FULLY STRESSED SEISMIC DESIGN OF DAMPERS IN FRAMED STRUCTURES

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Abstract

This paper presents an efficient and practical procedure for the optimal design of added damping in framed structures. The total added damping is minimized while inter-story performance indices for linear and nonlinear structures are chosen and restricted to allowable values under the excitation of an ensemble of realistic ground motion records. Optimality criteria are formulated based on fully stressed characteristics of the optimal solution and a simple analysis/redesign procedure is proposed for attaining optimal designs. Results of three examples presented compare well to those obtained using formal gradient based optimization.

Introduction

In the modern design of buildings to withstand strong earthquakes life safety is no longer the only concern but rather, performance whereby a prescribed level of damage is designed for. Hence, retrofitting of structures for a higher level of seismic protection may be needed. One of the means for achieving this enhancement to seismic performance is using supplemental damping which is the concern of this paper.

Various procedures for the design of added viscous damping, for *linear* behavior of damped 2D structures, were proposed by several researchers (for example Constantinou and Tadjbakhsh 1983; Zhang and Soong 1992; Fu and Kasai 1998; Inaudi *et al.* 1993; Gluck *et al.* 1996; Takewaki 1997; Lavan and Levy 2005a). Some of these procedures were extended to linear 3D structures (for example Wu *et al.* 1997; Takewaki *et al.* 1999). These methodologies, will usually require mathematics of stochastic processes, optimization methods, and/or variational mathematics – tools that are not that familiar to the practicing engineer. Garcia (2001) simplified the Sequential Search Algorithm, originally proposed by Zhang and Soong (1992) for stochastic models of the excitation, and made it appropriate for practical use. However, his method is restricted to linear structures under the excitation of a single deterministic record that does not guarantee optimal damping distribution of the dampers.

Procedures were proposed for the design of viscous dampers for *yielding* structures as well (Kim *et al.* 2003; Shen and Soong 1996; Lavan and Levy 2005b. The primary concern of the methodologies proposed by Kim *et al.* (2003), and by Shen and Soong (1996), was to estimate the total added damping needed rather than its distribution. The procedure proposed by Lavan and Levy (2005b) requires some nonlinear programming background and variational mathematics and may not be that easy to implement in the practicing design office. Allowable stress algorithms of the analysis/redesign procedures may be more suitable.

Allowable stress algorithms go back to the classical design of trusses, whereby the weight is minimized for a given allowable stress. Design problems of this type may be achieved iteratively using a two step algorithm in each iteration cycle. In the first step an analysis is performed for a given preliminary design, whereas in the second step the design is changed using a recurrence

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relationship which, for the truss problem is the ratio between the current stress and the allowable stress for each member. The algorithm possesses a fixed point (Levy 1991), i.e. fully stressedness and exhibits monotonic convergence properties. Convergence yields a *statically determinate fully stressed design, with members out of the design having strains smaller than the allowable*. This result appeared in the literature as early as 1900 (Cilley 1900). It was later shown (Levy 1985) that this design is a Karush–Kuhn–Tucker point and therefore, an optimal design. Algorithms closely related are the optimality criteria based algorithms as described by Khot *et al.* (1976), Venkayya (1978) and Rozvany (1989) to name only a few.

This paper presents a fully stressed design algorithm of the analysis/redesign type for the design of added viscous damping for linear as well as yielding structures for a given ensemble of realistic earthquake records and a specified target performance index. An algorithm of this kind is well liked by design engineers because its process is transparent and uses available and familiar dynamic time-history analysis programs that are common in civil engineering practice rather than unfamiliar mathematical optimization tools that need problem specific tailoring. Results were found to be in good agreement with optimal designs achieved using gradient based optimization. It should be noted in passing, that the fully stressed design described herein for viscous dampers has not been rigorously proven to be optimal yet.

Problem Formulation

This paper addresses the optimization problem of minimizing the added damping subject to constraints on local performance indices for framed structures excited by an ensemble of realistic ground motion records. The local performance indices *are interstory performance indices* for 2D frames, and interstory performance indices of the peripheral frames for 3D structures. Dampers are initially located at each story of the 2D frames or at each story of each peripheral frame in the 3D structures. As the optimization process progresses towards the optimum, however, some of the dampers will attain zero values.

The formulation of the optimization problem is comprised of the total added damping as an objective function, and an inequality constraint on the upper bound of each of the local performance indices which are computed based on the behavior of the structure, i.e., satisfying the equations of motion of the damped structure. The damping coefficients which are the design variables are required to be nonnegative.

The local performance indices are normalized by their allowable values such that a value of unity indicates that the local performance index is "fully stressed". Useful local performance indices are the maximal interstory drift, maximal interstory ductility, interstory hysteretic energy, combination of interstory ductility and interstory hysteretic energy such as the damage index of Park and Ang (1985), etc. or the maximal values of all of the above.

Linear Elastic Frames

A class of structures, usually regular ones, can be brought to behave elastically under an earthquake excitation, by the addition of a reasonable amount of damping. In these cases nonlinear analysis methodologies are not essential and some of the nonlinear performance indices are meaningless. Thus, linear tools are used.

Equations of motion: The equations of motion of a linear dynamic viscously damped system are given by:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + [\mathbf{C} + \mathbf{C}_d(\mathbf{c}_d)] \cdot \dot{\mathbf{x}}(t) + \mathbf{K} \cdot \mathbf{x}(t) = -\mathbf{M} \cdot \mathbf{e} \cdot \mathbf{a}_g(t); \quad \mathbf{x}(0) = \mathbf{0}, \quad \dot{\mathbf{x}}(0) = \mathbf{0} \tag{1}$$

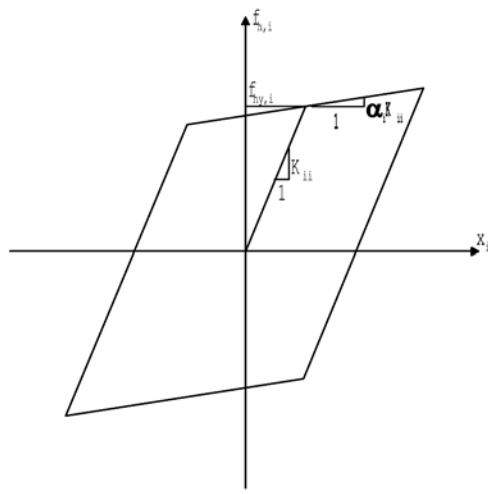

Fig. 1. Bilinear hysteresis diagram.

where $\mathbf{x}$ = the displacement vector of the degrees of freedom; $\mathbf{M}$ = mass matrix; $\mathbf{K}$ = stiffness matrix; $\mathbf{C}$ = inherent damping matrix; $\mathbf{c}_d$ = added damping vector; $\mathbf{C}_d(\mathbf{c}_d)$ = supplemental damping matrix; $\mathbf{e}$ = location matrix which defines location of the excitation, and $\mathbf{a}_g$ = vector of ground motion record. In the present work a damper is assigned to each story in the 2D frames, and at each story of the peripheral frames in the 3D structures.

Performance index: For linear structures, where the structure does not suffer structural damage, the maximal interstory drift becomes an important response parameter since it is a measure of nonstructural damage. Hence, the maximal interstory drift normalized by the allowed value, which is given as $pi_i = \max_t(|d_i(t)|/d_{all,i})$, is chosen as the local performance index for the 2D frames. Here $d_i(t)$ is the i-th story drift which is a linear function of $\mathbf{x}$ (such that $d_i(t) = \mathbf{L}_i\mathbf{x}(t)$ where $\mathbf{L}_i$ = transformation matrix), and $d_{all,i}$ is its allowable value. For the 3D structures a similar local performance index is used, but this time $d_i(t)$ is an interstory drift of a peripheral frame.

Optimization problem: The optimization problem is thus formulated as:

$$\text{minimize:} \quad J = \mathbf{c}_d^T \cdot \mathbf{1}$$

subject to:

$$\left.\begin{array}{l} pi = \max\limits_i \left(\max\limits_t (|\mathbf{L}_i \mathbf{x}(t)|/d_{all,i}) \right) \leq 1.0 \\ \text{where } \mathbf{x}(t) \text{ satisfy the equations of motion} \\ \mathbf{M}\ddot{\mathbf{x}}(t) + [\mathbf{C} + \mathbf{C}_d(\mathbf{c}_d)] \cdot \dot{\mathbf{x}}(t) + \mathbf{K} \cdot \mathbf{x}(t) = -\mathbf{M} \cdot \mathbf{e} \cdot a_g(t); \\ \mathbf{x}(0) = \mathbf{0}, \dot{\mathbf{x}}(0) = \mathbf{0} \\ 0 \leq \mathbf{c}_d \end{array}\right\} \forall a_g \in \text{whole ensemble.} \qquad (2)$$

Nonlinear Shear Frames

An addition of a reasonable amount of added damping may not be sufficient to result in elastic behavior of some classes of structures, usually irregular ones, under an earthquake excitation (Shen and Soong 1996; Uriz and Whittaker 2001). In these cases the damped structure develops plastic hinges. It is thus essential for the methodology to be based on nonlinear tools.

Equations of motion: The equations of motion of a nonlinear shear frame damped by linear viscous dampers are given by:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + [\mathbf{C} + \mathbf{C}_d(\mathbf{c}_d)] \cdot \dot{\mathbf{x}}(t) + \mathbf{T} \cdot \mathbf{f}_h(t) = -\mathbf{M} \cdot \mathbf{e} \cdot a_g(t); \quad \mathbf{x}(0) = \mathbf{0}, \dot{\mathbf{x}}(0) = \mathbf{0} \qquad (3)$$
$$\dot{\mathbf{f}}_h(t) = \mathbf{f}(\mathbf{L}\dot{\mathbf{x}}(t), \mathbf{f}_h(t)),$$

where $\mathbf{f}_h$ = interstory restoring force vector. In this work, a bi-linear hysteretic behavior is chosen (Figure 1); $\mathbf{T}$ =transformation matrix to transform $\mathbf{f}_h$ to the degrees of freedom coordinates, and $\mathbf{L}$ = matrix whose rows are $\mathbf{L}_i$. For a shear frame, the time derivative of the interstory restoring force of the i-th story, is a function of the i-th interstory restoring force and drift velocity only, i.e. $f_i = f_i(\dot{d}_i(t), f_{h,i}(t))$, where $f_{h,i}$ is the interstory restoring force of the i-th story.

Performance index: For nonlinear structures, where the structure is expected to suffer plastic deformations and dissipate energy by means of plastic behavior, the structural damage, that is measured by the damage index, becomes an important response parameter (see Williams and Sexsmith 1995 for a state of the art review on this topic), hence, it is chosen to represent the performance index. The story damage index is chosen to be energy based damage index due to its cumulative nature (Bannon *et al.* 1981). This damage index is chosen as the hysteretic energy dissipated by the restoring force divided by its allowed hysteretic energy. The total energy of the hysteretic component of the i-th story restoring force is given by $\int f_{h,i}(t)\dot{d}_i(t)dt$ and contains both its hysteretic and elastic energies. At the end of the ground motion, when the elastic displacements are small, the elastic energy is negligible, hence that integral represents the hysteretic energy dissipated by the i-th floor restoring force. Hence pi_i, the i-th element of $\mathbf{pi}$, is given by the expression:

$$pi_i(t_f) = \int_0^{t_f} f_{h,i}(t)\dot{d}_i(t)dt \Big/ \mathrm{H}_{\mathrm{all},i} \qquad (4)$$

where $\mathrm{H}_{\mathrm{all},i}$ is the allowable hysteretic energy of the i-th floor which is a fraction of its hysteretic energy at failure. The hysteretic energy at failure is taken proportional to the elastic energy at yielding (Bannon *et al.* 1981).

Optimization problem: The optimization problem takes the following formulation:

$$\text{minimize:} \quad J = \mathbf{c}_d^T \cdot \mathbf{1}$$

subject to:

$$\left.\begin{array}{l} pi = \max\limits_i (pi_i(\dot{\mathbf{x}}(t), \mathbf{f}_h(t), t)|_{t_f}) \leq 1.0 \\ \text{where } \dot{\mathbf{x}}(t) \text{ and } \mathbf{f}_h(t) \text{ satisfy the equations of motion} \\ \mathbf{M}\ddot{\mathbf{x}}(t) + [\mathbf{C} + \mathbf{C}_d(\mathbf{c}_d)] \cdot \dot{\mathbf{x}}(t) + \mathbf{T} \cdot \mathbf{f}_h(t) = -\mathbf{M} \cdot \mathbf{e} \cdot a_g(t); \\ \mathbf{x}(0) = \mathbf{0}, \dot{\mathbf{x}}(0) = \mathbf{0} \\ \dot{\mathbf{f}}_h(t) = \mathbf{f}(\dot{\mathbf{x}}(t), \mathbf{f}_h(t)) \\ 0 \leq \mathbf{c}_d \end{array}\right\} \forall a_g \in \text{whole ensemble.} \qquad (5)$$

Fully Stressed Design

Optimal seismic design of added damping for linear structures, as well as for yielding shear frames, has been achieved by the authors using formal nonlinear programming optimization techniques (Lavan and Levy 2005a, 2005b). Closer to the designer's heart, however are the analysis/redesign type techniques where an initial choice of the design variables is made (in this case damping coefficients, c_{d_i}) then, based on analysis results and a pre-defined recurrence relationship, the initial values are changed. A new analysis is made and the process continues until the designer is satisfied. Usually methods of this type converge very fast (up to five iterations for reasonable convergence of between 5–10% error in the examples of this paper). Moreover, in the classical FSD of trusses, for example, monotonic convergence is exhibited by the objective function (Spillers 1975). The designer is, thus, in full control and may stop at any iteration knowing that his results are the best up to that point.

Having studied the results achieved by the optimization methodologies mentioned above an analogy to the classical fully stressed design (FSD) of trusses seems to emerge. It is observed that, for 2D frames, the optimal design will attain nonzero values of c_{d_i} in stories for which the local performance index has reached the allowable, and zero values of c_{d_i} in stories for which the local performance index is less than the allowable. *In other words dampers are assigned only where the performance index is full*. Similarly, for the 3D structures the optimal design will attain nonzero values of c_{d_i} in stories of peripheral frames only, for which the local performance index has reached the allowable in at least one loading condition. In stories with no dampers the performance index is less than the allowable.

The recurrence relationship that is suggested in this work thus targets "fully stressedness" of the local performance index and is written as

$$c_{d_i}^{(k+1)} = c_{d_i}^{(k)} (pi_i^{(k)})^{1/q} \tag{6}$$

where $c_{d_i}^{(k)}$ = the i-th component of the damping vector at the k-th iteration, q is a convergence parameter and $pi_i^{(k)}$ = the actual i-th component of the performance index at the k-th iteration (using $\mathbf{c}_d^{(k)}$ as a damping vector). Note that $c_{d_i}^{(k)}$ and and $pi_i^{(k)}$ refer to the same location i.e. same story of the same peripheral frame. In case the active set which is the subset of the ensemble that is considered at a certain stage, as will be explained later on, is comprised of more than one ground motion, $pi_i^{(k)}$ is taken as the envelope of $pi_i^{(k)}$ for the records within the "active" set.

The choice of q in (6) affects the efficiency of the method. For larger values of q the method is more stable, i.e. the method is more likely to converge, however the convergence is slower. For the linear problem $q = 0.5$ may be used whereas for the nonlinear case a value of $q = 5$ seems to be appropriate.

Design Methodology

The design is done by first identifying the "active" ground motion and attaining the design for that record. Once this design is achieved, the performance of the damped structure is evaluated for the remaining records in the ensemble. If the damping for this ground motion violates constraints of other records in the ensemble, the ground motion for which $\max_i(pi_i)$ receives the largest value is added until an appropriate solution for the whole ensemble is reached.

The methodology is thus comprised of four main stages. These stages will be described in detail subsequently and are summarized as:

Stage 1. Select the "active" ground motion.

Stage 2. Attain the design for the records within the active set using the analysis redesign approach.
Stage 3. Apply the remaining records in the ensemble on the current design for feasibility check and go to Stage 2 if stopping criteria are not met.
Stage 4. Stop.

Stage 1: Selection of the "active" ground motion
In general, the active ground motion is chosen by first sketching a certain response quantity of a single degree of freedom system, having the natural frequency of the structure, versus the damping coefficient, for all records in the ground motion ensemble. For the linear problem, the response quantity sketched is the maximal displacement whereas, for the nonlinear case it is the input energy (according to Uang and Bertero 1990). The record for which the response quantity takes the largest values, for a reasonable range of damping, is chosen as the active record.

Stage 2: Analysis redesign
Following the discussion on fully stressed design an analysis/redesign approach using (6) is adopted.

As will be seen from the examples, the choice of a starting point does not have a large effect on the methodology since the methodology converges very fast to the region of the final solution. In the examples to follow a uniform distributed damping contributing a predetermined percentage of critical damping to the first mode shape is used, hence:

$$c_{d_i}^{(1)} = 2 \cdot \xi_{d_1} \cdot \omega_1 \cdot \frac{\boldsymbol{\phi}^{(1)^T} \mathbf{M} \boldsymbol{\phi}^{(1)}}{\boldsymbol{\phi}^{(1)^T} \boldsymbol{\phi}^{(1)}}, \tag{7}$$

where ξ_{d_1} = predetermined damping ratio of the first mode; ω_1 = circular frequency of the first mode, and $\boldsymbol{\phi}^{(1)}$ = first mode shape.

The analysis redesign stage is stopped when the constraint error, $\max_i(pi_i) - 1$ takes a small value and the changes in the objective function or in the damping vector for two subsequent iterations is small.

Stage 3: Feasibility check and stopping criteria
Once a design for the "active" ground motions is achieved in Stage 2, the performance of the damped structure for each of the remaining ground motions separately in the ensemble is evaluated using a time history analysis. If the design achieved in Stage 2 violates constraints of other records in the ensemble, i.e. $\max_i(pi_i) > 1$, the ground motion for which $\max_i(pi_i)$ receives the largest value is added to the active set.

In Examples 2 and 3 only one record is active. This record is easily tracked by the algorithm, and it is expected that the optimization scheme is likely to use, in general, only a few of the records and not whole ensembles. Therefore, the scheme becomes practical in the sense of the computational effort.

The methodology is terminated when no additional ground motion is added to the active set at this stage. If an additional ground motion is needed, then Stage 2 is repeated with the new active set.

Formal Optimization Methodology

The examples that follow were solved using the analysis/redesign technique that was introduced in the previous section. For the sake of comparison, these examples were also solved using a formal optimization methodology similar to the one introduced by Lavan and Levy (2005b). This methodology uses an appropriate first order optimization scheme that requires the derivation of the gradient of pi. The derivation of this gradient is done by first formulating the equations of motion and the performance indices in a differentiable equivalent *state space* formulation, i.e. as a differentiable set of first order differential equations, and then applying a variational approach for the gradient derivation.

Examples

Example 1 – linear 2 story shear frame

In order to demonstrate the proposed methodology, and the characteristics of the optimization problem the 2-story shear frame introduced by Lavan and Levy (2005a) is studied.

A 5% Rayleigh damping was assumed for the first and second modes. The constraint on the maximum drift given in (2) was set to 0.009m, which is 50% of the maximum drift of the bare frame. The two periods of the structure are 0.281 s and 0.115 s. The structure was excited by the record LA02 from the "LA 10% in 50 years" ground motions ensemble (Somerville *et al.* 1997), which is the N-S component of El-Centro 1940 scaled by a factor of 2.01 downloaded from "http://quiver.eerc.berkeley.edu:8080/studies/system/ground_motions.html". The mass, damping and stiffness matrices (displacements DOFs) to be used in (2) are:

$$\mathbf{M} = \begin{bmatrix} 25 & 0 \\ 0 & 25 \end{bmatrix} \text{ton}; \quad \mathbf{C} = \begin{bmatrix} 120.7 & -32.4 \\ -32.4 & 72.1 \end{bmatrix} \text{kN} \cdot \text{s/m};$$

$$\mathbf{K} = \begin{bmatrix} 62500 & -25000 \\ -25000 & 25000 \end{bmatrix} \text{kN/m}. \tag{8}$$

The contribution of the dampers to the damping matrix is:

$$\mathbf{C}_d = \begin{bmatrix} c_{d_1} + c_{d_2} & -c_{d_2} \\ -c_{d_2} & c_{d_2} \end{bmatrix} \text{kN} \cdot \text{s/m} \tag{9}$$

and

$$\mathbf{e} = \{1 \;\; 1\}^T. \tag{10}$$

The example was solved for a single record. Thus the "active" ground motion in Stage 1 is LA02. Applying the proposed analysis redesign procedure of Stage 2, with starting values obtained from (7) as $\mathbf{c}_d^{(1)} = 2795 \cdot \mathbf{1}$ kN·s/m and using $q = 0.5$ in (6), Figure 2 shows a contour map of the constraint, $\max_i(pi_i) \leq 1$, the objective function at the optimum value (straight line) and the iterative progress towards convergence of the analysis redesign.

The total added damping and the constraint's error ($\max_i(pi_i) - 1$) versus the iteration number are shown in Figure 3.

As can be seen from that figure, the convergence to the region of the optimum is quite fast (a constraint error of -0.35% and a total damping of 1543 occurred in 5 iterations), however, full convergence took 12 iterations. The final damping is $c_{d_1} = 1300.4$ kN·s/m and $c_{d_2} = 181.4$ kN·s/m. There are no remaining records to apply on the design (feasibility check of Stage 3) since only a single record is considered in this example. Hence, the design achieved is the final design. The value of the total added damping for the optimal design and the value of pi are compared in the first line of Table 1 with the ones achieved using the gradient based approach. As can be seen, the gradient based optimization leads to a, somewhat, lower value of the objective function, however, the violation of the constraint is a bit larger. It should be noted that the formal optimization technique yielded the same height distribution and practically equal values of the dampers (not shown).

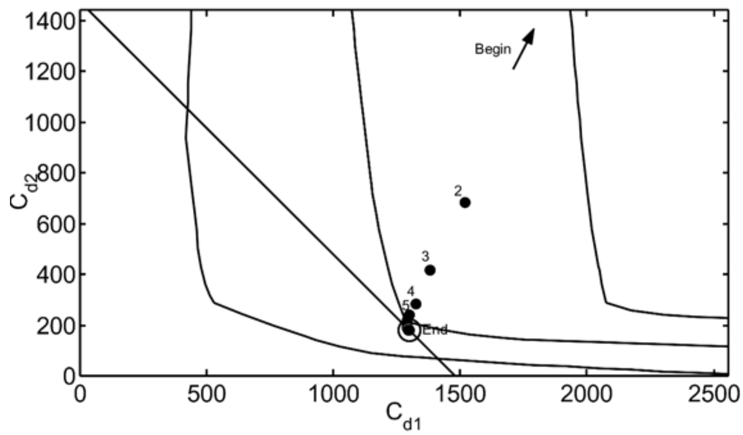

Fig. 2. Contour map of the constraint (curved lines), objective function at optimum (straight line) and iterative progress towards convergence (dots).

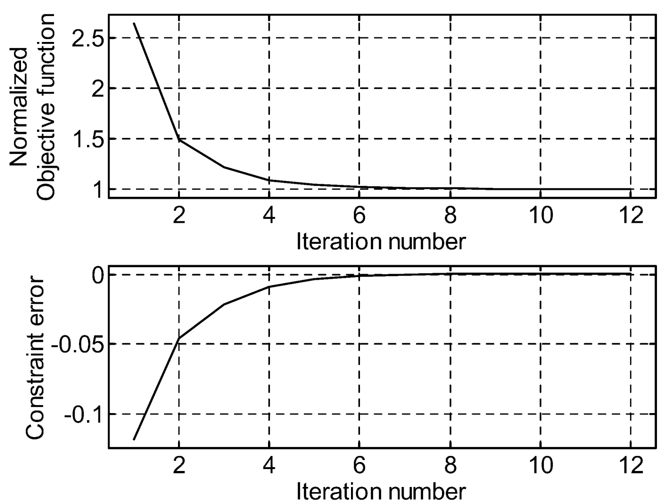

Fig. 3. Convergence to optimum.

Table 1. Optimal design values.

example	Analysis\Redesign		Gradient based optimization	
	Objective function	*pi*	Objective function	*pi*
1	1481.8 (101.94%)	1.0002	1453.6	1.0040
2	162005 (101.03%)	1.0000	160347.5	1.0065
3	418985.0 (100.00%)	1.0010	418972.0	1.0030

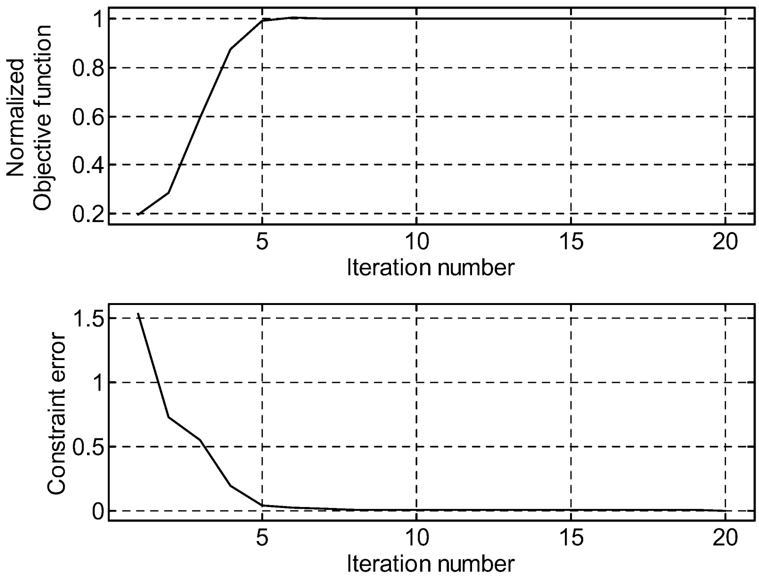

Fig. 4. Convergence to optimum.

Example 2 – linear 8 story 3 bay by 3 bay asymmetric framed structure

In order to demonstrate the applicability of the proposed methodology to 3D structures, the 8-story 3-bay by 3-bay asymmetric framed structure introduced by Tso and Yao (1994) is used. Inherent 5% Rayleigh damping in the first and second modes is assumed. The methodology was performed neglecting axial deformations, i.e. 3 degrees of freedom per floor were used (two horizontal displacements and torsional angle). The ground motion ensemble was chosen as the "LA 10% in 50 years" ensemble (Somerville *et al.* 1997), and the allowable drift at the peripheral frames was chosen as 1.0% of the story height. Stage 1 sketches the maximal displacement of a single degree of freedom system having the natural period of the frame (1.15 sec), versus the damping coefficient for each record in the whole ensemble. The record LA16 was chosen to start the process since its spectral displacement for all reasonable damping range had the largest value.

Applying the proposed analysis redesign procedure of Stage 2, with starting values obtained from (7) as $\mathbf{c}_d^{(1)} = 1000 \cdot \mathbf{1}$kN·s/m and using $q = 0.5$ in (6), Figure 4 shows the total added damping and the constraint's error $(\max_i(pi_i) - 1)$ versus the iteration number for the record LA16.

As can be seen, the convergence to the region of the optimum is quite fast (a constraint error of 3.9% and a total damping of 160121 kN·s/m occurred in 5 iterations), however, full convergence took 20 iterations. The final damping and the components of **pi** for the damped frame are show in Figure 5.

Applying the remaining records in the ensemble on the design (Stage 3) indicated that there was no record that led to greater performance indices than that of the active record, LA16. Hence, the optimization process was terminated. A comparison of the total added damping and *pi* with these of the gradient based optimization solution is given in the second line of Table 1 with the same behavior as the previous example, i.e. the same distribution and practically equal values of the dampers where achieved using the formal optimization technique (not shown).

As can be seen (Figure 5), the optimal solution assigns damping in stories of peripheral frames that fully utilized their local performance index (stories number 2–5 of frame 1 and stories number

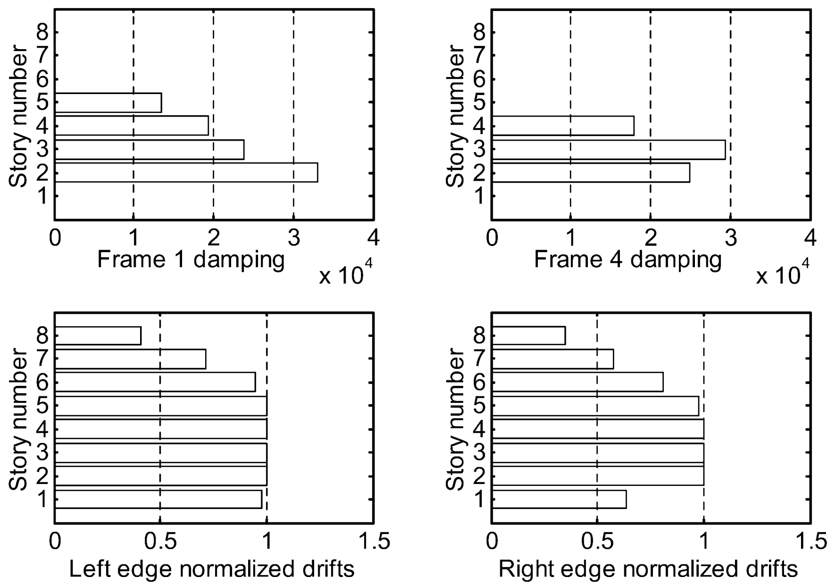

Fig. 5. (a) optimal damping of the damped structure for LA16: left edge (frame 1) and right edge (frame 4) and b) maximal drifts for LA16 (= envelope values): left edge (frame 1) and right edge (frame 4).

2–4 of frame 4), and no damping is assigned elsewhere.

Example 3 – yielding 10 story shear frame
In order to demonstrate the applicability of the proposed methodology to *yielding* shear frames under an ensemble of ground motion records, the 10-story shear frame introduced by Lavan and Levy (2005b) with inherent 2% Rayleigh damping in the first and second modes is used.

The fundamental period of the structure is 1.0 sec; $\mathrm{H}_{\mathrm{all},i}$, which appears in (4), is taken as the elastic energy at yielding of the i-th story multiplied by $0.2 \times 16 = 3.2$. The secondary slope ratio for all floors was chosen as 0.02. The ground motion ensemble was chosen as the "SE 10% in 50 years" ensemble (Somerville *et al.* 1997). The record SE19 started the process (Stage 1) since its spectral input energy for the fundamental period of the structure had the largest value for a reasonable damping range (see Lavan and Levy, 2005b). A nonlinear analysis was performed on the bare frame for this record and revealed values much larger than 1 for the components pi_i. Applying the proposed analysis redesign procedure of Stage 2, with starting values obtained from (7) as $\mathbf{c}_d^{(1)} = 12095 \cdot \mathbf{1}$ kN·s/m and using $q = 5$ in (6), leads to the optimal damping and the components for the damped frame excited by SE19 are shown in Figures 6(a) and 6(b).

The total added damping and the constraint's error ($\max_i(pi_i) - 1$) versus the iteration number are shown in Figure 7.

As can be seen from that figure full convergence took 12 iterations. Applying the remaining records in the ensemble on the design indicated that the design is feasible, thus, the design achieved is the final design. A comparison of the total added damping and pi with those of the gradient based optimization solution is given in the third line of Table 1. In this case of yielding structure, the same result observed in the linear cases is repeated, i.e. here too, the formal optimization technique yielded the same distribution and practically equal values of the dampers (not shown).

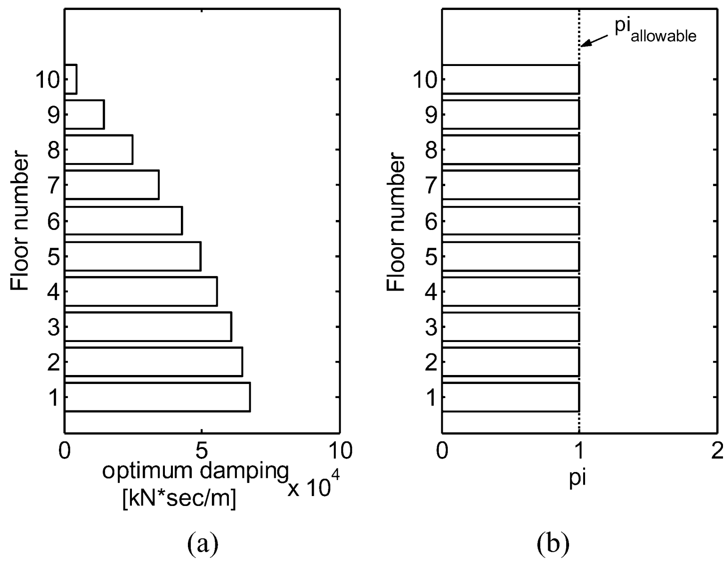

Fig. 6. (a) Design supplemental damping and (b) damage indices envelope for the designed damped frame excited by the SE 10% in 50 years ensemble.

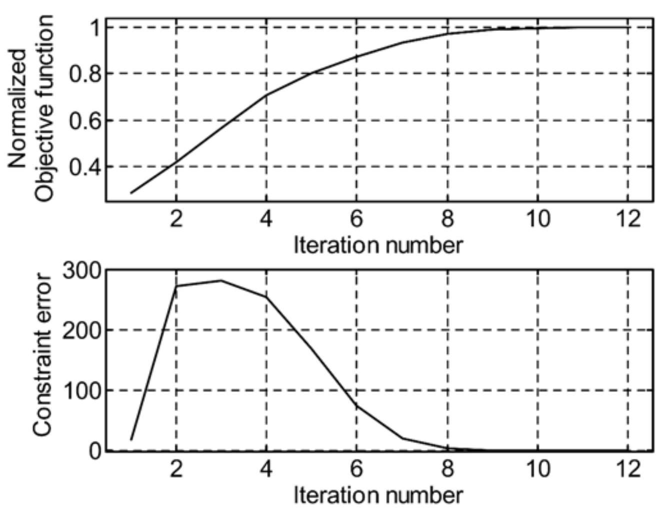

Fig. 7. Convergence to optimum.

Conclusions

A methodology for the optimal design of added viscous damping for an ensemble of realistic ground motion records with a constraint on the maximum drift for linear structures, and on the maximum energy based damage index for nonlinear shear frames, was presented.

The optimization methodology is based on an iterative procedure of the analysis/redesign type that is appropriate for engineering practice. This scheme seems to converge very fast to the region of the final design and is applicable to nonconvex problems.

The final solution coming from formal optimization is characterized by equal maximal drifts in the linear case, and equal maximal damage indices in the nonlinear case, for floors with assigned dampers, and lower maximal drifts/damage indices for floors with no assigned damping. This fully stressed result was targeted by the iterative procedure and is desired in structures due to the uniform distribution of damage (structural and nonstructural) throughout the structure.

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