EVOLUTIONARY OPTIMIZATION OF BRACED STEEL FRAMEWORKS FOR TALL BUILDINGS USING A HYBRID OC-GA METHOD

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Abstract

Having many attractive advantages, genetic algorithms (GAs) have been applied to many design optimization problems. However, the practical application of GAs to realistic tall building design is still rather limited, since GAs require a large number of structural reanalyses and perform poorly in local searching. While the Optimality Criteria (OC) method can be applied effectively to the element sizing optimization of tall buildings, there is no guarantee that the OC method can always lead to the global optimum. In this paper, the so-called hybrid OC-GA method is presented to fully exploit the merits of both OC and GA for topology and element sizing optimization of braced tall steel frameworks. While the GA is particularly useful in the global exploration for optimal topologies, the OC technique serves as an efficient local optimizer for resizing elements of selected topologies. The effect of population size and the importance of the local OC search operator have been investigated. The applicability and efficiency of the hybrid OC-GA method were tested with two braced steel building examples. Results indicate that the incorporation of the OC operator into the GA has remarkably improved the efficiency and robustness of the evolutionary algorithm and thus make the hybrid method particularly useful for topology optimization of practical tall building structures involving a large number of structural elements and the use of numerous structural forms.

1. Introduction

For the past few decades, genetic algorithms (GAs) first developed by Holland (1975) have gained wide popularity and demonstrated their advantages over the conventional gradient-based optimization techniques. GAs are stochastic search methods, which mimic the principle of the survival of the fittest in natural selection. Due to their generality, GAs have been applied to a wide range of design problems especially those with discrete sizing variables, geometrical and topological variables. Unlike conventional optimization techniques, GAs are able to explore simultaneously the entire design space with a population of designs and therefore is capable of seeking for the global optimum. GAs can be applied directly and conveniently to structural design problems with both discrete and continuous design variables.

GAs can be regarded as a type of zero-order method, which requires numerous functional evaluations for achieving solution convergence. Consequently, as the scale and complexity of building structures increase, the required computational effort also increases, thus making GAs prohibitively difficult in solving practical large design problems. Another shortcoming of the GA approach is its lack of precision in searching for the definitive global optimum point. Since GAs are stochastic techniques, they are incapable of determining the precise global optimum and converge generally only to a near-optimum point.

One effective approach for the element sizing optimization of building structures has been based on the Optimality Criteria (OC) method, which has been shown to suit particularly well for tall building design with many design variables (Chan, 2001; Chan 2004). In the OC method, a set of necessary optimality criteria for the optimal design is first derived and a recursive algorithm is then applied to resize the element sizing design variables to indirectly satisfy the optimality criteria. For the lateral stiffness design of tall buildings, the OC method generally converges quite rapidly in a few design

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cycles indicating a weak dependence of the computational efficiency on the number of design variables. Although the OC method can be remarkably efficient in sizing optimization, they cannot be applied to topological optimization problems with addition and removal of discrete structural elements. Furthermore, there is no guarantee that the OC method can always lead to the global optimum (Kirsch 1993).

To overcome the problems associated with GAs while maintaining their merits, hybrid methods incorporating local search techniques into GAs, which provide great flexibility for hybridization, were proposed. Sakamoto and Oda (1993) proposed a hybrid method comprising a genetic algorithm and the generalized OC method to optimize both the layout and cross sectional area of simple trusses with nine nodes. Element sizing designs were refined locally by a proportional scaling in their proposed algorithm. Yeh (1999) inserted the fully stressed OC method to GAs for optimising element sizes of truss structures subject to stress and displacement constraints. The best design amongst a population size of 60 individuals in any generation was optimised by the fully stressed OC method. The results suggested that the hybrid GA method is more superior to a pure GA in terms of both quality of the optimal design and convergence behaviour. Chan et al. (2003) developed a hybrid method which combines GA with a rigorously derived OC technique. The method has the advantage over the conventional GAs and is capable of solving element sizing design problems of practical tall building structures in which the conventional OC method has encountered the problem of achieving solution convergence. Espinoza et al. (2005) demonstrated that a hybrid GA method with local search algorithm required significant reduction in the number of function evaluations for obtaining the optimal design when compared to simple GAs alone in solving a groundwater remediation problem. Fawaz et al. (2005) presented an evolutionary algorithm with a globally stochastic but locally heuristic search strategy. Considerably fewer computational operations have been found in the shape optimization of a simple 18-bar truss with stress constraints.

In this paper, the so-called hybrid OC-GA method is further extended to both topological and element sizing optimization of skeletal steel building frameworks. A local search operator based on a rigorously derived OC technique is developed and embedded in the framework of a GA. While the GA is used to explore the entire design space and generate improved topologies, the OC operator is applied as an efficient local optimizer for element resizing of selected topologies. The hybrid OC-GA method works in concert with the global GA searching method and the local OC optimizer to provide better optimal topological and element sizing design of building frameworks than that GA could provide alone. Different rates of the local OC search operator have been investigated for different population sizes with the aim of determining the most appropriate value of the probability of an OC operation for the topological and element sizing optimization of tall steel frameworks using the proposed hybrid OC-GA method.

2. General Design Problem Formulation for steel frameworks

Consider a general steel building framework having initially i = 1,2,…,N elements (or element fabrication groups), the minimum cost design the optimal topology and element sizing design in terms of material cost can be formulated as:

Minimize:
$$W(t_i, A_i) = \sum_{i=1}^{N} w_i \cdot t_i \cdot A_i \tag{1a}$$

Subject to:
$$d_j = \frac{\delta_j - \delta_{j-1}}{h_j} \le d_j^U \qquad (j = 1,2,\ldots, M) \tag{1b}$$

$$\left(\frac{\sigma_i}{\sigma_i^U} - 1\right) t_i \le 0 \qquad (i = 1,2,\ldots, N) \tag{1c}$$

$$A_i \in \boldsymbol{A_i} = (a_{i1}, a_{i2}, \ldots, a_{in_i}) \qquad (i = 1,2,\ldots, N) \tag{1d}$$

$$t_i = 0 \text{ or } 1 \qquad (i = 1,2,\ldots, N) \tag{1e}$$

Eq. (1a) defines the material cost of the building framework in which w_i denotes the unit material cost per unit cross sectional area of element *i* and the design variables t_i, A_i represent the Boolean variable and the cross sectional area of the element, respectively. The Boolean variable t_i defines the presence or absence of the element *i*. If $t_i = 1$, then the element *i* exists; otherwise $t_i = 0$ and the element is removed from the structural model. The element sizing variable A_i is selected from a specified set of discrete commercial sections $\boldsymbol{A_i}$ as given in Eq, (1d), where n_i represents the number of available discrete sections for the i^{th} element. Eq. (1b) defines the set of $j = 1,2,..,M$ serviceability lateral drift criteria, where δ_j, and δ_{j-1} are the lateral deflections of two adjacent floor levels *j* and *j*-1; h_j is the corresponding *j*th story height; d_j, and d_j^U are the drift ratio and its corresponding allowable limit. Eq. (1c) defines a set of element strength constraints, where σ_i, and σ_i^U are the individual element stress and its allowable limit. Since the design of tall buildings is predominately controlled by serviceability lateral stiffness requirements, element strength design constraints are secondary design considerations that only a small number of them are critical to a tall building design.

3. Hybrid OC-GA Method

3.1 Hybridization Strategy

In the proposed OC-GA method, a local search Optimality Criteria algorithm is incorporated into the GA process and thus called as an OC operator, which has the ability of performing element sizing optimization for a selected topology. The GA framework first starts with a randomly generated initial population and then produces new offspring designs with random changes in both the topology and element sizing by crossover and mutation. Unlike the genetic operators, the OC operator is a deterministic gradient based algorithm which improves a design with predefined topology subject to the specified structural design constraints. The OC technique is an efficient local search method which can resize rapidly the element sizes through the use of a recursive algorithm that satisfies a set of prescribed necessary optimality conditions. The balance between the global exploration of topology by the GA and the exploitation of efficient local element optimization by the OC is crucial to the success of achieving progressively improved designs whilst avoiding the occurrence of premature convergence. To achieve a satisfactory cooperation between GA and OC, the proposed hybridization strategy involves promoting frequent topology changes in the GA and precluding premature dominant designs generated by the OC at the early generations.

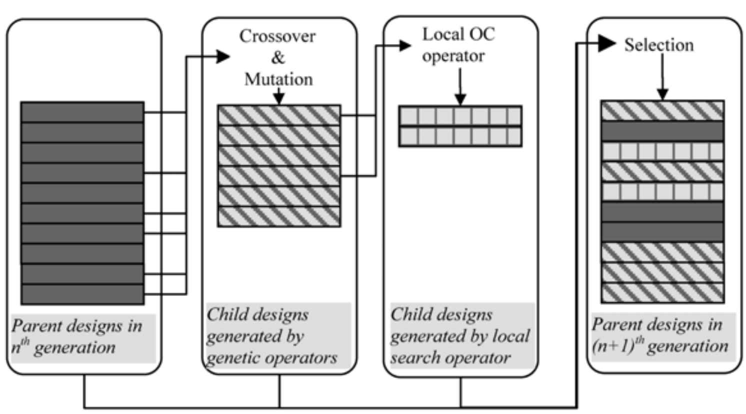


Figure 1. Schematic of the hybrid OC-GA method

An application of the OC operator can be illustrated in Fig.1. At the n^{th} generation, child designs are first recombined by crossover and mutation from the parent population. The OC operator is then applied to a portion of the child designs stochastically to undertake the local search OC element sizing optimization. The rate of OC, p_{oc}, denoting the probability of a child design that takes on the OC operation, is used to control the application of the local search operator. If a randomly generated real number ranging from 0 to 1 is found to be smaller than a prescribed value of p_{oc}, the OC operator will then be invoked. In theory, the OC operator may be applied to every child design (i.e. when p_{oc}=1). However, it may become impractical to do so for realistic structures with a large population due to the excessive computation required. It is important that an appropriate value of the probability of OC be determined to strike a right balance between computational efficiency and the quality of the optimised designs. After the fitness of each design is evaluated, an enlarged sampling selection (Gen and Cheng, 1997) is employed such that the population size is temporarily enlarged to contain both the parent and offspring designs during the selection process. Based on their fitnesses, the offspring designs are set to compete with their parents and the surviving candidates then form the next parent population in the $(n+1)^{th}$ generation.

3.2 Local OC search operator

The Optimality Criteria (OC) approach has long been recognised as a highly efficient method for element sizing optimization of large-scale structures. The rigorously derived OC method is shown to be particular suitable for lateral stiffness optimization problems associated with tall buildings subject wind-induced serviceability design constraints (Chan, 2001; Chan, 2004). Generally speaking, the design of a tall building with a height-to-width aspect ratio larger than 5 is likely to be governed by wind-induced drift and motion perception serviceability design criteria. The rigorously derived OC method is herein adopted as a local search operator for the lateral drift design of tall skeletal steelworks.

To commence the rigorously derived OC method, the lateral drift constraints Eqs.(1c and 1d) must be formulated explicitly in terms of the design variables A_i. Using the principle of virtual work, the collective set of lateral top and interstory drift constraints can be expressed explicitly in terms of A_i:

$$d_j(A_i) = \sum_{i=1}^{N}\left(\frac{e_{ij}}{A_i} + e'_{ij}\right) \le d_j^U \qquad (j = 1, 2, .., M) \tag{2}$$

where e_{ij} and e'_{ij} are the respective virtual strain energy coefficient and its correction factor of the i^{th} steel element of the structure associated with the j^{th} drift constraint (Chan, 1997). Once a finite element analysis is carried out for a randomly selected child design with a given topology under the actual and virtual loading conditions, the internal element forces and moments are obtained and the element virtual stain energy coefficients are then readily calculated.

Upon establishing the design constraints into explicit functions, the constrained optimization problem can then be transformed into an unconstrained Lagrangian function which involves the objective function Eq.(1a) and the explicit drift constraints Eq.(2) associated with corresponding Lagrangian multipliers. Based on the stationary conditions derived from the Lagrangian function, the following recursive linear relations can be used to resize the active sizing variables A_i (Chan, 1997) :

$$A_i^{v+1} = A_i^{v} \cdot \left\{1 + \frac{1}{\eta}\left(\sum_{j=1}^{M} \frac{\lambda_j e_{ij}}{w_i A_i^2} - 1\right)_v\right\} \tag{3}$$

where λ_j denotes the Lagrangian multiplier for the corresponding j^{th} drift constraint, v represents the current iteration number; and η is a relaxation parameter. During the recursive resizing iteration process, any element found to reach its size bounds is deemed an inactive element having its size set at its corresponding size limit. Before Eq. (3) can be used to resize A_i, the Lagrangian multipliers λ_j must first be determined. Considering the sensitivity of the drift constraints due to the changes in the design variables, one can derive a set of M simultaneous equations to solve for M number of λ_j. Having the current design variables A_i^v, the corresponding λ_j^v values are readily determined by solving the simultaneous equations. Having the current values of λ_j^v, the new set of design variables A_i^{v+1} can then be obtained by the respective recursive relations Eq. (3). Therefore, the recursive applications of the simultaneous equations to find the λ_j^v and the resizing formula Eq. (3) to find the design variables constitute the OC algorithm (Chan, 1997). By successively applying the recursive OC algorithm until convergence, a local optimal solution for the design optimization problem is then found. The local optimum with improved element sizes will then complete for survival with all parent designs and other child designs generated from crossover and mutation in the selection process.

3.3 Overall design procedure

Fig. 2 shows the flowchart of the hybrid OC-GA method. The framework of a simple GA is adopted and an OC operator is added after the crossover and mutation operations. An initial population of designs is randomly generated to commence the design process. Structural analysis using SAP2000 (CSI, 2001) is then carried out on each individual design to determine the response performances. Once internal element forces and the drift responses of each individual design within a generation are found, fitness evaluations according to the design problem formulation Eq. (1) are conducted. In the fitness evaluation process, a penalty-based modified objective function is used. Any constraint violation found will reduce the fitness of a design and is reflected by a penalty function which is imposed on the objective function. The selection operator will determine survival candidate designs from the current generation and pass them to the next generation. The tournament selection with elitism mechanism is implemented in this hybrid method. If the convergence criteria are not satisfied, the genetic crossover and mutation operators together with the OC operation will then be applied to produce new designs. The hybrid OC-GA process can be repeated until the optimum solution is found.

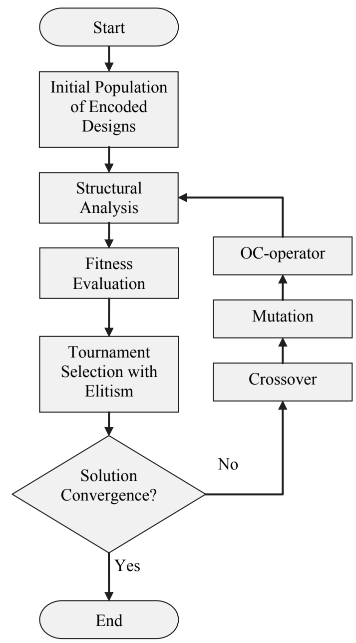


Figure 2. Flowchart of the hybrid OC-GA method

4. Illustrative examples

Simultaneous topological and element sizing optimization of a braced tall steel frameworks subject to lateral drift constraints is considered in two examples. Cross diagonal bracing elements are allowed to be added to the central bay of a 40-story, 3-bay planar building frame as shown in Fig. 3. The effectiveness of the hybrid OC-GA method on performing the optimal form searching as well as element sizing for the building framework was assessed by varying the different values of the probability of the OC operator and population sizes. The results produced by the hybrid method are compared with that generated by the GA method alone.

4.1 Example 1: Continuous addition of diagonal braces to a 40-story 3-bay framework

In this example, cross diagonals are assumed to hide from view behind the central lift core and are added only to the central bay of the 40-story 3-bay planar frame. These diagonal braces are added continuously from the ground level up to a specific story level denoted by a topological variable. The topological variable determines a story level which separates the frameworks along the height into two parts: stories below which are braced in the central bay; and stories above which use an unbraced rigid frame composed of only beams and columns. The optimal stopping level of the central bracing and optimal element sizes are sought by the hybrid OC-GA so as to minimize the total cost of the structure. The optimization involves searching for the most cost effective structural framework ranging from a pure rigid frame to a fully braced frame. Wind loads, as shown in Fig. 3, are derived based on the Hong Kong Wind Code (1983) using a general terrain wind profile. No gravity loading is applied to this framework example. Element strength design constraints and second-order $P-\Delta$ effects are not considered.

For the example framework having the cross diagonal braces as the topological and sizing variables and cross sectional sizes of the beams and columns as element sizing variables, the minimum cost function can be stated as follows:

$$W = \sum_{i=1}^{N_{column}+N_{beam}} \left(w_i \cdot A_i\right) + \sum_{j=1}^{N_{brace}} \left(t_j \cdot w_j \cdot A_j\right) \quad \text{where } t_j \begin{cases} =0 & if\ s_{t_j} > T \\ =1 & if\ s_{t_j} \le T \end{cases} \tag{4}$$

in which T defines the stopping level of the diagonal braces. The variable t_j denotes the on/off status of the j^{th} pair of diagonal cross braces at j^{th} story. For the story levels above story T, the diagonal cross braces of these levels are removed from the central bay. However, for the story levels below or equal to story T, pairs of diagonal braces are added to the central bay of these levels and their associated costs are included in the cost function of the structure as t_j =1. It should be noted that all beams are rigidly connected to columns such that the stability of the framework is always maintained whenever any diagonal brace is removed from the structure.

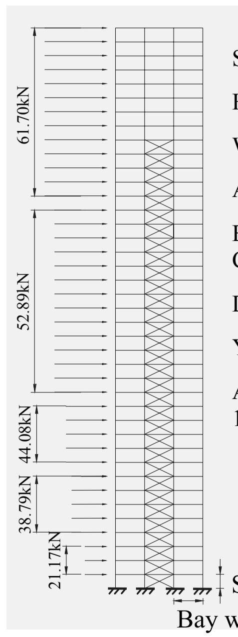


Figure 3. A 40-story 3-bay building frame under applied wind loads

All columns and diagonal braces are selected from among 36 discrete sections over the range of American AISC W14X22 to W14X730; beams are limited to W18 and W24 shapes selected from among 47 discrete sections over the range of W18X35 to W24X492. To account for symmetry and reversal of wind loads, exterior columns and beams are grouped together over two adjacent stories, as were interior ones. Diagonal braces are also grouped similarly to have the same size once over every adjacent two stories. The numbers of sizing variables for beams, columns and braces are 40, 40 and 20 respectively. Together with the topological variable T, the structure has a total of 101 design variables.

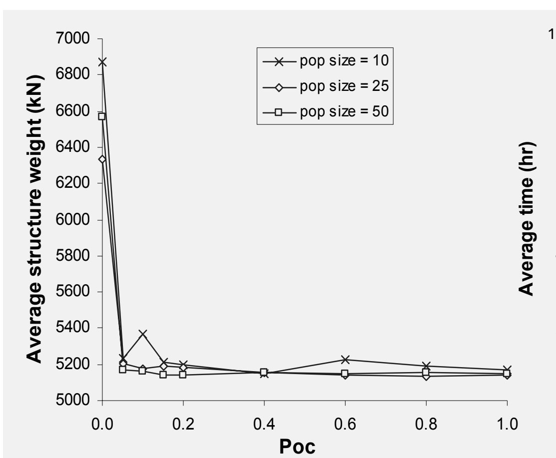
Two major parameters affecting the performance of the hybrid OC-GA, the population size and the probability of OC-operator, are studied in this example. Fixed population sizes of 10, 25, and 50 are used with a value of p_{oc} ranging from 0.0 (i.e., a pure GA without any OC operation) to 1.0 (i.e. all child designs undertake the OC operation).

Integer representation is adopted. Uniform crossover is applied with a probability of 80% such that 8 out of 10 of the parent designs are chosen to produce offspring designs. The mutation rates for the topological and element sizing variables are 20% and 5%, respectively. The reason for using a higher mutation rate for the topological variables is to increase the exploration of new creation of different forms of the braced frame. A quadratic penalty function is used for all design constraints. Binary tournament selection with elitism is employed in this example. The OC-GA algorithm is set to stop either when the maximum generation reaches 200 or when the same best-fit design is found for 20 consecutive generations. The optimization is carried out using a Pentium 4 3.0GHz computer with 512 MB memory.

Since the hybrid OC-GA is a stochastic algorithm, five independent runs are conducted with randomly generated initial designs. The average final structure weight produced by the hybrid OC-GA method for three population sizes is plotted with the different values of p_{oc} as shown in Fig. 4. For all runs wherein the probability of OC-operator is non-zero, the average final structure weights are found to be significantly over 20% less than those obtained by the pure GA (where p_{oc}=0).

With the use of the local OC operator, most of the final least-weight designs are found to satisfy almost all specified drift constraints, with a slight violation of less than 1% in the drift constraints being found in only a few final designs. For the case of the pure GA runs (where p_{oc} =0), over half of the final designs are found to be infeasible with a maximum violation of 10% in lateral drift constraints. Based on the results obtained for this example, the hybrid OC-GA method is able to produce more superior designs than the pure GA method.

Considering the results of all OC-GA runs using a value of p_{oc} ranging from 0.05 to 1.0, the average structural weights are fairly uniform among themselves with a small variation within 5% of the smallest value as shown in Figure 4,. This implies that under the current settings of the hybrid OC-GA method for this problem, the performance of the OC-GA algorithm was quite insensitive to the value of p_{oc}. When the population size is 10, the variation of the average structure weight against the p_{oc} is more fluctuating as compared with that of the population sizes of 25 and 50. It is evident that when a small population size is used, the likelihood of resulting in premature convergence to a local optimum becomes higher.



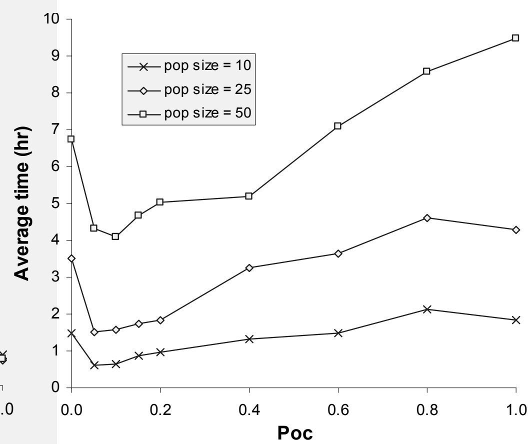


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Figure 4. Effect of p_{oc} on average structure weight **Figure 5.** Effect of p_{oc} on average computer time

Since the values of the best-fit designs generated by the hybrid OC-GA method with different values of p_{oc} are similar, the computational time required for convergence becomes a crucial factor to determine the most appropriate value of p_{oc} to be applied for this example problem. Figure 5 shows the average computation time used for the runs with different values of p_{oc}. Apparently the least computation time required is found when the value of p_{oc} is around 0.05-0.1. It is evident that the use of a relatively small value of p_{oc} can cause a significant reduction in the computational effort required to produce a reasonable final design using the hybrid OC-GA method. However when p_{oc} is larger than 0.1, a gradual increase in the computational time is observed with an increasing p_{oc} as shown in Fig. 5. In general, experience indicates that the best value of p_{oc} for topological and element sizing optimization of building frameworks is found to be about 0.1, meaning that only 10% of the offspring designs are needed to undertake the local OC sizing optimization in order to achieve the best-fit designs by the OCGA method with the least computational effort.

4.2 Example 2: Design of a 40-story 3-bay framework allowing random addition or removal of diagonal bracings

The effectiveness of the hybrid OC-GA method is further investigated in this more complex example 2 design problem in which pairs of diagonal cross bracing members are randomly added to or removed from the central bay of any two adjacent floor levels of the same 40-story, 3-bay steel framework used in example 1. In addition to 80 element sizing design variables for columns and beams, 40 grouped topological and element sizing variables for braces were introduced to form a design problem having a total of 120 design variables. This design problem involves searching an optimal structure from 2^{20} possible topologies, where 2 represents the on/off topology choices and 20 represents the number of grouped stories. All optimization parameters used for this example are kept the same as that of example 1. Four different population sizes of 10, 25, 50 and 100 are considered with the use of different values of p_{oc} ranging from 0.05 to 1.0 for this example.

Fig. 6 shows the effect of the p_{oc} on the quality of the final designs obtained by the OC-GA method. The structure weights of all final designs of four different population sizes generated by the hybrid OC-GA method are given as point marks; the average values of these runs of each population size are joined by lines as given in Fig. 6. Similar to the first example, much superior designs are generally found by the hybrid OCGA when compared to that of the pure GA (where p_{oc} = 0). Regarding lateral drift performance of these runs, almost all final designs of totally 140 OC-GA runs with various values of p_{oc} are found feasible with only a few designs within the smallest population of 10 having slight 2% violation in drift. On the other hand, all the final designs generated by the pure GA (where p_{oc}=0) are found to be infeasible with some significant violations exceeding 100% in the specified lateral drift constraints.

As shown in Fig. 6, the larger the population size is used, the better the average results of the optimum designs are generally produced. However, the benefit of using a larger population size is apparently not significant particularly when $p_{oc} > 0.2$. It should be noted that the OC-GA method is able to produce steadily optimum designs with smaller spread of scattered final designs for the larger population sizes of 50 and 100. When the population size is relatively small (10 or 25), a larger spread of scattering of final designs is found as $p_{oc} < 0.2$. To improve the robustness and quality of the optimum designs, a larger value of p_{oc} (greater than 0.2) is needed to be applied for smaller population sizes of 10 and 25. When the population size is comparable to the similar order of the number of design variables (i.e. 50 or 100 for this example with 120 design variables), small values of p_{oc}, in the range of 0.05 to 0.2, is generally found to produce good quality designs.

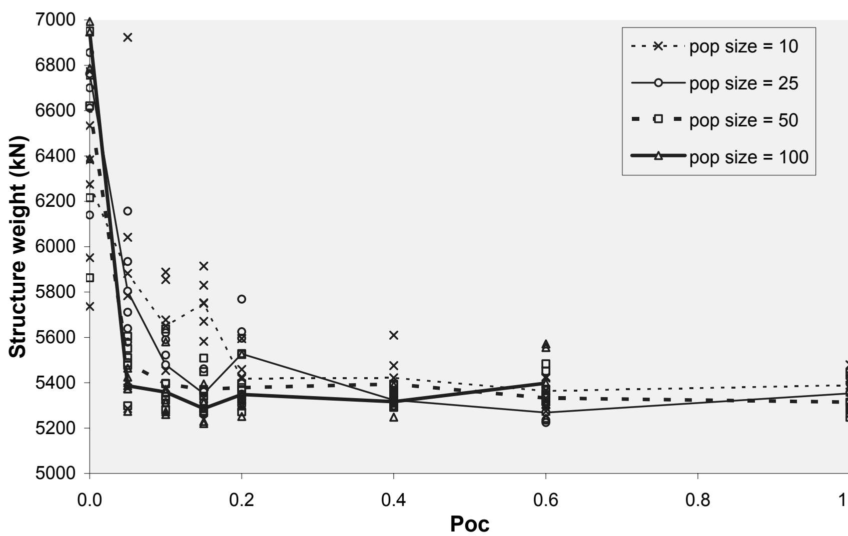

Figure 6. Effect of p_{oc} on structural weight

The scattering of the final design of all four population sizes diminishes with increasing p_{oc} values. This suggests that more frequent application of the OC-operator improves the quality of final designs and produce equally good results in terms of the objective function. Although much heavier designs are generally found by the pure GA method (where p_{oc} = 0), the final designs generated by GA are found all infeasible with significant drift violations.

For all the OC-GA runs, the design optimization process is terminated when the maximum generation reaches 200 or when almost the same best-fit design is maintained for 20 consecutive generations. The computational time required for solution convergence of each run is given in Fig. 7. For a given value of p_{oc}, the average computer time required is generally found to be linearly proportional to the population size. In other words, the time required for a population size of 100 is more or less two times that of a population size of 50. For all the OC-GA runs of a given population size, the average computer time required increases generally with an increasing value of p_{oc}. In general, the larger the population size, the smaller the value of p_{oc} should be adopted in terms of computational efforts. For this example, a relatively small value of p_{oc} ranging from 0.05 to 0.2 is found adequately sufficient to produce the best-fit designs with the least computational effort.

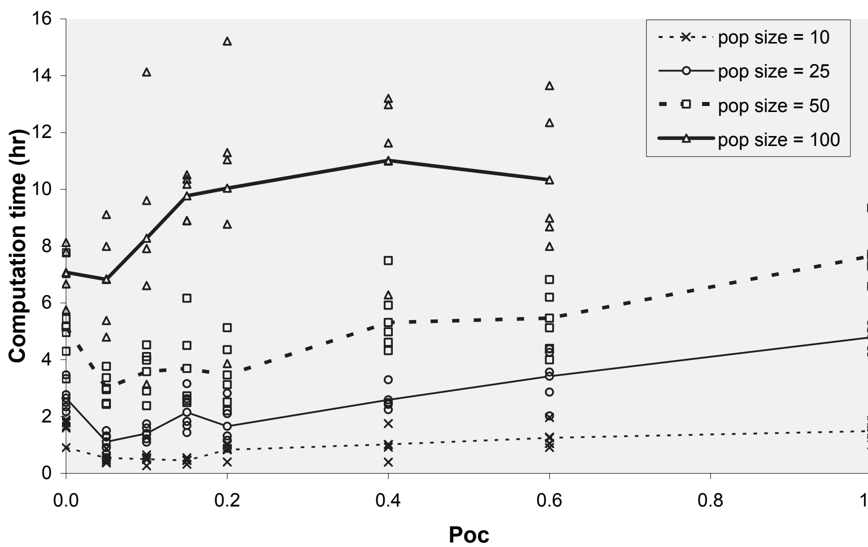

Figure 7. Effect of p_{oc} on computation time

5. Conclusion

In this paper, a novel hybrid OC-GA method incorporating a local search OC algorithm is presented for simultaneous topological and element sizing design optimization of tall steel building frameworks. The hybridization involves the exploration of topologies by the GA method and the refinement of element sizes by the local search OC algorithm. Two 40-story, 3-bay framework examples were tested to demonstrate the effectiveness of the hybrid OC-GA method on generating more superior optimal designs than conventional simple GAs. The incorporation of the OC operator into the GA process has remarkably improved the quality of the best-fit designs in terms of structural material consumption and computational efficiency. Results of the two examples indicate that robust and rapid solution convergence can be found by the hybrid OC-GA method. The use of a relatively small value of the probability of OC operator ranging from 0.05 to 0.2 is generally found to produce good quality best-fit designs with the least computational effort for a population size with a similar order of the number of design variables. The hybrid OC-GA method promises to become a useful tool for optimizing both the topology and element sizes of large-scale practical tall building structures.

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