The plane strain/stress boundary conditions for the problem are:

- The surfaces at $r = a$ ($\hat{\mathbf{n}} = \hat{\mathbf{e}}_r$) and $r = b$ ($\hat{\mathbf{n}} = -\hat{\mathbf{e}}_r$) are traction-free ($\mathbf{t} = \mathbf{0}$), which gives

$$\sigma_{rr} = \sigma_{r\theta} = 0 \quad \text{on } r = a \text{ and } r = b \; .$$

- The surface at $\theta = \pi/2$ is fixed:

$$u_r = u_\theta = 0 \quad \text{on } \theta = \frac{\pi}{2} \; .$$

- The traction distribution on the surface $\theta = 0$ ($\hat{\mathbf{n}} = -\hat{\mathbf{e}}_\theta \implies t_r = -\sigma_{\theta r} \; , \; t_\theta = -\sigma_{\theta\theta}$) is statically requivalent to $\mathbf{P} = -P\hat{\mathbf{e}}_1 = -P\hat{\mathbf{e}}_r$:

$$\begin{aligned}
F_1 &= \int_a^b t_r \, dr = -P && \implies && \int_a^b \sigma_{\theta r}|_{\theta=0} \, dr = P \; , \\
F_2 &= \int_a^b t_\theta \, dr = 0 && \implies && \int_a^b \sigma_{\theta\theta}|_{\theta=0} \, dr = 0 \; , \\
M_0 &= \int_a^b r t_\theta \, dr = 0 && \implies && \int_a^b r\sigma_{\theta\theta}|_{\theta=0} \, dr = 0 \; .
\end{aligned}$$

For the proposed Airy stress function of the form

$$\Phi = \left(Ar^3 + \frac{B}{r} + Cr \ln r \right) \sin\theta \; ,$$

check first that the compatibility condition $\nabla^4 \Phi = 0$ is satisfied:

$$\begin{aligned}
\nabla^2 \Phi &= \frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} = \left(8Ar + \frac{2C}{r} \right) \sin\theta \; , \\
\nabla^4 \Phi &= \nabla^2(\nabla^2 \Phi) = \left(\frac{4C}{r^3} + \frac{8A}{r} - \frac{2C}{r^3} - \frac{8A}{r} - \frac{2C}{r^3} \right) \sin\theta = 0 \; .
\end{aligned}$$

Next, the components of stress are

$$\begin{aligned}
\sigma_{rr} &= \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2} = \left(2Ar - \frac{2B}{r^3} + \frac{C}{r} \right) \sin\theta \; , \\
\sigma_{\theta\theta} &= \frac{\partial^2 \Phi}{\partial r^2} = \left(6Ar + \frac{2B}{r^3} + \frac{C}{r} \right) \sin\theta \; , \\
\sigma_{r\theta} &= -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \Phi}{\partial \theta} \right) = -\left(2Ar - \frac{2B}{r^3} + \frac{C}{r} \right) \cos\theta \; .
\end{aligned}$$

The boundary conditions $\sigma_{rr} = \sigma_{r\theta} = 0$ on $r = a$ and $r = b$ are satisfied if and only if

$$\left. \begin{aligned}
2Aa - \frac{2B}{a^3} + \frac{C}{a} = 0 \\
2Ab - \frac{2B}{b^3} + \frac{C}{b} = 0
\end{aligned} \right\} \implies B = -Aa^2b^2 \; , \quad C = -2A(a^2 + b^2) \; .$$

Now the components of stress are

$$\sigma_{rr} = 2A\left(r + \frac{a^2b^2}{r^3} - \frac{a^2+b^2}{r}\right)\sin\theta \ ,$$
$$\sigma_{\theta\theta} = 2A\left(3r - \frac{a^2b^2}{r^3} - \frac{a^2+b^2}{r}\right)\sin\theta \ ,$$
$$\sigma_{r\theta} = -2A\left(r + \frac{a^2b^2}{r^3} - \frac{a^2+b^2}{r}\right)\cos\theta \ .$$

For the average boundary conditions at $\theta = 0$, since $\sigma_{\theta\theta}|_{\theta=0} = 0$,

$$\int_a^b \sigma_{\theta\theta}|_{\theta=0}\,dr = 0 \ , \qquad \int_a^b r\sigma_{\theta\theta}|_{\theta=0}\,dr = 0 \ ,$$

are identically satisfied. Finally, since

$$\int_a^b \sigma_{r\theta}|_{\theta=0}\,dr = -2A\int_a^b \left(r + \frac{a^2b^2}{r^3} - \frac{a^2+b^2}{r}\right)dr = -2A\left[b^2 - a^2 - (a^2+b^2)\ln\frac{b}{a}\right] = P \ ,$$

it follows that

$$\boxed{A = -\frac{P}{2N} \ , \quad B = \frac{Pa^2b^2}{2N} \ , \quad C = \frac{P(a^2+b^2)}{N} \ ,}$$

where

$$\boxed{N \equiv b^2 - a^2 - (a^2+b^2)\ln\frac{b}{a} \ .}$$

Note that the fixed-displacement boundary condition at $\theta = \pi/2$ is not satisfied—the solution is approximate in the sense of Saint-Venant's principle.