency.

We put this point at the end of our list, since this consequence is *not* inevitable. By abolishing a Hilbert type Lagrangian and going over to a Lagrangian quadratic in torsion and/or in curvature ("Poincaré gauge theory"), one could ameliorate this situation, see, e.g., Itin and Kaniel [46, 47], Obukhov [54], and Heinicke et al. [43]. However, we won't do that because the reasons given above exclude an approach à la Evans. Still, for more than 20 years it is known of how to make torsion a propagating field, see Sezgin and van Nieuwenhuizen [64] and Kuhfuss and Nitsch [50]. Evans' theory just documents that his author is not clever enough to propose a nontrivial model.

6 Conclusion

Around the year 2003, Evans grafted his ill-conceived $O(3)$-electrodynamics on the viable Einstein-Cartan theory of gravity, calling it a unified field theory. The hybrid that he created has numerous genetic defects; some of them are lethal.

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7 Appendix: Decomposing the trace of the first field equation

We start from (70), namely

$$\Sigma_\alpha = \sigma_\alpha - e_\beta \rfloor (T^\beta \wedge \mu_\alpha) + D\mu_\alpha \,, \tag{88}$$

and from $\tau^{\alpha\beta} = \vartheta^{[\alpha} \wedge \mu^{\beta]}$. The inverse of the latter relation reads [37]

$$\mu_\alpha = -2 e_\beta \rfloor \tau_\alpha{}^\beta + \frac{1}{2}\, \vartheta_\alpha \wedge (e_\beta \rfloor e_\gamma \rfloor \tau^{\beta\gamma}) \,, \tag{89}$$

and its contraction is

$$\vartheta^\alpha \wedge \mu_\alpha = 2 e_\alpha \rfloor (\vartheta^\beta \wedge \tau_\beta{}^\alpha) \,. \tag{90}$$

Now we recall, see (68), that we only need the contraction of (88) with ϑ^α:

$$\vartheta^\alpha \wedge \Sigma_\alpha = \vartheta^a \wedge \sigma_\alpha + e_\beta \rfloor \left(\vartheta^\alpha \wedge \mu_\alpha \wedge T^\beta\right) - d\left(\vartheta^\alpha \wedge \mu_\alpha\right) \,. \tag{91}$$

This will be substituted in (68). By using (90), we find

$$\begin{aligned} R^{\alpha\beta} \wedge \eta_{\alpha\beta} \;\; = \;\; & \kappa \left\{ \vartheta^\alpha \wedge \sigma_\alpha + 2 e_\alpha \rfloor \left[T^\alpha \wedge e^\beta \rfloor (\vartheta^\gamma \wedge \tau_{\gamma\beta}) \right] \right. \\ & \left. -2d \left[e_\alpha \rfloor (\vartheta^\beta \wedge \tau_\beta{}^\alpha) \right] \right\} \,. \end{aligned} \tag{92}$$

Obviously, we can now eliminate the spin $\tau_{\alpha\beta}$ by contracting the second field equation (64),

$$\kappa\, \vartheta^\beta \wedge \tau_{\beta\alpha} = \eta_{\alpha\beta} \wedge T^\beta \,, \tag{93}$$

and substituting it in (92). This yields

$$\begin{aligned} R^{\alpha\beta} \wedge \eta_{\alpha\beta} \;\; = \;\; & \kappa\, \vartheta^\alpha \wedge \sigma_\alpha + 2 e_\alpha \rfloor \left[T^\alpha \wedge e^\beta \rfloor (\eta_{\beta\gamma} \wedge T^\gamma) \right] \\ & -2d \left[e_\alpha \rfloor (\eta^\alpha{}_\beta \wedge T^\beta) \right] \,. \end{aligned} \tag{94}$$

Some algebra shows that the second term on the right-hand-side vanishes and that $e_\alpha \rfloor (\eta^\alpha{}_\beta \wedge T^\beta) = \vartheta^\alpha \wedge {}^\star T_\alpha$. Thus,

$$R^{\alpha\beta} \wedge \eta_{\alpha\beta} = \kappa\, \vartheta^\alpha \wedge \sigma_\alpha - 2d\left(\vartheta^\alpha \wedge {}^\star T_\alpha\right) \,. \tag{95}$$

This is a remarkably simple formula. The first term $\kappa\, \vartheta^\alpha \wedge \sigma_a$ is the Einsteinian trace, the second one represents a correction by torsion and hence by spin.

We can now study the effect of spin on the Riemannian piece $\widetilde{R}$ of the curvature scalar R. For that purpose, we start from the geometrical decomposition formula [37, 42, 54]

$$\begin{aligned} R^{\alpha\beta} \wedge \eta_{\alpha\beta} &= \widetilde{R}^{\alpha\beta} \wedge \eta_{\alpha\beta} + K^{\alpha\mu} \wedge K_{\mu}{}^{\beta} \wedge \eta_{\alpha\beta} - K^{\alpha\beta} \wedge T^{\gamma} \wedge \eta_{\alpha\beta\gamma} \\ &\quad -2d(\vartheta^{\alpha} \wedge {}^{\star}T_{\alpha}) \, . \end{aligned} \tag{96}$$

The second and the third terms on the right-hand-side can be collected. Then,

$$R^{\alpha\beta} \wedge \eta_{\alpha\beta} = \widetilde{R}^{\alpha\beta} \wedge \eta_{\alpha\beta} - \frac{1}{2} K^{\alpha\beta} \wedge T^{\gamma} \wedge \eta_{\alpha\beta\gamma} - 2d(\vartheta^{\alpha} \wedge {}^{\star}T_{\alpha}) \, . \tag{97}$$

The latter equation is substituted into (95). The derivatives drop out and we are left with

$$\widetilde{R}^{\alpha\beta} \wedge \eta_{\alpha\beta} = \kappa \, \vartheta^{\alpha} \wedge \sigma_{\alpha} - \frac{1}{2} \eta_{\alpha\beta\gamma} \wedge T^{\gamma} \wedge K^{\alpha\beta} \, . \tag{98}$$

Clearly, the second field equation can be re-substituted and we arrive[16] at

$$\widetilde{R}^{\alpha\beta} \wedge \eta_{\alpha\beta} = \kappa \, \left(\vartheta^{\alpha} \wedge \sigma_{\alpha} + K^{\alpha\beta} \wedge \tau_{\alpha\beta} \right) \, . \tag{99}$$

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