



# The influence of real-world rail head roughness on railway noise prediction

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## Abstract

Harmonised methods for the prediction of noise from roads, railways, aviation and industry will eventually be available as a result of the HARMONOISE and IMAGINE EC projects, but not in the immediate future. As the first round of noise mapping under Directive 2002/49/EC is required to be completed in 2007, alternative models are required. The options for railways under the Directive are either to use the Dutch national procedure as the approved EC “interim method” or for Member States to use their own method if it can be adapted to produce results in terms of the indicators required by the Directive. In the UK it is therefore likely that the national method “Calculation of Railway Noise 1995” (CRN) will be applied. This paper discusses a shortcoming in CRN that results from its assumption that the rail head is comparatively smooth. In reality this is often not the case, leading to increased rolling noise levels. A statistically based study has been carried out for the UK Department for Environment, Food and Rural Affairs into the effects of rail head roughness on rolling noise prediction. Correction factors have been devised to account for the true roughness of rails in the UK, either at a local level or across the entire network. The acoustic effectiveness of rail grinding strategies has also been examined.

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## 1. Introduction

The European Directive 2002/49/EC [1] relating to the assessment and management of environmental noise requires that noise mapping be carried out by 2007 for railways with 60,000+ movements per year and also for railways in “agglomerations” (urban areas) of over 250,000 people. The Directive also requires maps to be produced of aviation, roads and, in urban areas, industry. Further mapping is required by 2012 for railways with 30,000+ movements per year and also for those in agglomerations of over 100,000 people. These mapping exercises are to be followed by Action Plans to manage noise. Although harmonised methods for prediction of noise from road, rail, aviation and industry are expected from the Harmonoise and Imagine EC projects, these will not be available in time to be used in the 2007 mapping. The options for railways in the absence of harmonised methods are either to use the Dutch national procedure as the approved EC “interim method” or for Member States to use their own method if it can be adapted to produce results in terms of the

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indicators required by the Directive. In the UK it is therefore likely that the national method “Calculation of Railway Noise 1995” (CRN) [2] will be applied. It is also likely that CRN will be used in connection with the railway noise mapping element of the current pilot exercise for England funded by the UK Department for Environment, Food and Rural Affairs (Defra).

However, this method does not allow the variability of rail head roughness that occurs over the network to be taken into account when predicting the rolling noise. The reason for this is that CRN was developed for application under the Noise Insulation (Railways and other Guided Transport Systems) Regulations for Railways 1996 [3] for new or additional railways, where the assumption is that rails will be new and therefore smooth. It is well known that rolling noise is a function of the combined roughness at the wheel/rail interface. It is also known that rough track, especially where “corrugations” (periodic wear patterns with a wavelength typically ranging from 30 to 80 mm) are present, can lead to an increase of 20 dB or more in *A*-weighted rolling noise. If, therefore, CRN were to be used in its standard form to produce the noise maps required by the Directive, it is possible that specific locations where rail head corrugation is present may be 20 dB or more noisier than the procedure would indicate, which could seriously discredit the process. Of equal concern is the fact that a rail head grinding strategy to remove corrugations and maintain smooth rails may form part of an Action Plan as required by the Directive. Predictions of noise before and after grinding using the current version of CRN would show no change.

Defra therefore commissioned a study into the effects on noise prediction of the true distribution of rail head roughness across the network. The study was required to determine whether it was feasible, and of use, to derive roughness corrections for CRN to apply at the end of the standard calculation process. These corrections would be designed (a) to account for the prevailing real levels of rail head roughness in the UK and (b) to allow for the effects of rail grinding strategies to be catered for in the modelling.

## 2. The database of rail roughness used within the study

It is possible to measure the roughness of the rail head with high precision using devices that either consist of static frames attached to the rail, within which are displacement transducers that pass along the rail head, or by using trolleys with probes coupled to accelerometers that can gather data at walking pace [4–6]. Neither of these approaches is practical, however, when it is required to quantify rail head roughness over large sections of the railway network. Traditionally, track recording vehicles have used accelerometers attached to wheel axle boxes (bearing housings) to assess the level of rail head corrugation. However, these tend not to produce useful results when considering roughness at the wavelengths that affect rolling noise because the wheel modes will modify the measured vibration. A more useful approach for the purpose of considering the effects of rail head roughness on rolling noise over a large section of track is to measure the rolling noise directly beneath a vehicle in the vicinity of the wheelset. AEA Technology has developed a system based on this concept (NoiseMon) that can be installed on passenger trains routinely traversing the network, with Global Positioning System (GPS) location and cell phone connection to a base station, allowing the track owner or maintainer to monitor the system continuously. By using a vehicle with comparatively smooth (disc-braked) wheels, combined roughness at the wheel–rail interface (and hence rolling noise level) tends to be dominated by rail roughness.

Fig. 1 shows the nature of noise levels measured over a range of track sections with this vehicle-mounted system. It can be seen that at any given speed there is a range of values, with a lower bound representing the smoothest combined wheel–rail roughness likely to be encountered. The highest value at each speed will represent rough, or corrugated, track. For a section of track with a known roughness the speed dependence of the under-floor noise level can be represented by

$$L_2 = L_1 + 10 \times n \times \log_{10} \left( \frac{v_2}{v_1} \right), \quad (1)$$

where  $L_1$  is the noise level at speed  $v_1$ ,  $L_2$  is the noise level at speed  $v_2$  and  $n$  is the “speed exponent”.

The value of  $n$  can be determined for each under-floor installation by considering the slope of the lower bound of data of the type shown in Fig 1. This approach has a degree of uncertainty associated with it because of the scatter of data and the fact that the sound spectrum shape is affected by speed, but for the purposes of

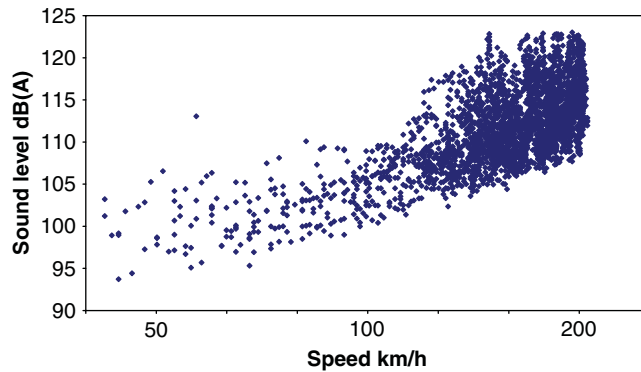


Fig. 1. Example characteristics of under-floor rolling noise level vs speed.

the current exercise, where statistical trends are considered, this is an acceptable approximation.  $n$  is normally found to have a value of around 3, which also applies to wayside values when considering the maximum pass-by level.

If  $L_1$  and  $v_1$  are measured values, then, by fixing  $v_2$  (usually at 160 km/h), the equation can be used to calculate the level ( $L_2$ ) “normalised” to this standard speed.

A large database of rail head roughness information over the UK network has been acquired by these means, and this, combined with track-side measurements at locations where on-train measurement data were available, was used as the basis for this study.

### 3. The study approach

CRN predicts the noise generated from rolling and also from locomotives when on power. The rolling noise element is based on a “Baseline Sound Exposure Level (SEL)” at 25 m, which is a function of speed. This baseline SEL is then corrected for vehicle type from a table of values for a range of vehicles in operation in the UK in 1995 when the Procedure was published. As rolling noise is a function of combined wheel and rail roughness, and as CRN assumes track in comparatively good condition, the wheel roughness is a major factor in determining this correction. Therefore, a typical cast-iron tread-braked vehicle, the Mk 2 coach, has a correction factor of +14.8 dB, while the correction for a disc-braked, and hence smooth-wheeled, Mk 3 coach is +6 dB, i.e. 8 dB quieter on “CRN” track. CRN, in its standard form, enables the  $L_{eq}$  from segments of track to be calculated, taking into account traffic speed, flow and composition, and also propagation of sound over various terrains and structures. These  $L_{eq}$  values can then be used as the basis for calculating the  $L_{den}$  (day, evening, night) and  $L_{night}$  as required under the Environmental Noise Directive [1] as follows:

$$L_{den} = 10 \lg \frac{1}{24} (12 \times 10^{L_{day}/10} + 4 \times 10^{(L_{evening}+5)/10} + 8 \times 10^{(L_{night}+10)/10}), \quad (2)$$

where  $L_{day}$  is the  $A$ -weighted long-term average sound level determined over all the day periods of a year,  $L_{evening}$  is the  $A$ -weighted long-term average sound level determined over all the evening periods of a year,  $L_{night}$  is the  $A$ -weighted long-term average sound level determined over all the night periods of a year, and the default day, evening and night periods are 0700–1900, 1900–2300 and 2300–0700, respectively.

In order to use the rolling noise data acquired under vehicles over the UK network to establish the effects of rail roughness on the prediction of  $L_{den}$  and  $L_{night}$ , it was necessary to obtain the transfer function between under-floor levels and track-side levels.

Fig. 2 is the measured relationship between the speed-adjusted  $L_A$  at 25 m from acoustically similar vehicles and the speed-adjusted  $L_A$  measured under the vehicle at that location. The noise measured at the track side comprises the noise originating from a length of track and not just a short section nearest to the measurement position. Therefore, the under-floor data were averaged over a 200 m length of track.

Fig. 2 also includes a straight line that has a slope of 1 and a constant equal to the average difference between the speed-adjusted under-floor and track-side measurements. The slope of 1 means that for a specific

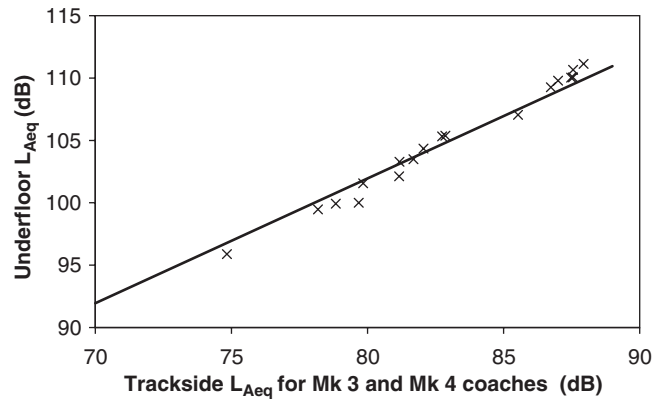


Fig. 2. Measured transfer function for  $L_{eq}$  measured at a microphone under a disc-braked train to whole vehicle  $L_{eq}$  measured at the track side, 25 m from the nearest rail from acoustically similar disc-braked trains: ×, measured; —, straight line with a slope of 1 and an intercept of 22 dB.

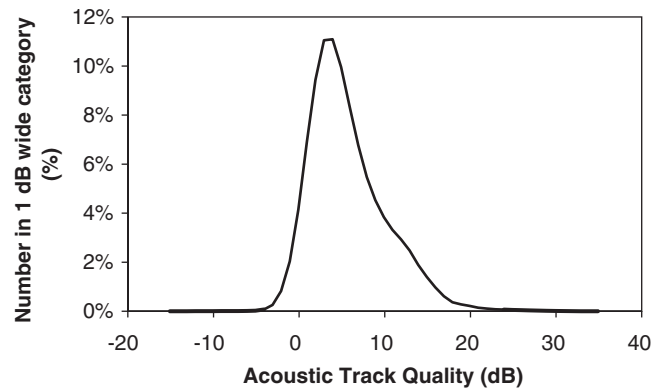


Fig. 3. The distribution of ATQ on a typical UK track.

change in the noise levels measured under the train there is an identical change in the noise at the track side. As the measured data closely follow this line, the intercept of 22 dB can safely be taken as the transfer function for this particular under-floor microphone position.

Having established the relationship between the under-floor and track-side noise data, the under-floor noise level that produces a pass-by noise level equal to the value produced by CRN can be calculated. This was done by calculating the Transit Exposure Level (TEL) from the SEL predicted by CRN:

$$TEL = SEL - 10 \log_{10}(T_p), \tag{3}$$

where  $T_p$  is the time the train takes to pass (“buffer to buffer”).

The advantage of using the TEL is that it is independent of the number of vehicles in the train and approximates to the measured pass-by  $L_{Aeq}$  of part of the train. If the train comprises identical vehicles, then the TEL can be derived from the measured SEL.

By applying CRN, the *A*-weighted TEL at 25 m from the track for a rake of Mk 3 coaches travelling at 160 km/h is 84 dB. Using the difference calculated from the data shown in Fig. 2, the *A*-weighted under-floor level that gives a value at the track side that would agree with CRN is 106 dB. Because a change in the noise under the train produces a corresponding change in the rail/wheel noise at the track side, the amount by which the speed-normalised (to 160 km/h) under-floor level exceeds 106 dB is a measure of how much an Mk 3 or Mk 4 coach would exceed the level predicted by CRN at that location. This difference is known as the acoustic track quality (ATQ), and is plotted in Fig. 3 in terms of its distribution over a large section of the UK network.

The aim of this study was to consider the implications on noise predictions of a level of rail roughness different from that assumed in CRN. However, because a very noisy track occurs comparatively infrequently, is often only one of two or more tracks at that location, and has trains passing over it at a range of speeds, the implications are complex. In this study a statistical approach has been followed, where the measured current variation in the condition of the running surface of the rail is combined with the types and speeds of trains found at 18 example locations within the UK. These locations were chosen to represent the wide range of railway traffic types found in the UK and to include (a) sites with only diesel trains, (b) those where multiple units dominate and (c) those where electric trains dominate.

Fig. 4 is a flow diagram of the basic steps involved in the calculations.

The initial train speeds and type of trains were obtained from the traffic previously observed at a number of sites. These were supplemented by information on other trains that are subsequently known to pass a site.

Because CRN contains data for a limited number of types of railway vehicle, additional archive data, measured under appropriate conditions, were used when available. However, there remained some vehicles with no CRN, or measured, data available (or where the measured data were not acquired under the CRN conditions). In this case, levels were predicted. For rolling noise, this was done by using the fact that the noise level depends largely on the number of wheels and whether or not the train has cast-iron tread brakes.

At each site, the condition of the rail was selected at random from the distribution in Fig. 3, representing major sections of the UK network. Using a technique developed in connection with the West Coast Main Line upgrade, the available train noise source data were adjusted for the condition of the rail. These adjusted data were then used to predict the noise levels at each site by applying the techniques in CRN to obtain  $L_{den}$  and  $L_{night}$  for a location 25 m from one side of the railway. These predicted levels were then compared with the level obtained from the standard application of CRN. This process was repeated over a million times at each site so that a statistically significant measure of the average effect of the condition of the rail could be obtained. From these data a correction that allows CRN predictions to be adjusted so that they reflect the levels that would be found at an 'average' location in the UK could be derived.

For vehicles with smooth wheels (such as the Mk 3 and Mk 4 coach), the ATQ curve could be used to adjust the CRN source term simply by adding the ATQ value to the CRN source term. For wheels that have a roughness that makes a significant contribution to the total surface roughness, the situation is more complex. At low levels of rail roughness, the difference between smooth and rough wheels will remain relatively constant. However, at very high levels of rail roughness, when the surface roughness of the rail dominates, the

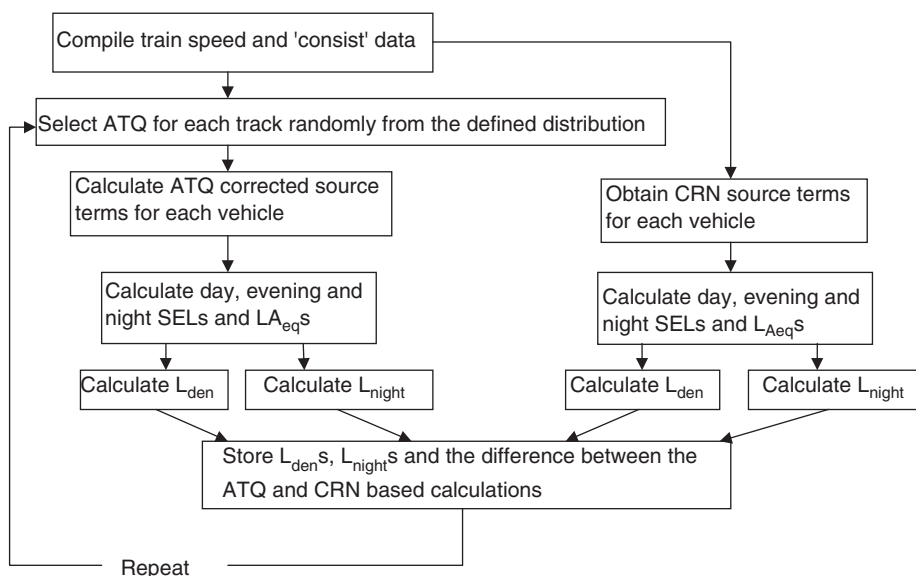


Fig. 4. Flow diagram of steps used in calculations.

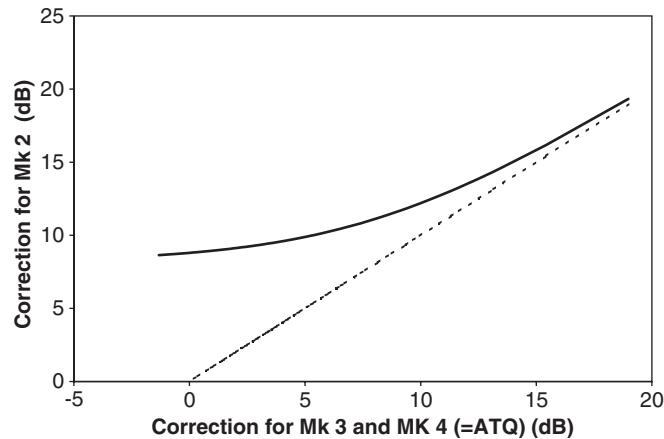


Fig. 5. The relationship between the CRN correction required for a cast-iron tread-braked coach (Mk 2) and the correction required for a disc-braked (Mk 3 or Mk 4) coach: —, measured line; - - - - - , straight line with a slope of 1.

roughness of the wheel is no longer significant and the noise from smooth and rough wheels will be approximately the same.

Fig. 5 shows the relationship between the CRN correction for disc-braked vs cast-iron tread-braked stock used in this study. The “measured line” (the best fit line to available data) crosses the vertical axis at 8.8 dB, which is the difference between the CRN corrections for an Mk 2 and Mk 3 coach. Because there are few measured noise data available for a track with very low roughness, some of the data shown have been derived from directly measured roughness. The ‘straight line with a slope of 1’ represents the relationship that occurs when rail roughness dominates (for example, when rail roughness is very high). It can be seen in Fig. 5 that the ‘measured line’ approaches the ‘straight line’ when the correction level is high.

Fig. 5 can be considered to show the relationship between the ATQ and the CRN-type correction for an Mk 1 or Mk 2 cast-iron tread-braked coach. In practice, similar curves can be derived for other types of vehicles.

#### 4. Derivation of post-calculation roughness correction to CRN

Using the data from the predictions, a range of parameters was examined to see how well they correlated with the ATQ-corrected predictions minus the CRN predictions. These parameters included the  $L_{den}$  and  $L_{night}$  predicted using CRN, the average speed of trains past the site, the number of wheels with cast-iron tread brakes, the number of wheels that were powered, the number of diesel locomotives and the number of multiple units. Fig. 6 shows data for the parameter that has the best correlation with the ATQ prediction minus the CRN prediction, namely the “flow-weighted” speed at each site. The  $L_{night}$  data produce very similar results.

Although Fig. 6 does show a clear trend with average site speed, the spread is large. Attempts to improve the correlation by using multiple dependent variables produced no statistically significant improvement in the correlation.

The range of  $\pm 1$  dB contains over 70% of the data and a range of  $\pm 2$  dB includes 95% of the data. Given the other likely errors in any form of prediction, this is an acceptable tolerance. Carrying out the same exercise with the  $L_{night}$  produces essentially the same result.

Based on these findings, the following single roughness correction to the complete CRN prediction was derived for all the data:

$$\text{Correction} = 8.33 \log_{10}(\bar{v} + 21) - 15 \text{ dB} \quad (\text{above } 42 \text{ km/h}),$$

$$\text{Correction} = 0 \text{ dB} \quad (\text{below } 42 \text{ km/h}),$$

where  $\bar{v}$  is the average speed of all the individual trains passing the site in km/h. At low speeds, the noise from the traction equipment will dominate and this is why there is no correction below 42 km/h. Clearly, the largest correction occurs at the highest speeds because rolling noise will dominate at these locations.

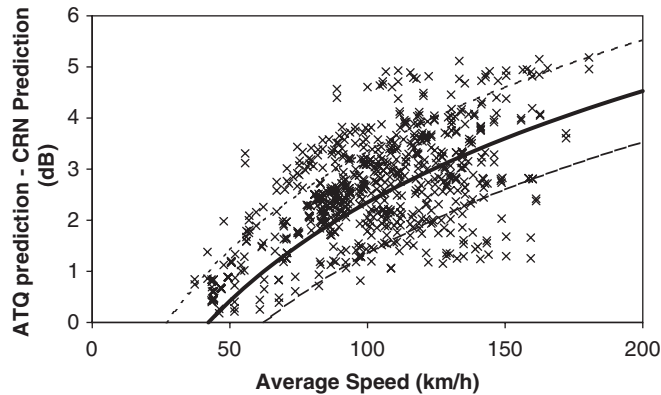


Fig. 6. The relationship between the average speed of the trains and the ATQ-corrected prediction minus the CRN prediction for the  $L_{den}$  with 'best fit' line: x, predicted data; ---, 'best fit' line  $-1$  dB; —, 'best fit' line; - - - - -, 'best fit' line  $+1$  dB.

The roughness correction provides a global indicator of the effects of typical current UK rail condition on the overall noise emission from the existing fleet under typical operating conditions.  $L_{den}$  and  $L_{night}$  can therefore be predicted for the entire country using the correction and will provide, on average, a significantly closer representation of the population's noise exposure than CRN would predict in its standard form. The statistical reliability of the approach can be improved somewhat (depending on the speed distribution at the site) by calculating the CRN values at a receiver position separately for each "speed group" of trains passing the site and then roughness-correcting before combining, rather than defining a single "flow-weighted" speed value at the site.

## 5. Effect of rail grinding strategies

By grinding the rail head, it should be possible to modify the shape of the ATQ distribution. This is because the track can be ground to produce a smoother rail head. The simulations could then be re-run to find the overall effect of this grinding. However, a difficulty arises in establishing the consequences of grinding and how quickly roughness will subsequently re-develop. The rate of rail roughness growth is very variable and cannot be easily predicted. One "worst case" assumption is that after a short period of time the newly ground rails will have levels of ATQ with the highest levels removed and the remainder distributed throughout the rest of the existing distribution. This is probably what would happen if grinding were based purely on a trigger value of ATQ.

When the track is ground initially, the rail roughness will be determined by the grinding marks. These grinding marks quickly roll out and the rail head becomes smooth. However, the roughness soon starts to develop at different rates. For this study it was necessary to decide what will be the distribution of the ATQ for ground track over a period of time. One scenario is to assume that the ground track has the same distribution as all the unground track below the trigger point. An alternative scenario is that the ground track remains smooth.

When deciding to grind based on the ATQ, there will be some deterioration between the time at which the track reaches the trigger point and the point at which it is ground. This will mean that, instead of the distribution being truncated at the trigger level, there will be a smooth transition towards the  $x$ -axis. The point where the distribution meets the  $x$ -axis will depend on the maximum increase in the ATQ that can occur in the time between the trigger point being reached and the track being ground. To assess the impact of this delay, the following two cases have been considered: (a) the ATQ deteriorates by up to 3 dB before grinding, (b) the ATQ deteriorates by up to 10 dB before grinding. In each case, it is assumed that there will be a relatively smooth transition between the trigger point and the upper limit. The shape of the distributions will depend on the grinding trigger point. Fig. 7 shows the distributions for a trigger point of  $ATQ = 6$  dB, which represents grinding 33% of the noisiest track. In practice, it may prove difficult to grind this amount of track. However, it does illustrate the effect of the different grinding strategies.

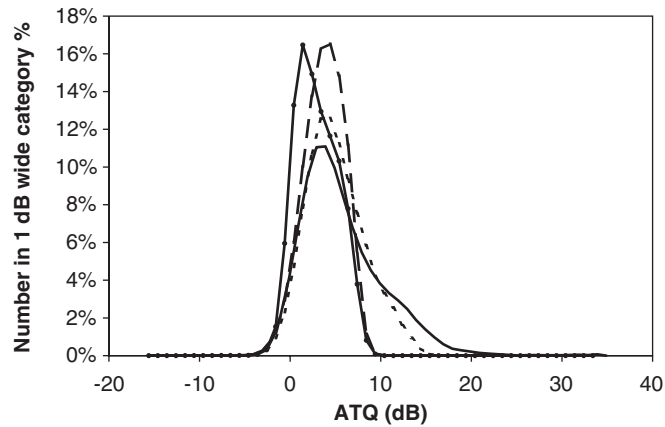


Fig. 7. Distribution of the ATQ for different rail grinding strategies with an ATQ threshold of 6 dB (33% of the current track would require grinding): —, original; ---, even distribution 3 dB deterioration; - · - ·, even distribution 10 dB deterioration; —◇—, all  $\pm 2$  dB of ATQ of 0 dB.

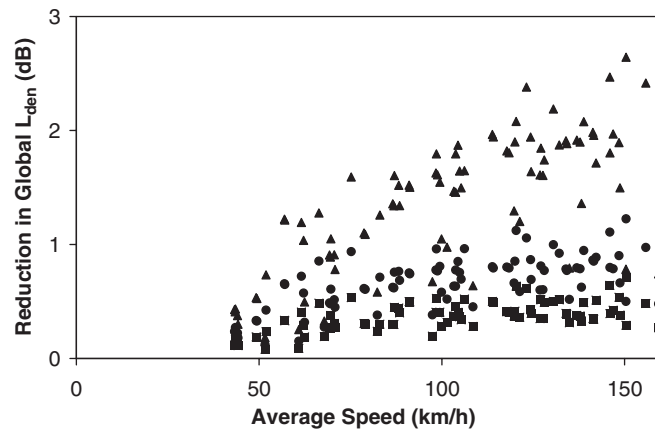


Fig. 8. The reduction in the  $L_{den}$  achieved with different trigger levels for the ‘even distribution, 3 dB deterioration’: ■, worst 5%; ●, worst 10%, ▲, worst 33%.

Clearly, grinding the track with the highest ATQ and maintaining it at around 0 dB moves the distribution to the left and is likely to produce the maximum benefit. However, as this would require frequent light grinding to maintain the track in this condition, this is the strategy that is the most difficult to apply in practice. Fig. 8 shows the impact of different trigger levels on the ‘even distribution, 3 dB deterioration’.  $L_{night}$  shows a similar tendency.

Use of the other scenarios shown in Fig. 7 produces a distribution of data similar to that shown in Fig. 8. However, the grinding strategy does have some effect on the average reduction. At around 160 km/h the annual reductions for  $L_{den}$  and  $L_{night}$  are shown in Table 1.

Consideration of grinding has shown that, in reality, a practical strategy comprising the grinding of the worst 10% of national track miles is likely, at best, to reduce the global noise exposure from the railway system, when quantified in terms of  $L_{den}$  and  $L_{night}$ , by around 1 dB. (These figures are for a typical system speed of 160 km/h.) Even grinding the worst 33% of track only increases this to around 2 dB. The largest reductions occur at the highest speeds. This is to be expected as it is at these higher speeds that rolling noise dominates and there will be the greatest benefit in grinding the rail. Of course, local effects can be significantly greater, e.g. by grinding a severely corrugated site to a low level of rail head roughness, where a 20 dB reduction or more could be achieved, although the reduction will be lower if there is a high percentage of



Table 1  
The effect of grinding strategies on reductions in annual  $L_{den}$  and  $L_{night}$  at around 160 km/h

	Even distribution, 3 dB deterioration between trigger and grind (dB)	Even distribution, 10 dB deterioration between trigger and grind (dB)	All within $\pm 2$ dB of an ATQ of 0 dB (dB)
Grind worst 5%	0.5	0.2	0.6
Grind worst 10%	0.9	0.4	1.2
Grind worst 33%	2.2	1.2	2.7

vehicles with cast iron tread brakes. At the sites considered, reducing the ATQ from 20 to 0 dB typically reduces noise levels by between 10 and 16 dB. The reduction will always be less than the reduction in the ATQ, even at high speeds, because the CRN corrections for multiple units with disc brakes tend to be slightly higher than Mk 3 and Mk 4 coaches on which the ATQ is based. This is thought to be because the powered wheels have a slightly rougher running surface (and are therefore slightly noisier) than those that are unpowered.

An alternative to using the global approach investigated above would be to produce noise predictions with actual levels of rail head roughness at specific locations, and then to modify this actual roughness by the amount that a targeted grinding would produce. ATQ could be obtained at the specific location using a vehicle-mounted measurement. For prevailing levels, the rolling noise element of the CRN prediction could then be roughness-corrected for disc-braked passenger vehicles via the value of ATQ obtained from the vehicle-mounted approach or by measuring the pass-by noise from known (preferably disc-braked) stock. The correction for the rolling noise from tread-braked passenger vehicles would also be available via the approach of Fig. 5. The predictions could then be re-run with a reduction in ATQ predicted as resulting from a particular grinding technique.

## 6. Conclusions

Rail head roughness can have significant effects on the levels of rolling noise from trains. This can be a major issue locally, as some trains can be 20 dB or more noisier on very rough, or corrugated, rail than on smooth rail. A roughness correction has been developed that allows predictions made using the Calculation of Railway Noise Procedure to reflect typical UK rail conditions rather than conditions of comparatively smooth rail for which the procedure was designed. This approach will enable the global noise exposure in the UK, quantified in terms of  $L_{den}$  and  $L_{night}$  as required under the Environmental Noise Directive, to be more accurately modelled with CRN than would have otherwise been the case.

The study has also considered the effects of rail grinding strategies on the overall railway noise exposure. A very large number of simulations of typical UK situations with characteristic traffic levels and mixes, including diesel-powered stock, were carried out, allowing a roughness correction to account for various grinding strategies to be derived.

If local rail head conditions are taken into account rather than national average characteristics, however, techniques are available to correct CRN appropriately. The local rail head roughness can be quantified by measuring rolling noise under a disc-braked vehicle passing over the site. The ATQ thus derived can be used to correct the predicted rolling noise at that site to indicate prevailing environmental levels, either directly for passenger stock with disc-braked wheels, or via derived relationships for other types of braking and classes of stock. Overall, prevailing noise levels, including noise from diesel traction, can then be calculated. The process can be repeated for an assumed reduction in ATQ resulting from a particular grinding exercise.

At first sight, the conclusion could be drawn that there is little overall benefit from practical grinding strategies, but this is partly a function of the assessment method ( $L_{den}$  or  $L_{night}$ ). At locations where roughness is an issue, there would clearly be local benefit. Importantly, though, a method that can take account of rail head roughness is available.

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