COMPUTATIONAL ANATOMY: COMPUTING METRICS ON ANATOMICAL SHAPES

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ABSTRACT

An important area of research in Computational Anatomy is to assign a metric space structure to 2D/3D images of anatomical structures. The images are registered in the non-rigid dense large deformation setting by computing a diffeomorphic transformation between the given images. The metric distance on the images follows from the Lie Group structure of diffeomorphisms, which allows measurement of lengths of curves on the manifold of diffeomorphisms. We present here a gradient-based method to compute the diffeomorphism matching the given images and estimating the metric distance for the pair. We show results for matching 2D sections of canine heart images.

1. INTRODUCTION

Whole brain anatomical atlases at sub-millimeter resolution are emerging from the Human Brain Mapping and National Partnership in Advanced Computational Infrastructure (NPACI) initiatives. Analysis and inference based on these anatomical volumes present a challenge in the emerging field of computational anatomy [8]. Specifically, we wish to compute metric distances between images of anatomical configurations, with the aim that these will provide information on the differences in shape and size of the anatomical configurations present in the images. Such information is desired with the end-application of being useful in aiding decisions on "close" and "far" for clinical applications.

In the deformable template model, the anatomical configurations are an orbit under the group $\mathcal{G}$ of diffeomorphic transformations acting on the coordinate space of the configurations. With this model, given two images I_0, I_1 in the orbit of a template, there exists an element of the diffeomorphisms matching the images $I_1 = \phi I_0$. In the small deformation dense volume matching and sparse landmark matching pioneered by Bajcsy et. al., Bookstein et. al. [1, 3, 4], the transformations are computed by perturbations of the underlying coordinate space arising from elasticity models. The mappings generated by this method are invertible in the small-deformation setting, but are not so when registrations of images requires large deformations. They are not diffeomorphims in general and do not support a metric space structure. Christensen et. al [6] introduced the large deformations setting constraining the mappings for dense image data registration to be the solutions of an Eulerian ODE forcing the transport to be a flow of diffeomorphisms. Dupuis et. al. [7] proved the conditions for the existence of solutions in the group of diffeomorphisms for the dense image matching problem.

Simultaneously, the connection to metric distances on the space of diffeomorphisms was established via work of Trouvé [11, 12, 13] utilizing the property of the Lie group structure of diffeomorphisms, which is a Riemannian manifold. By a suitable choice of a Riemannian metric in the tangent space of this manifold, the space of diffeomorphisms is converted into a metric space via measurement of geodesic length between points on the manifold. In this short paper, we review the framework for assignment of metrics on the space of images in an orbit of the group of diffeomorphisms, present the gradient of a cost functional to estimate the diffeomorphism that matches the given images in an orbit and provide an estimate of the metric distance

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between the images. The derivation of the gradient, its implementation and numerical issues with more detailed results are presented in [2].

2. REVIEW OF FRAMEWORK FOR METRICS ON ORBITS OF IMAGES

Let the background space Ω be a bounded domain with piecewise C^1 boundary on $\mathbb{R}^n$ on which the image functions $I : \Omega \to \mathbb{R}^d$ are defined. Let $\mathcal{G}$ be a subgroup of $\mathrm{Hom}(\Omega)$ (for instance the set $\mathrm{Diff}(\Omega)$). The set $\mathrm{Hom}(\Omega)$ is a group with the law of composition $\psi \cdot \phi \doteq \psi \circ \phi$ and for any image I, $\phi I \doteq I \circ \phi^{-1}$ defines an action of $\mathrm{Hom}(\Omega)$ on the set of images. Given a template I_{template}, an anatomical ensemble is the orbit

$$\mathcal{I} \doteq \{ \phi I_{\text{template}} \mid \phi \in \mathcal{G} \}$$

of I_{template} under the action of $\mathcal{G}$. Given two anatomical images I_0 and I_1 in the orbit, identify the first image with the identity element in $\mathcal{G}$ and the second image with an element $\phi_1 \in \mathcal{G}$. The unknown diffeomorphism $\phi_1 \in \mathcal{G}$ registering the given images $I_1 = \phi_1 I_0$ is the end-point of a flow associated to a smooth compactly supported time-dependent vector field $v \in C_c^\infty((0,1) \times \Omega, \mathbb{R}^n)$. The flow $\phi : [0,1] \to \mathcal{G}$ satisfies for any $x \in \Omega$:

$$\frac{\partial \phi_t}{\partial t}(x) = v_t(\phi_t(x)) \quad \text{with} \quad \phi_0(x) = x. \quad (1)$$

The notation ϕ_t^v is used to make explicit the dependance of ϕ_t with its associated velocity field v and $\phi_{s,t} : \Omega \to \Omega$ is used to denote the composition $\phi_{s,t} = \phi_t \circ (\phi_s)^{-1}$. The interpretation of $\phi_{s,t}(y)$ is that it is the position at time t of a particle that is at position y at time s.

Consider a Hilbert space V, with dot product $\langle\, ,\, \rangle_V$, containing the space $C_c^\infty(\Omega, \mathbb{R}^n)$ of smooth compactly supported vector fields on Ω. The length and the energy of the curves $t \to \phi_t^v$ in the space of smooth diffeomorphisms on Ω are given by:

$$\text{Length} = \mathrm{L}(\phi^v) = \int_0^1 \|v_t\|_V \mathrm{d}t = \int_0^1 \sqrt{\langle v_t, v_t \rangle_V} \mathrm{d}t \quad (2)$$

$$\text{Energy} = \mathrm{E}(\phi^v) = \int_0^1 \|v_t\|_V^2 \mathrm{d}t = \int_0^1 \langle v_t, v_t \rangle_V \mathrm{d}t \quad (3)$$

The dot product $\langle\, ,\, \rangle_V$ is defined through a differential operator L on $C_c^\infty(\Omega, \mathbb{R}^n)$ such that:

$$\langle u, v \rangle_V \doteq \langle Lu, Lv \rangle_2,$$

where $\langle\, ,\, \rangle_2$ is the usual L^2-product for square integrable vector fields on Ω and that V is the completion of $C_c^\infty(\Omega, \mathbb{R}^n)$ for the chosen dot-product. The flow of $v \in L^1([0,1], V)$ generates the diffeomorphisms in $\mathcal{G} \doteq \{ \varphi \mid \varphi = \phi_1^v, \ v \in L^1([0,1], V) \}$ making $\mathcal{G}$ a sub-group of C^1-diffeomorphisms of $\mathrm{Hom}(\Omega)$ and further,

$$\rho_{\mathcal{G}}(\varphi_0, \varphi_1) \doteq \inf\{ \int_0^1 \|v_t\|_V dt \mid \varphi_1 = \phi_1^v \circ \varphi_0 \} \quad (4)$$

defines a distance on $\mathcal{G}$ for which $\mathcal{G}$ is complete. Moreover, from the definition (4) of the distance $\rho_{\mathcal{G}}$, we get immediately the right-invariance property $\rho_{\mathcal{G}}(\varphi_0, \varphi_1) = \rho_{\mathcal{G}}(\varphi_0 \circ \varphi, \varphi_1 \circ \varphi)$. The metric distance on the space of images is defined by:

$$\rho_{\mathcal{I}}(I_0, I_1) \doteq \inf\{ \rho_{\mathcal{G}}(\mathrm{Id}, \varphi_1) \mid I_1 = \varphi_1 I_0, \ \varphi_1 \in \mathcal{G} \}, \quad (5)$$

and this metric is positive, symmetric and satisfies the triangle-inequality [14, 9]. The distance presented only deals with the deformation aspects, and is not invariant by rigid transformations, which is not an issue for many application, for which rigid registration is given, or at least may be recovered by standard algorithms. If necessary, the metric can be modified to incorporate rigid deformation invariance [5].

3. GRADIENT-DESCENT BASED ESTIMATION OF DIFFEOMORPHISMS FOR IMAGE MATCHING

In the present framework, computation of the metric distance between two given images requires the estimation of the diffeomorphism that matches the given images. This estimation involves finding the point ϕ_1 that matches the given images and finding the infimum length curve joining this point with the identity in the manifold of diffeomorphisms. The infimum of the length functional (2) over all possible such paths is computed by the optimization of the energy functional of the flow (3), as the critical points of the two coincide. Another term that measures the mismatch of the image I_0 composed with the estimate of the matching diffeomorphism ϕ_1 from the image I_1 provides the term in the cost, the optimization of which drives the matching towards the desired solution ϕ_1. This gives

$$E(v) \doteq \int_0^1 \|v_t\|_V^2 dt + \frac{1}{\sigma^2}\int_\Omega |I_0 \circ \phi_{1,0}^v(y) - I_1(y)|^2 dy. \quad (6)$$

as the cost whose optimization in the space V provides the simultaneous estimation of the matching diffeomorphism and the estimation of the shortest length path to that point, and falls in the category of "inexact" matching schemes. The variational gradient of this cost function is calculated by perturbing velocity $v \in L^2([0,1], V)$ an ϵ amount along direction $h \in L^2([0,1], V)$ giving the Gâteaux variation $\partial_h E(v)$, related to its Fréchet derivative $\nabla_v E$ by

$$\partial_h E(v) = \lim_{\varepsilon \to 0} \frac{E(v+\varepsilon h) - E(v)}{\varepsilon} = \int_0^1 \langle \nabla_v E, h \rangle_V dt. \quad (7)$$

The gradient of the cost, in this form, becomes

$$\nabla_v E = 2v_t - K\left(\frac{2}{\sigma^2}|D\phi_{t,1}^v| \nabla J_t^0 \left[J_t^0 - J_t^1\right]\right) \quad (8)$$

where $J_t^0 = I_0 \circ \phi_{t,0}^v, J_t^1 = I_1 \circ \phi_{t,1}^v$, $|D\phi_{t,1}^v|, \nabla J_t^0$ are the determinant of the Jacobian and the gradient respectively of the functions $\phi_{t,1}^v$ and J_t^0 respectively. The operator $K : L^2(\Omega, \mathbb{R}^d) \to L^2(\Omega, \mathbb{R}^d)$ is a compact self-adjoint operator such that when V is defined from a differential operator L, one gets for any smooth vector field $a \in C_c(\Omega, \mathbb{R}^d)$, $K(L^\dagger L)a = a$ where $L^\dagger$ is the adjoint of L. This variational gradient is used in a standard gradient based scheme exploiting the vector space structure of V yielding the update

$$v^{n+1} = v^n - \epsilon \nabla_{v^n} E \quad (9)$$

where n denotes the simulation number.

4. NUMERICAL RESULTS

The experiment presented involved computing the metric distance between two corresponding 2D transverse sections from two canine heart images. The images were first registered using the software "Analyze" [10] to remove rigid rotation and translation. The time interval $[0,1]$ of the flow is discretized into 20 steps, with each step being of length $\delta t = 0.1$. The Cauchy-Navier operator was chosen to be $L = -0.01\nabla^2 + I$. The estimated diffeomorphism is shown in 1(g) which gives the image mismatch error to be 7.12% of the error without the mapping, and the metric distance estimated for this image pair is 6.1277.

5. DISCUSSION AND CONCLUSION

The main contribution of this work is to present a gradient descent based method for computing the diffeomorphism in non-rigid dense image registration and estimation of metrics for images. We present an application of this algorithm to compute the metric distance between two canine heart images. As this is a short paper, we refer the reader to [2] for a detailed discussion of the numerical issues, comparison to previous work and more examples of applications of this technique. While the metric distance between two images in isolation does not provide much useful information regarding the shape and size of anatomical configurations present therein, however when considered with many such metric distances for images mapped to a common anatomical reference image, these metrics may provide useful information quantifying relative "close" and "far", information that may be of potential use in aiding clinical diagnosis and treatment.

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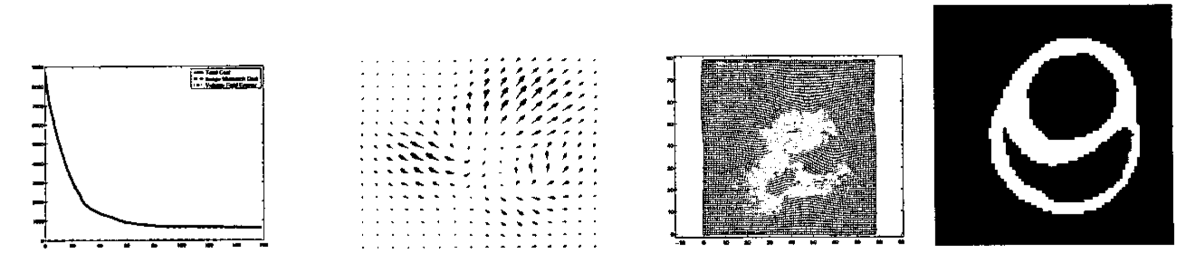
(a) I_0 Metric (a) 0 — (b) $I_0 \circ \phi_{t_6,0}$ (b) 2.1447 — (c) $I_0 \circ \phi_{t_{12},0}$ (c) 3.9830 — (d) $I_0 \circ \phi_{t_{19},0}$ (d) 6.1277



(e) Cost vs Simulation-Step — (f) Superposed Velocity Field — (g)$\phi_{1,0}$ — (h) I_1

Fig. 1. "Heart Mapping Experiment". Top Row: (a)Image I_0, (b,c,d)I_0 composed with estimated mappings for along the discretized flow $t_j \in [0,1], j \in [0,19]$. The estimated diffeomorphism ϕ_1 is shown in (g), and $\phi_1 I_0$ is shown in (d), which is to be compared to the image I_1 in (h). The cost as a function of simulation number is shown in (e) and a plot of the velocity field along time superposed on a single plot is.shown in (f).

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