A NEW METHOD FOR ANALYSIS OF STRUCTURES INCLUDING NONLINEAR SEMIRIGID CONNECTIONS

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Abstract

Semirigid connections show nonlinear behavior even due to small loadings. Therefore linear analysis is not a proper solution algorithm for structures that have such connections; rather a nonlinear analysis should be done. The conventional methods of nonlinear analysis of frames are inherently iterative, and their final results include some small order of approximation. They usually are done through modification of the stiffness matrix of structure and/or load vector.

In this paper, a new method of nonlinear analysis has been presented that contrary to iterative methods, it is non-iterative. It does the analysis in one step without change in the initial model and stiffness matrix of the structure or its load vector. Theoretically it does not include approximation and gives exact results. In this method to force internal moments follow their nonlinear moment-rotation curves, some virtual moments (that are primarily unknown) are imposed to the structure at semirigid connections. To find the unknown virtual moments, a quadratic programming problem is formulated and solved. After finding the values of virtual moments, employing superposition principle, exact nonlinear response of structure is obtained and internal forces and moments of members are calculated.

The method is capable to model semirigid connections with multilinear moment-curvature relations. The formulation of the problem for bilinear and trilinear moment-curvature relations has been presented here. Two examples are presented to demonstrate the robustness, capability and validity of the method.

Keywords: Semirigid connection, multilinear moment-rotation relation, nonlinear analysis, mathematical programming.

Introduction

In general connection of a beam to column can be categorized in three groups. The first group are rigid connections in which, theoretically saying, the angle between the two connected members does not change due to applied moments. The second group are hinged connections in which the connected members can have relative rotation without any resistance. In reality there is neither solid rigid connection, nor theoretical hinge connection, i.e. every rigid connection admits some rotation and every hinge connection tolerates some moment. Semi-rigid connections that have situation between the two groups constitute the third group. Their characteristics are determined by moment rotation relations that are usually nonlinear. To simplify nonlinear analysis, nonlinearity of materials is usually modelled as multilinear relations between stress and strain, the first part of which characterizes linear relations. The most simplified nonlinear relation is the bilinear elastic-plastic relation in which there is an elastic relation between stress and strain up to yield point.

If the structure is stressed up to this stress limit a linear analysis is sufficient for stress-strain calculations. However for further stress or strain a nonlinear analysis is necessary.

Literature is almost mature of nonlinear analysis methods. Crisfield (1991) and Owen and Hinton (1980) have cited good summaries of classical nonlinear analysis techniques. Among the major

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nonlinear analysis techniques the *Incremental Scheme, Initial Stiffness method, Newton-Raphson method* and combination of these methods can be mentioned

There are also some other techniques, that have been established for inelastic analysis of structures based on theorems of Structural Variation. Structural variation theory studies the effect of change of properties, or even removal, of a member on the entire structure. It takes advantage of linear analysis and sensitivity of structure to some self equilibrating unit loads that are applied at the end nodes of changing members. This technique has been applied to analysis of several types of inelastic skeletal structures including space trusses (Saka & Celik 1985), frames (Majid & Celik 1985) and grids (Saka 1997), etc. It has been also extended to nonlinear finite elements analysis (Abu Kassim & Topping 1985, and Saka 1991). Although this method takes advantage of initial stiffness matrix and does not require change in the stiffness matrix of structure during the analysis process, it is a historical and step by step method of analysis in which every step uses information from the previous step.

Nonlinear analysis of structures by mathematical programming is another field of research in this ground. De Donato (1977) presented fundamentals of this method for both holonomic (path independent) and nonholonomic material behaviours. In this method, it is assumed that displacement of nodes of an elastic-plastic structure comprises two parts namely elastic and plastic parts. Then, the problem of finding total displacement vector of a structure is formulated in the form of a quadratic programming (QP) problem with some complementarity yield constraints. These yield constraints state that individual members either are stressed within elastic limits and do not accept plastic deformations or they are stressed up to yield limit and, as a result, undergo some plastic deformations. The output of this sub-problem is linear and nonlinear deformation of structure. Despite its robustness, this method suffers from the considerable number of variables that enter in the QP sub-problem.

The goal of this research work is to bypass iterative techniques in the analysis of nonlinear structures and build up a method based on simple equilibrium relations to conduct analysis in one step. The idea of this technique has been initially examined by Moharrami et al (2000) for nonlinear analysis of structures including tension only and compression-only truss-type elements. Here in this paper it is extended to elastic-plastic flexural connections. This method holds simplicity of structural variation theorem, advantages and robustness of mathematical programming and precision of the results with less computation effort.

Formulation of the Solution Procedure

For simplicity of formulation of the problem, first the elastic-plastic model of nonlinear behaviour is considered and formulated. Then the formulation is extended to more complicated behaviours. In general the elastic-plastic behaviour of a semi-rigid connection is defined as in Fig.(1) in which moment in the connection is proportional to rotation up to certain limits. Beyond these limits, despite increase in rotation, moment remains constant. In the elastic-plastic semi-rigid connections the Moment-Rotation relation in each direction can be defined by two parameters M_{lim} and R_0 . R_0 is the stiffness of the connection and for rigid connections it tends to be infinity. It is zero for hinge connections and accordingly in this case M_{lim} is zero. To formulate the solution procedure, it is noted that the elastic-plastic behaviour of the connection can be mathematically written as :

$$M_{lim}^{-} \le M \le M_{lim}^{+} \quad \textbf{(1)}$$

in which M_{lim}^{-} and M_{lim}^{+} are limits of negative and positive moments in the connection respectively. This is equal to the two following relations:

$$\begin{aligned} M_{lim}^{-} &\le M^{-} \\ M^{+} &\le M_{lim}^{+} \end{aligned} \quad \textbf{(2)}$$

Noting that the Moment in the connection is either positive (M^{+}) or negative (M^{-}), the above relation can be written as follows:

$$M_{lim}^{-} \le M^{+} - M^{-} \le M_{lim}^{+} \quad \textbf{(3)}$$

To formulate the new method, we first focus on a condition that one of the inequalities in Eq.(2) is governing. Later on the formulation will be extended for the case that both are concerned.

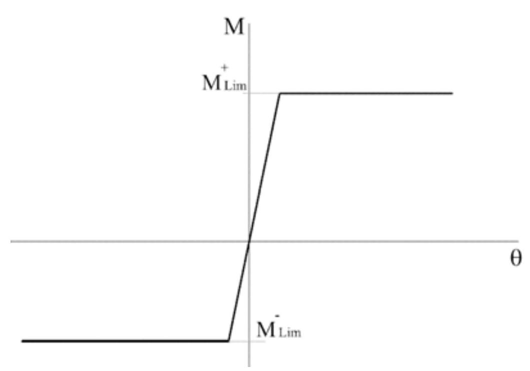
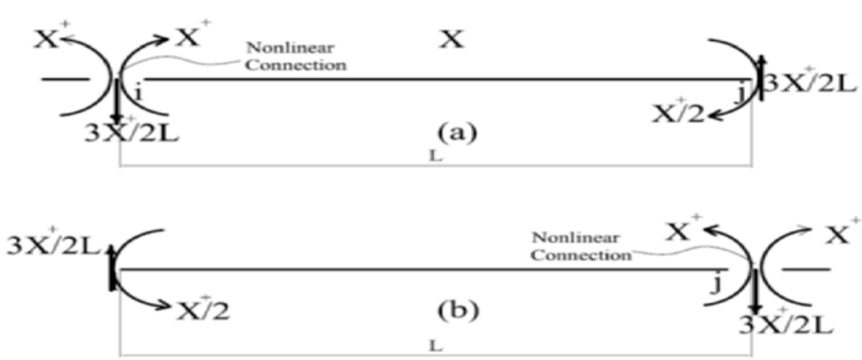


Fig.(1): Elastic-Plastic Behaviour

Elastic-Plastic Connections with Limited Capacity of Negative Moment

Consider a connection in which the second inequality in Eq.(2) is ignored, i.e. it is assumed that the behaviour of the connection is such that there is only a limit on negative moment but there is not any limit on positive moment, i.e. $M_{lim}^{+} = \infty$. In this case if a linear analysis is done and the value of moment exceeds the capacity of the connection it is necessary to add a positive moment to the connection such that after its distribution in the structure, it reduces the moment in the connection to the capacity limit M_{lim}^{-}. Let us assume that this unknown positive moment is X^{+}. To maintain the equilibrium and compatibility in the structure, it is necessary that this moment be accompanied by a moment at the other end and a pair of shear forces at the two ends of connected member.



Fig(2): Application of Self-Equilibrated Positive Moment X^{+} at two ends of a member and its related moment and shear forces. (a) Connection at **i** (b) Connection at **j**

Fig.(2) shows the application of self equilibrated moment and its accompanied moment and shear forces to a connection. From this point forward whenever it is talked about addition of a moment, it is meant a self equilibrated set of moments and shear forces.

Due to the applied moment X_i^+ in a connection or a joint named **i**, a moment $M_{ji} = \overline{m}_{ji} X_i^+$ will be produced in joint **j**. In this equation $\overline{m}_{ji}$ is the moment produced in joint/connection **j** due to unit moment in **i**. Considering that there are n such artificial moment X_i^+ on the structure, the resultant moment M_{ri} in a typical connection **i** can be obtained from the following equation:

$$M_{ri} = \hat{M}_i + X_i^+ + \sum_{j=1}^{n} \overline{m}_{ij} X_j^+ \quad \textbf{(4)}$$

In Eq.(4), $\hat{M}^-$ is the moment obtained from linear elastic analysis. It should be pointed out that since in applying unit moments at the connection **i** at member **i-j** two external moments M_i=1 and M_j=-.5 are applied, these moments should be added to $\overline{m}_{ii}$ and $\overline{m}_{ji}$ values. This why the X_i^+ appears in Eq. (4). From this point forward, m_{ij} that comprises M_i=1 and M_j=-.5, is replaced for $\overline{m}_{ij}$ to include the effect of external unit moments and corresponding shear forces. Therefore Eq.(4) will be written as

$$M_{ri} = \hat{M}_i + \sum_{j=1}^{n} m_{ij} X_j^+ \quad \textbf{(5)}$$

Provided that X^+ are known, Eq. (5) can be used for calculation of moment in all joints/connections. Therefore It remains to find the values of X^+'s for all nonlinear connections. To that end substitute the M_{ri} from Eq. (5) into Eq. (2) for all nonlinear connections to have:

$$M_{ri} = \hat{M}_i + \sum_{j=1}^{n} m_{ij} X_j^+ \geq M_{lim}^- \qquad ,i = 1,2,...,n \quad \textbf{(6)}$$

Eq. (6) provides a set of n inequalities that can be used for determination of n unknowns. However there is a special condition that should be observed during solution of this set of inequalities.

Considering that due to applied loads the value of M_{ri} from Eq.(5) does not exceed the moment capacity limit M_{lim}^-, then it would not be necessary to apply the unknown moment X_i^+, i.e. in this case X_i^+=0. Otherwise the artificial moment $X_i^+ \geq 0.$ should be applied to the joint/ connection to make M_{ri}=M_{lim}^-. These mutual conditions which are in fact the complementary conditions for the set of inequalities (6) can be written in the following form:

$$X_i^+ (M_{ri\,ri}^- - M_{lim}^-) = 0. \qquad ;i = 1,2,\ldots,n \quad \textbf{(7)}$$

To solve the set of inequalities (6) with the conditions in Eq. (7), a Quadratic Programming (QP) problem as follows is established in which Eqs. (7) constitute its objective function and the set of inequalities in Eq. (6) comprise its constraints. It is noted that for arbitrary values of $X_i^+ \geq 0.$ that satisfies Eq. (6) the value of $X_i^+(M_{ri}-M_{lim}^-)$ is always positive. Therefore minimization of the objective function reduces it to zero.

$$\begin{aligned} &\text{Minimize } \sum_{i=1}^{n} X_i^+ (M_{ri} - M_{lim,i}^-) \\ &\text{Subject to } \quad M_{ri} = \hat{M}_i + \sum_{j=1}^{n} m_{ij} X_j^+ \geq M_{lim}^- \qquad ,i = 1,\ldots,n \end{aligned} \quad \textbf{(8)}$$

The solution of QP problem in Eq. (8) while reduces the objective function to zero provides the values of X_i^+'s. Then the Eq. (5) can be used to determine the moments in every joint of the structure regardless that there are or not any artificial moment.

Elastic-Plastic Connections with Limited Capacity of Positive Moment

In this type of connections it is assumed that positive moment is limited to M_{lim}^+ but it is not limited in negative direction. If we use the same unit moments that was used in the previous section, similar formulation can be derived except that to keep unique coordinate system there will be a negative sign in behind of X^-. In this case the sub-problem Eq. (8) will be as in Eq. (9)

$$\begin{aligned} &Minimize \ \sum_{i=1}^{n} X_i^- (M_{lim,i}^+ - M_{ri}) \\ &Subject\ to \quad M_{ri} = \hat{M}_i - \sum_{j=1}^{n} m_{ij} X_j^- \le M_{lim}^+ \qquad , i = 1, \ldots, n \end{aligned} \tag{9}$$

Elastic-Plastic Connections with Limited Capacity in both directions

The formulation of the problem in this case which provides the general elastic-plastic behaviour can be extracted from the formulation of previous sections. Noting that the artificial moment should be applied in either positive or negative direction, a general formula for evaluation of moment can be written as follows:

$$M_{ri} = \hat{M}_i + \sum_{j=1}^{n} m_{ij} (X_j^+ - X_j^-) \qquad , i = 1, \ldots, n \tag{10}$$

in which in any time one of the artificial moments can be present. i.e. in the presence of either of X_i^+ or X_i^-, the other one will have zero value. This can be mathematically stated as: $X_i^+ \times X_i^- = 0$. Substituting the general M_{ri} from Eq.(10) into Eq.(3) for M^+ -M^- and using a combination of objective functions in Eq.(8) and Eq.(9) and the complementary condition of $X_i^+ X_i^- = 0$ yields the following problem for the general elastic-plastic behaviour of a nonlinear connection.

$$\begin{aligned} &Minimize \ \sum_{i=1}^{n} [X_i^+ (M_{ri} - M_{lim,i}^-) + X_i^- (M_{lim,i}^+ - M_{ri}) + X_i^+ X_i^-] \\ &Subject\ to \quad M_{lim,i}^- \le \hat{M}_i + \sum_{j=1}^{n} m_{ij} (X_j^+ - X_j^-) \le M_{lim,i}^+ \qquad , i = 1, \ldots, n \end{aligned} \tag{11}$$

Example 1.

To illustrate the solution procedure, and capabilities of the method a one-bay, one-story portal frame as shown in Fig.(3-a) is considered. In this example the moment-curvature relation of connections of beam to columns are assumed to be elastic-plastic in both positive and negative directions. All members are assumed to have the similar cross-sections. It is assumed that the frame will be loaded laterally up to 40 KN. The Moment capacity of connections in each direction has been intentionally assumed to be 85% of the members' plastic limit as shown in Fig (3-b).

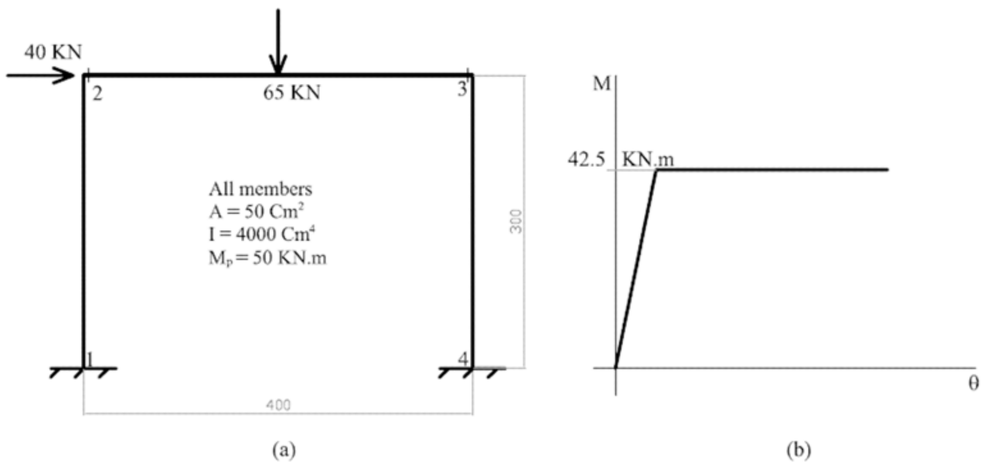


Fig.(3): Example 1, a) Properties of a Portal Frame, b)M- relations for connections 2 and 3

To have a comparison tool in hand, first the non-linear analysis of the frame will be done in a historical manner and then it is compared to the results of proposed solution procedure. Assuming that the structure behaves linearly, it is analysed under total lateral load (40-H & 65-V KN). The result of linear analysis has been reported in the first row of Table 1.

Table 1: Results of analysis of Example 1

Loading	M_1	M_2	Midspan	M_3	M_4
Total (Linear)	-23.892	0.961	41.456	-48.050	47.098
First Yield	-21.132	.850	36.668	-42.5	41.658
Load increment	-6.715	0.904	7.959	0	6.240
Total Nonlinear	-27.847	1.755	44.627	-42.5	47.898

Since value of M_3 =-48.05 exceeds the plastic limit of the connection (42.5 KN.m), the structure will experience inelastic behaviour under this intensity of load. The values of moments in all joints/ connections at the threshold of inelastic behaviour can be obtained by scaling the initial results by $42.5/48.050$. The results have been reported at second row of Table1. In the next stage, for the rest of loading (i.e. 4.620-H and 6.507-V), the frame model will contain a hinge at connection 3. The result of this load increment has been reported in the third row of the Table 1. The total moments due to nonlinear behaviour is the sum of second and third row of the Table and has been reported in the fourth row.

The analysis of the problem via proposed method requires analysis of the structure under unit moments that are applied in all elastic-plastic connections. The results of such analysis for unit loads at joints 2 and3 have been shown in Table 2.

Employing Eq. (10), the resultant moments at connections 2 and 3 can be written as follows:

$$M_{r2} = 0.961 + (1 - 0.6823)(X_2^+ - X_2^-) + (-0.5 + 0.5454)(X_3^+ - X_3^-)$$

$$M_{r3} = -48.050 + (-0.5 + 0.5454)(X_2^+ - X_2^-) + (1 - 0.6823)(X_3^+ - X_3^-)$$

Now according to Eq. (11) a quadratic programming problem is established as follows:

$$\text{Minimize } \sum_{i=1}^{2} [X_i^+(M_{ri} - (-42.5)) + X_i^-(42.5 - M_{ri}) + X_i^+ X_i^-]$$

$$\text{Subject to } M_{r2} = 0.961 + 0.3177(X_2^+ - X_2^-) + 0.0454(X_3^+ - X_3^-) \le 42.5$$

$$M_{r2} = 0.961 + 0.3177(X_2^+ - X_2^-) + 0.0454(X_3^+ - X_3^-) \ge -42.5$$

$$M_{r3} = -48.050 + 0.0454(X_2^+ - X_2^-) + 0.3177(X_3^+ - X_3^-) \le 42.5$$

$$M_{r3} = -48.050 + 0.0454(X_2^+ - X_2^-) + 0.3177(X_3^+ - X_3^-) \ge -42.5$$

Solution of this QP problem gives the following results:

$X_3^+ = 17.46783 \quad \text{and} \quad X_2^+ = X_2^- = X_3^- = 0$

Table 2: Results of analysis of Example 1 for unit loads and Proposed Method.

Loading	M_1	M_2	Midspan	M_3	M_4
Unit moment at 2	+0.0458	1-0.6823	+0.1816	+0.5454-.5	-0.2264
Unit moment at 3	-0.2264	+0.5454-.5	+0.1816	1-0.6823	+0.0458
Total external loads	-23.892	0.961	41.456	-48.050	47.098
Unit moment at 3 × 17.46783	-3.955	0.793	3.172	5.55	0.800
Sum of the Two Above Rows	-27.847	1.754	44.628	-42.5	47.898

It remains to obtain internal forces in all sections using Eq. (10). The results of moment calculations have been shown in fifth and sixth rows of Table2. If the results of Table 1 and Table 2 are compared, it is observed that there is a very small difference between the two results which may have been because of numerical round off error in either of the algorithms/methods or both. As a conclusion, in this example it was shown that the proposed method is capable of solving the nonlinear problem and theoretically saying, it results in exact nonlinear solution.

Nonlinear Analysis with Tri-linear M-Y Curve

In this section the formulation of nonlinear analysis for tri-linear type of connection is presented. Fig. (4) shows such a typical relation. A tri-linear relation usually fits better on an M-Y curve. It can also model strain-hardening of a typical connection. This type of relation may be applicable for both negative and positive moment directions

As shown in Fig. (4) the tri-linear relation may be characterized by four parameters: M_1, M_{lim} ,R_0 and R_1. In which M_{lim} is the limit of bending moment capacity, R_0 and R_1 are initial and secondary flexural stiffness of the connection and M_1 is the first yield in the **M-Y** curve. Similar to the previous procedure for formulation of the problem, first the case of negative moment is considered.

Depending on the value of moment or rotation in a connection, the M-Y relation may coincide one of the three linear relations shown in Fig. (4).

If $\theta_i \le \theta_{i1}$, then the connection behaves linearly. In this case: $M_r = \hat{M}_i$ and there will be no need for external virtual moment.

If $\theta_{i1} \le \theta_i \le \theta_{i2}$, the actual value of $\overline{M}_i^-$ for a given value of Y can be obtained from Eq. (12).

$$\overline{M}_i^- = M_{1i} + (\theta_i - \theta_{1i}) * R_{1i} \quad \textbf{(12)}$$

From linear analysis we have: $\hat{\bar{M}}_i = R_{0i}\theta_i$ and $M_1^- = R_{0i}\theta_1$, therefore it looks possible to substitute for values of Y_i and Y_{1i} to find:

$$\overline{M}_i^- = M_{1i} + (\hat{M}_i^- - M_1^-) * \frac{R_{1i}}{R_0} \tag{13}$$

Eq. (13) stipulates that in the connection with reduced stiffness, only partial part of the moment in excess of M_1 is stored and some part of the excess moment should be distributed in the structure.

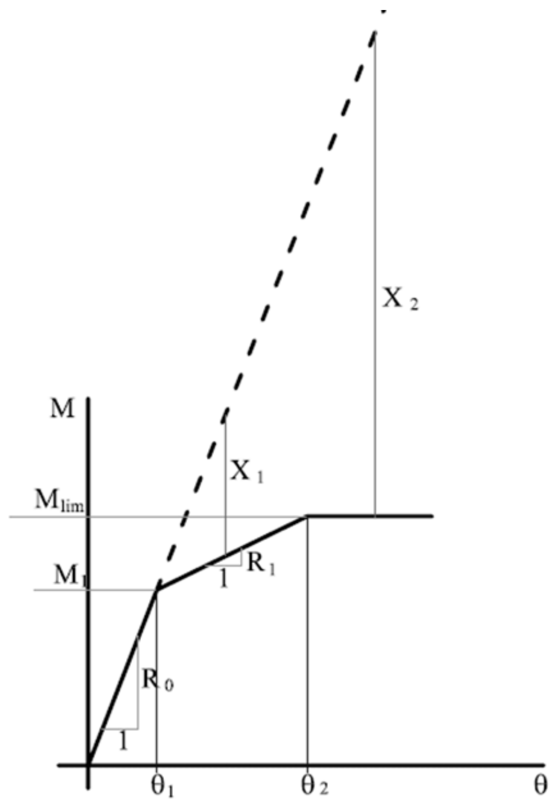


Fig.(4): Atypical Tri-linear M-Y relation for a connection

From the above explanation it is understood that the $(\hat{M}_i^- - M_1^-) * (1 - R_{1i} / R_0)$ part of the excess moment should be distributed in the structure. However any moment redistribution from connection into the structure not only affects the value of rotations of connection and corresponding moments in connected members but also affects the moment distribution in the structure which in turn affects again the value of excess moment in the connection. The study on this mutual moment redistribution between softened connection and the structure shows that there is a reciprocal relation as in Eq. (13) between remained moment in the connection and other parameters such as the moment distribution factor m_{ii} , the ratio of stiffness of member to the connection (EI/L)/R0 and the rate of stiffness softening R0/R1.

$$remained\ Moment\ in\ connection\ i = \gamma_i \times Moment\ in\ excess\ of\ M_1$$

$$\frac{1}{\gamma_i} = 1 + m_{ii}(\frac{R_0}{R_1} - 1)(\frac{4EI/L}{R_0}) \tag{14}$$

in which $\mathbf{m_{ii}}$ is the measured moment at connection **i** due to application of a unit moment at **i**. The statement in Eq. (14) can be mathematically written as the following equation for the softened range of behaviour of the connection.

$$M_{ri}^{-}=\hat{M}_{i}^{-}+\sum_{j=1}^{n}m_{ij}(X_{1j}^{+}+X_{2j}^{+}-(X_{1j}^{-}+X_{2j}^{-}))\geq M_{1,i}^{-}+\Delta M_{i}*\gamma_{i} \qquad \textbf{(15)}$$

The value of ΔM in the above equation includes all transferred moments from all virtually loaded connections onto the connection in question i.e.

$$\Delta M_{i}^{-}=\hat{M}_{i}^{-}-M_{1}^{-}+\sum_{j=1,\,j\neq i}^{n}m_{ij}(X_{1j}^{+}+X_{2j}^{+}-(X_{1j}^{-}+X_{2j}^{-})) \qquad \textbf{(16)}$$

Substituting for ΔM from Eq. (16) into Eq. (15) and rearranging for unknowns gives the yield condition for the case of stiffness softening in the connection as follows:

$$m_{ii}X_{1i}^{+}+(1-\gamma_{i})\sum_{j=1,j\neq i}^{n}m_{ij}(X_{1j}^{+}+X_{2j}^{+}-(X_{1j}^{-}+X_{2j}^{-}))\geq(M_{1,i}^{-}-\hat{M}_{i}^{-})*(1-\gamma_{i}) \qquad \textbf{(17)}$$

Similarly when $\theta_i \geq \theta_{i2}$ we should add the external virtual moment X_{i2} to the connection to reduce the moment to M_{lim}. In this case the following relation can be written between resultant moment in joint **i** and limit moment:

$$M_{ri}^{-}=\hat{M}_{i}^{-}+\sum_{j=1}^{n}m_{ij}(X_{1j}^{+}+X_{2j}^{+}-(X_{1j}^{-}+X_{2j}^{-}))\geq M_{lim}^{-} \qquad \textbf{(18)}$$

It should be noted that, although X_{i1} and X_{i2} as shown in Fig.(4), are independent virtual moments, they can not exist simultaneously. Since it is not known a priori that how much is the θ_i, both of Eqs.(17&18) should be considered in the solution of the problem. In general the QP problem for calculation of unknown X's become as follows:

$$\textit{Minimize}\ \sum_{i=1}^{n}\left\{\begin{array}{l}X_{i1}^{+}[(\hat{M}_{i}^{-}-M_{1i}^{-})(1-\gamma_{i})+m_{ii}X_{1i}^{+}+(1-\gamma_{i})\sum_{j=1,j\neq i}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))]\\ +X_{i2}^{+}[\hat{M}_{i}^{-}-M_{lim,i}^{-}+\sum_{j=1}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))]\end{array}\right\}$$

$$\begin{array}{ll}\textit{Subject to:}\ m_{ii}X_{1i}^{+}+(1-\gamma_{i})\sum_{j=1,j\neq i}^{n}m_{ij}(X_{1j}^{+}+X_{2j}^{+}-(X_{1j}^{-}+X_{2j}^{-}))\geq(M_{1,i}^{-}-\hat{M}_{i}^{-})*(1-\gamma_{i}) & \\ \sum_{j=1}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))\geq M_{lim}^{-}-\hat{M}_{i}^{-} \qquad ;i=1,\ldots,n & \end{array} \qquad \textbf{(19)}$$

Similar QP problem can be established for reverse direction i.e. the case of a tri-linear connection in positive moment direction and unlimited linear relation in negative moment direction. If R_0, R_1, M_1 and M_{lim} properties in positive direction are assumed to be the same as their counterpart in negative direction, the following QP problem can be obtained. However one may use different properties for two different directions of moments.

$$\textit{Minimize}\ \sum_{i=1}^{n}\left\{\begin{array}{l}X_{i1}^{-}[(M_{1i}^{+}-\hat{M}_{i}^{+})(1-\gamma_{i})+m_{ii}X_{1i}^{-}+(1-\gamma_{i})\sum_{j=1,j\neq i}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))]\\ +X_{i2}^{-}[M_{lim,i}^{+}-\hat{M}_{i}^{-}+\sum_{j=1}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))]\end{array}\right\}$$

$$Subject\ to:\ m_{ii}X_{1i}^{-}+(1-\gamma_i)\sum_{j=1,j\neq i}^{n}m_{ij}(X_{1j}^{+}+X_{2j}^{+}-(X_{1j}^{-}+X_{2j}^{-}))\geq(\hat{M}_i^{+}-M_{1,i}^{+})*(1-\gamma_i)$$

$$\sum_{j=1}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))\geq\hat{M}_i^{+}-M_{lim}^{+}\qquad;i=1,\ldots,n \tag{20}$$

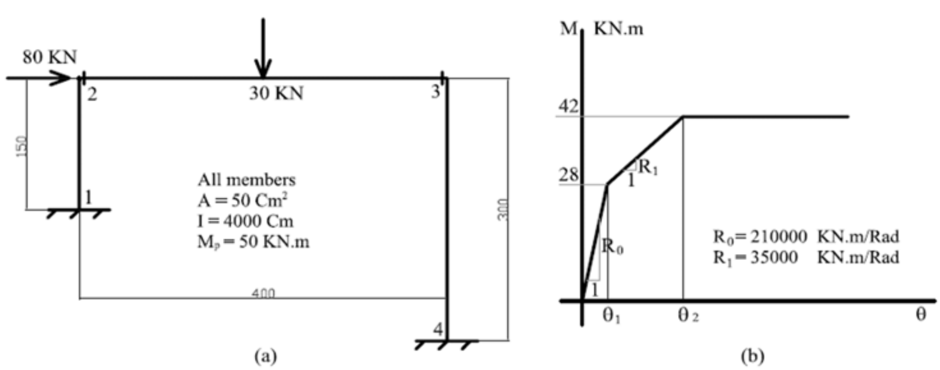
Combining equations (19) and (20) and noting that one of $X_{1i}^{+}, X_{1i}^{-}, X_{2i}^{+}$ and X_{2i}^{-} can possess nonzero value at a time, and the others are zero, yields the following QP sub-problem from which all unknown Virtual Moments can be evaluated.

$$Minimize\quad \sum_{i=1}^{n}\left\{\begin{array}{l} X_{i1}^{+}[(\hat{M}_i^{-}-M_{1i}^{-})(1-\gamma_i)+m_{ii}X_{1i}^{+}+(1-\gamma_i)\sum_{j=1,j\neq i}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))] \\ +X_{i2}^{+}[\hat{M}_i^{-}-M_{lim,i}^{-}+\sum_{j=1}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))]+ \\ X_{i1}^{-}[(M_{1i}^{+}-\hat{M}_i^{+})(1-\gamma_i)+m_{ii}X_{1i}^{-}+(1-\gamma_i)\sum_{j=1,j\neq i}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))] \\ +X_{i2}^{-}[M_{lim,i}^{+}-\hat{M}_i^{-}+\sum_{j=1}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))]+(X_{i1}^{+}+X_{i2}^{+})\times(X_{i1}^{-}+X_{i2}^{-}) \end{array}\right\}$$

$$\begin{array}{l} Subject\ to:\ m_{ii}X_{1i}^{+}+(1-\gamma_i)\sum_{j=1,j\neq i}^{n}m_{ij}(X_{1j}^{+}+X_{2j}^{+}-(X_{1j}^{-}+X_{2j}^{-}))\geq(M_{1,i}^{-}-\hat{M}_i^{-})*(1-\gamma_i) \\ \sum_{j=1}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))\geq M_{lim}^{-}-\hat{M}_i^{-} \\ m_{ii}X_{1i}^{-}+(1-\gamma_i)\sum_{j=1,j\neq i}^{n}m_{ij}(X_{1j}^{+}+X_{2j}^{+}-(X_{1j}^{-}+X_{2j}^{-}))\geq(\hat{M}_i^{+}-M_{1,i}^{+})*(1-\gamma_i) \\ \sum_{j=1}^{n}m_{ij}((X_{1j}^{+}-X_{1j}^{-})+(X_{2j}^{+}-X_{2j}^{-}))\geq\hat{M}_i^{+}-M_{lim}^{+}\qquad;i=1,\ldots,n \end{array} \tag{21}$$

Example 2

To illustrate the capability of the proposed method in the analysis, a portal frame with un-equal column lengths under vertical and lateral load as shown in Fig.(6-a) is considered. The M-Y relation for connections 2 and 3 is shown in Fig(6-b). It is assumed that the members can be loaded up to their plastic moment Mp. Beyond this moment, there will establish a plastic hinge that is assumed to be bilinear.



Fig(6): Example 2, A Portal Frame with Tri-linear Connections at 2 and 3.

It is to be noted that in general a connection is separate than a member and its role should be modeled as a torsional spring with rigidity of R_0 , R_1 and 0 at the three ranges of rotations. In this example the connections in 2 and 3 had to be modeled as in Figure 7-a. This required special analysis software. Instead the model in figure 7-b which can be analyzed by most of commercial softwares was used. In this model torsional rigidity of the members 2-6 and 3-7 are equal to the flexural stiffness of connection. Note that in nodes 1 and 4 there is no connection and the capacity of the structure at these nodes is determines by capacity of the members.

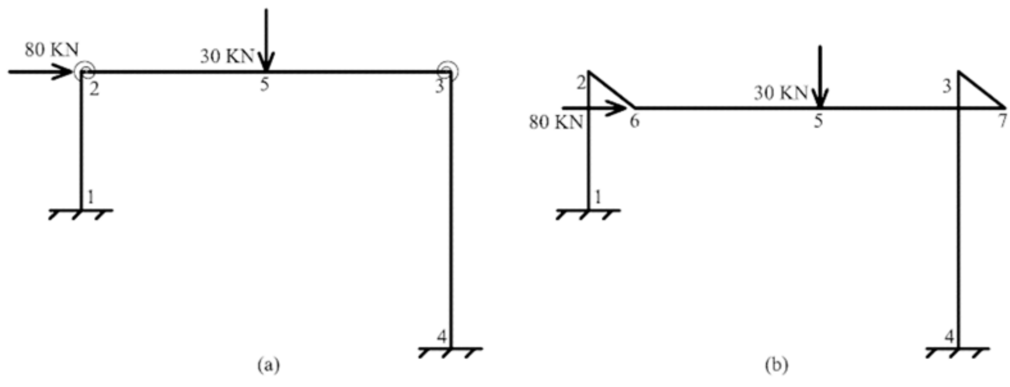


Fig.(7): Actual Analysis model (a) and its equivalent 3-D model (b)

To solve the problem we need to perform a linear elastic analysis for the applied loads and do some analysis for unit moments. The number of virtual unit moments depends on the number of potentially nonlinear points in the structure. Here we assume that all nodes will have nonlinear behavior. However the behavior of joints 1 and 4 will be assumed to be the same as elastic-plastic behavior of members. The moment at midspan does not seem to exceed its proportionate limit. As a result unit moment has not been applied there. Therefore there will be four virtual loadings. The results of aforementioned analyses have been reported in Table 3.

Table 3: Results of Analysis of Frame of Example 2 for Various Types of Loadings

Moment Position	Full External Load	Unit M at 1 in member 1-2	Unit M at 2 in member 2-3	Unit M at 3 in member 2-3	Unit M at 4 in member 3-4
Joint No. 1	-92.815	1.0-0.8503	0.12693	-0.10811	0.32734
Connection 2	7.726	0.5476-0.5	1.0-0.85089	0.4961-0.5	-0.00362
Midspan	28.256	0.00353	0.07260	0.07694	-0.03709
Connection 3	-11.213	-0.04054	0.4961-.5	1.0-0.84222	0.46202-0.5
Joint No. 4	27.206	+0.16367	-0.04828	-0.05064	1-0.31091

According to the results in Table 3, a QP sub-problem, can be established as follows. In the following QP problem a combination of Eq.(11) and Eq.(21) is used. This is because the joints 1 and 4 have bilinear behavior and obey the rules of Eq.(11) and joints 2 and 3 have tri-linear behavior and obey Eq.(21). In addition since in this simple problem we know the direction of applied virtual moments, it is evident that $X_1^- = X_{21}^+ = X_{22}^+ = X_{31}^- = X_{32}^- = X_4^+ = 0$. Therefore only those virtual moments that potentially are nonzero have been participated in the formulation. Because of this decision, only the complementary condition on $X_{i1}^{\pm} \times X_{i2}^{\pm} = 0.$ has been considered.

$$Minimize\ \{ X_1^+ [(\hat{M}_1^- - M_{lim,1}^-) + m_{11}X_1^+ - m_{12}(X_{12}^- + X_{22}^-) + m_{13}(X_{13}^+ + X_{23}^+) - m_{14}X_4^-) +$$

$$X_{12}^- [(1-\gamma_2)(M_{1,2}^+ - \hat{M}_2^+) + m_{22}X_{12}^- - (1-\gamma)(m_{21}X_1^+ - m_{23}(X_{31}^+ + X_{32}^+) + m_{24}X_4^-)] +$$

$$X_{22}^- [(M_{lim,2}^+ - \hat{M}_2^+) - [m_{21}X_1^+ - m_{22}X_{22}^- + m_{23}(X_{31}^+ + X_{32}^+) - m_{24}X_4^-)] + (X_{12}^- \times X_{22}^-) +$$

$$X_{13}^+ [(1-\gamma_3)(\hat{M}_3^- - M_{13}^-) + m_{33}X_{13}^+ + (1-\gamma_3)(m_{31}X_1^+ - m_{32}(X_{21}^- + X_{22}^-) + m_{34}X_4^-)] +$$

$$X_{23}^+ [(\hat{M}_3^- - M_{lim,3}^-) + m_{31}X_1^+ - m_{32}(X_{21}^- + X_{22}^-) + m_{33}X_{23}^+ - m_{34}X_4^-)] + (X_{13}^+ \times X_{23}^+) +$$

$$X_4^- [(M_{lim,4}^+ - \hat{M}_4^+) - m_{41}X_1^+ + m_{42}(X_{21}^- + X_{22}^-) - m_{43}(X_{31}^+ + X_{32}^+) + m_{44}X_4^+]\}$$

$$Subject\ to:\quad m_{11}X_1^+ - m_{12}(X_{12}^- + X_{22}^-) + m_{13}(X_{13}^+ + X_{23}^+) - m_{14}X_4^-) \ge M_{lim,1}^- - \hat{M}_1^-$$

$$m_{22}\, X_{12}^- - (1-\gamma_2)(m_{21}X_1^+ + m_{23}(X_{13}^+ + X_{23}^+) - m_{24}X_4^-)) \ge (1-\gamma_2)(\hat{M}_2^+ - M_{12}^+)$$

$$- m_{21}X_1^+ + m_{22}X_{22}^- - m_{23}(X_{13}^+ + X_{23}^+) + m_{24}X_4^- \ge \hat{M}_2^+ - M_{lim,2}^+$$

$$m_{33}X_{13}^+ + (1-\gamma_3)(m_{31}X_1^+ - m_{32}(X_{21}^- + X_{22}^-) - m_{34}X_4^- \ge (1-\gamma_3)(M_{13}^- - \hat{M}_3^-)$$

$$m_{31}X_1^+ - m_{32}(X_{21}^- + X_{22}^-) + m_{33}X_{23}^+ - m_{34}X_4^- \ge M_{lim,3}^- - \hat{M}_3^-$$

$$- m_{41}X_1^+ + m_{42}(X_{21}^- + X_{22}^-) - m_{43}(X_{31}^+ + X_{32}^+) + m_{44}X_4^+ \ge \hat{M}_4^+ - M_{lim,4}^+$$

$$X_{12}^- \times X_{22}^- = 0$$

$$X_{13}^+ \times X_{23}^+ = 0$$

In this sub-problem m_{ij} values are given in Table 3. Other parameters are as follows:

$M_{lim,1}^+ = -M_{lim,1}^- = M_{lim,4}^+ = -M_{lim,4}^- = 50 \quad KN.m$

$M_{1,2}^+ = -M_{1,2}^- = M_{1,3}^+ = -M_{1,3}^- = 28 \quad KN.m$

$M_{lim,2}^+ = -M_{lim,2}^- = M_{lim,3}^+ = -M_{lim,3}^- = 42 \quad KN.m$

$\gamma_2 = 0.788776 \quad and \quad \gamma_3 = 0.7593655$

Solution of the problem yields the following results:

$X_1^+ = 574.1396 \quad X_{21}^- = 54.6801 \quad X_{31}^+ = 10.6389 \quad X_4^- = 107.0615$

According to these results evaluation of internal forces in members is quite simple. Table 4 shows the calculation of final nonlinear response for moments.

It can be seen from the Table 4 that the position of vertical point load (Midspan) is still in its proportionate range and therefore, as it was foreseen, there is no need to apply external virtual moments. On the other hand except Node No. 1 it did not seem that other nodes can reach to nonlinear stage. But results show that Node No. 4 has reached to its plastic limit; and the moments at connections No. 2 & 3 are a little beyond the first yield of the connection. It is obvious that if the load is increased, the moments at connections 1 and 4 will remain constant and the moments of Midspan, Node 2 and Node 3 will increase.

Table 4: Calculation of nonlinear response of Example 2, Proposed method.

Effect of X_j at joints and connections	Values of Virtual Moments				Total Moment
Moment Position	X_1^+	X_{21}^-	X_{31}^+	X_4^-	$\hat{M} + X_1^+ + X_{21}^-$ $+ X_{31}^+ + X_4^-$
	574.1396	54.6801	10.6389	107.0615	
Node No. 1	85.9506	-6.9404	-1.1502	-35.0454	-50.0000
Connection No. 2	27.3277	-8.1531	-0.0415	3.8752	30.7340
Midspan	2.0254	-3.9699	0.8186	3.9705	31.1010
Connection No. 3	-23.2768	0.2134	1.6787	4.0658	-28.5319
Node No. 4	93.9691	2.6389	-0.5387	-73.7753	50.0000

Conclusion

In this paper, a new, non-iterative method of nonlinear analysis was proposed. In the proposed method to make internal moments follow their nonlinear moment-rotation curves, some virtual moments (that are primarily unknown) were imposed onto the structure at semirigid connections. To find the unknown virtual moments, a quadratic programming problem was formulated and solved. In the proposed method, the exact nonlinear response of structure including internal forces and moments of members are calculated by employing the values of virtual moments and using the superposition principle. Compared to the classical methods of nonlinear analysis, the following preferences can be mentioned for the proposed method.

- In the classical methods, the nonlinear analysis is done through iterative procedures which consist of modification of the stiffness matrix of structure and/or load vector, but in the proposed method the nonlinear analysis is performed in one step without change in the initial model and stiffness matrix of the structure or its load vector.
- the final result of iterative methods are somewhat dependant on the start point and the convergence criteria while in the proposed method it is obtained by solution of a Quadratic programming method and therefore not only it does not require any initial point, but also it does not require convergence criterion and mathematically saying it gives exact results.
- To increase the accuracy of the results in the classical methods it is necessary to decrease the incremental loading and tighten the convergence criterion. This increases the number of required analysis. However in the proposed method the results are exact and only preventing round off error in calculations increases the accuracy.

The method is capable to model semirigid connections with multilinear moment-curvature relations. The formulation of the problem for bilinear and trilinear moment-curvature relations was presented. Two examples were solved to demonstrate the robustness, capability and validity of the method. It was shown that the method not only can be used for nonlinear connections but also provided that the critical sections are pre-specified, it can do the plastic analysis in a flexural frame as well.

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