STORY BASED DUCTILITY MODELS FOR DISPLACEMENT BASED DESIGN OF STEEL FRAMES

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Abstract

Estimation of ductility demand distribution through the height of the structure is a very hard task for seismic design engineers working on performance based design of buildings. In this paper a modified direct displacement based design procedure has been proposed. In this method the design force distribution among the height of the structure is obtained based on various ductility demand distributions derived from modal characteristics of the structure and mathematical formulations. The method has been applied to the moment steel frames in low, medium and high rise buildings and the results of various ductility distributions have been compared. The plastic mechanism has also been modeled and the efficiencies and deficiencies of each have been discussed through various numerical examples. The effect of yield mechanisms and ductility demand patterns for various building types on the equivalent SDOF parameters have been investigated compared to the time history analysis results to find the sensitive parameters.

Introduction

The purpose of Performance Based Design is to design the structure with sufficient and proportioned stiffness and strength in the structural members so as to develop inelastic action in the ductile designed members and to have appropriate over strength in the brittle members. Then the structure must be checked so that the demands do not exceed the existing capacities. This is best performed using a set of nonlinear dynamic analyses under earthquake with appropriate characters. Different design methods have been proposed based on performance criteria such as, Capacity Spectrum Method [1,2,3], N2 Method [4,5], Energy Based Methods and Displacement Based Design (DBD) Methods. In the last four decades the idea of DBD has been introduced and developed by different researchers started by introducing the concept of substitute structure [6]. This idea has been adopted for a direct displacement design of SDOF and MDOF reinforced concrete bridges [7,8,9]. Capacity Spectrum Method and the N2 Method have also been used to create other direct DBD procedures [1,2,3,4,9,10]. In all of these researches, seismic demand is specified as either a displacement spectrum or an acceleration-displacement response spectrum. Generally nonlinear inelastic behavior of a structural system can be accounted for either by an equivalent elastic response spectrum or an inelastic response spectrum. The former is associated with effective viscous damping and the latter is directly constructed based on relations between reduction factors and ductility.

In this paper the direct DBD method is briefly reviewed for multi story steel buildings. In this method the design force distribution among the height of the structure is obtained based on various ductility demand distributions derived from modal characteristics of the structure and mathematical formulations. The method has been applied to the steel braced frames with concentric and eccentric bracing systems in low, medium and high rise buildings. The plastic mechanism for each system has also been modeled and the efficiencies and deficiencies of each have been discussed through various numerical examples. The effect of yield mechanisms and ductility demand patterns for various

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building types on the equivalent SDOF parameters have been investigated compared to the time history analysis results to find the sensitive parameters. A design displacement spectrum has also been created for the parametric study based on the Iran earthquakes. Various factors affecting the dynamic response have also been provided in the procedure. It has been shown that this method is capable of predicting the response of braced frames especially high rise buildings in an efficient and robust manner.

Displacement Based Design of Steel Frames

Direct Displacement Based Design of multi story buildings is based on the generation of equivalent SDOF system or substitute structure concept. For this purpose, it is assumed that the structure vibrates in a pre-defined harmonic displaced shape. The base shears and the works developed by lateral external forces are also assumed the same for both equivalent and main structures [7-15]. Consider the relative displacement vector $\{\delta(h,t)\}$ for the multistory building with total height of H expressed in a decomposed form of displacement and time and assume a harmonic response with amplitude Δ for the system. We can write,

$$\{\delta(h,t)\} = \Delta.Sin(\omega.t).\{\Phi(h)\}\ ,\ 0 \le h \le H \tag{1}$$

which results in an acceleration vector $\{a(h,t)\}$ proportional to the assumed normalized displacement vector $\Phi(h)$ as follows,

$$\{a(h,t)\} = -\Delta.\omega^2.Sin(\omega.t).\{\Phi(h)\} = -\omega^2.\{\delta(h,t)\} \tag{2}$$

In order to obtain the equivalent system parameters, we define the normalized displacement vector $\{c(h,t)\}$ as,

$$\{c(h,t)\} = \frac{1}{\delta_{eff}}\{\delta(h,t)\} \tag{3}$$

where δ_{eff} is called the effective displacement. From equation 2 and 3 we may have,

$$c_i(h,t) = \frac{\delta_i(h,t)}{\delta_{eff}} = \frac{a_i(h,t)}{a_{eff}}, i = 1,2,...,n \tag{4}$$

in which n stands for number of stories, a_{eff} is called the effective acceleration of the equivalent SDOF system and δ_i , a_i are the story displacement and acceleration respectively. Using equation 4, the base shear can now be determined in terms of the multi story structure and the equivalent system parameters as,

$$V_b = \sum_{i=1}^{n} f_i = \sum_{i=1}^{n} m_i.a_i = \left\{\sum_{i=1}^{n} m_i.c_i\right\}.a_{eff} = m_{eff}.a_{eff} \tag{5}$$

which leads to the definition for the effective mass as $m_{eff} = \sum_{i=1}^{n} m_i.c_i$. The lateral force at each level, f_i may also be determined using equations 4 and 5 as,

$$f_i = \frac{m_i.\delta_i}{\sum_{j=1}^{n} m_j.\delta_j} V_b \tag{6}$$

Equating the external works for the two systems, $V_b.\delta_{eff} = \sum_{i=1}^{n} f_i.\delta_i$ and using equation 6, we can obtain the definition for effective displacement as,

$$\delta_{eff} = \frac{\sum_{i=1}^{n} m_i.\delta_i^2}{\sum_{i=1}^{n} m_i.\delta_i} \tag{7}$$

The effective stiffness of the substitute SDOF system may also be obtained by entering the effective displacement into the displacement response spectrum with appropriate damping value and then substituting the obtained effective period and effective mass from equation 5 into the following equation,

$$K_{eff} = \frac{4\pi^2 m_{eff}}{T_{eff}^2} \tag{8}$$

The effect of story ductility may be considered substituting δ_i defined in equation 4 with the following relationship,

$$\delta_i = \mu_i.\delta_{yi} \tag{9}$$

where μ_i is the story ductility demand and δ_{yi} is the story yield displacement. The problem is now how to determine these two parameters. The story yield displacement δ_{yi} may be obtained by defining the story yield mechanism and has been discussed later in section 3. Determination of the ductility distribution through the height of the structure has also been discussed in section 3. Finally for detail design of the structure the base shear is obtained as $V_b = K_{eff}.\delta_{eff}$ and then the story forces f_i are computed using equation 6. Then the capacity design of the structure can be started considering the ductility capacities. This capacity-designed structure may then be verified using the time history or static push over analyses.

Design Displacement Spectrum

In this paper, a displacement response spectrum has been obtained through a deterministic procedure based on acceleration data for Iran earthquakes. These accelerograms were selected form more than 2000 records for different stations and earthquakes in Iran. The near field records were omitted and the accelerograms with medium to high magnitude (minimum 5 degrees in Richter scale) were selected. Using an Artificial Neural Network simulator (a committee neural simulator including competitive and back error propagating networks), prepared by the authors [17], the records were categorized according to their shapes (duration, sequence of peaks and their amplitude) by the competitive network to four categories. Each category represented a soil type thus the design displacement spectra for each soil type was obtained. The spectrum with 5 percent damping for soil type C (or II according to Iranian seismic code) has been presented in Fig. 1. A four-degree polynomial function has been matched to the data with a 0.98 standard deviation as shown in the figure. This equation was used to calculate the effective displacement in the numerical examples.

Definition of Effective Damping

Assuming a single displacement cycle based on the ultimate displacement the following well-known relationship between ξ_{eff} and the ductility demand μ for Elastic-Perfectly Plastic (EPP) behavior is

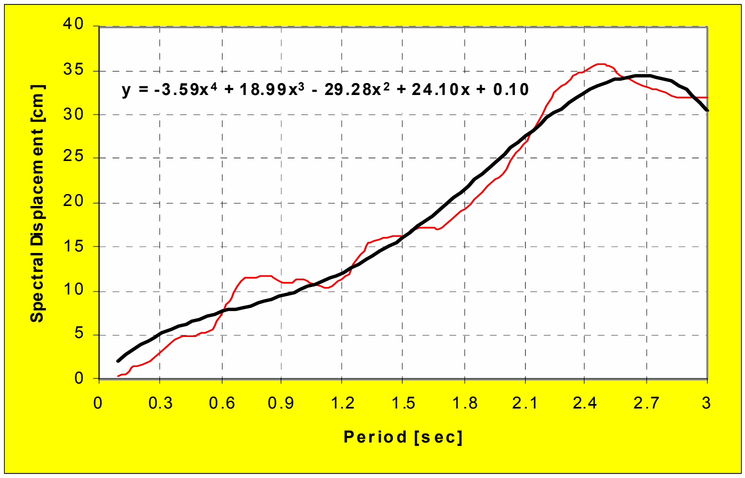


Figure 1: The displacement spectrum based on filtered Iran earthquakes and the design curve

obtained (Equivalent Energy Method [16]),

$$\xi_{eff} = \frac{2}{\pi}\left(1 - \frac{1}{\mu}\right) + \xi_{elastic} \tag{10}$$

where $\xi_{elastic}$ stands for the damping of the elastic structure. The equivalent viscous damping for bilinear systems with strain hardening ratio α and ductility μ may also be determined using the following equation [5],

$$\xi_{eff} = \frac{2}{\pi}\left(\frac{(1-\alpha).(\mu-1)}{\mu - \alpha\mu + \alpha\mu^2}\right) + \xi_{elastic} \tag{11}$$

The effective damping obtained above which is greater than the elastic viscous damping due to the hysteretic behavior is then used to get the effective period from the displacement spectrum. The presented spectrum shown in fig. 1 may be modified for other damping values using the EC8 [18] factor $\sqrt{7/(2+\xi)}$. Greater value of damping results in a greater effective period in the displacement spectrum and thus less base shear for the design.

Ductility Demand Models

In recent years many researchers have tried to find engineering procedures to do this task but each had some limitations. Studies have shown that Push Over Analysis is limited to low to medium rise buildings and is not recommended for high rise or flexible structures in which higher mode effects are not negligible and may govern the dynamic response [19]. Other existing methods such as the Capacity Spectrum Method and the N2 Method have similar limitations. In the first part of this section, simple plastic models for the story or local ductility of braced steel building with concentric and eccentric bracing systems have been proposed. In the second part, various functions for ductility demand distribution over the height of the structure have been introduced and then they have been verified by comparing to dynamic analysis results through various numerical examples.

Story Plastic Mechanism Models

In the presented DBD method, the story mechanisms are presumed to occur due to the yielding of the lateral resisting system. Therefore, the failure mechanism of the stories that depends on the structural geometry, yield strains and lateral load resisting system may be determined. In the case of a capacity-designed structure, the lateral mechanism can be predicted with good approximation. For steel braced frames considered in this study, the mechanisms are assumed to form by yielding of braces in concentric systems and by flexural yielding of link beams in eccentric systems. In these models one column lift is assumed for each story, which means that no rotational column plastic hinge is produced.

For concentric X bracing systems as shown in fig. 2-a the lateral story displacement δ_i can be written as a function of story shear V_i, brace span L_s, brace length L_b, brace sectional area A_b and elastic section modulus E as,

$$\delta_i = \frac{V_i.L_b^3}{E.A_b.L_s^2} \tag{12}$$

Using equation 12 and considering the relation $L_s / L_b = V_{yi} / \sigma_y.A_b$ from figure 2, the story yield displacement δ_{yi} in a concentric braced frame can obtained as,

$$\delta_{yi} = \frac{V_{yi}.L_b^3.\varepsilon_y}{\sigma_y.A_b.L_s^2} = \frac{L_b^2.\varepsilon_y}{L_s} \tag{13}$$

which does not depend on the brace sectional area. Similarly, for the Chevron bracing shown in fig. 2-b, the same equations may be obtained as follows,

$$\delta_i = \frac{2V_i.L_b^3}{E.A_b.L_s^2} \tag{14}$$

$$\delta_{yi} = \frac{2V_{yi}.L_b^3.\varepsilon_y}{f_y.A_b.L_s^2} = \frac{2L_b^2.\varepsilon_y}{L_s} \tag{15}$$

It is again clear that for Chevron bracing the displacement shape in yielding case of the structure is independent of the brace sectional properties. Thus for the concentric systems, an equal span and story height result in a linear deformed shape of the structure over the height at yield condition.

For eccentric bracing systems (fig. 2-c), the lateral displacement depends on plastic rotation capacity of the link beam, θ_p, and can be expressed through the following equations,

$$\delta_{yi} = H_i.Sin(\theta) - l_l.(1 - Cos(\theta_p - \theta)) \tag{17}$$

$$\theta = \pi - \beta - ACos\left[\frac{L_s^2 - l_l^2 + 2l_l.L_s.Sin(\beta - \theta_p)}{2L_s.L_b}\right] \tag{18}$$

where θ_p is defined as a function of section plastic moment M_p and plastic stiffness K_p as,

$$\theta_p = \frac{M_p}{K_p} = \frac{V_i.H_i}{2K_p} \tag{19}$$

For small length link beams, the shear mechanisms are formed while in moderate length link beams moment plastic hinges are generally formed. The latter case has been considered here. The value of θ_p is generally related to the connection and link beam details.

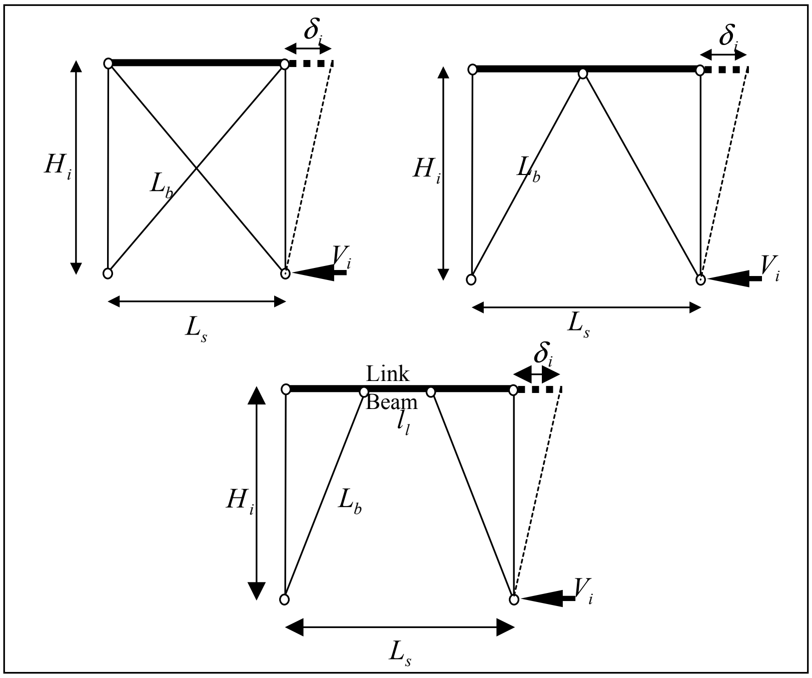


Figure 2: Structural models for (a) X bracing, (b) chevron bracing and (c) eccentric bracing systems

Ductility Demand Distribution Patterns

In the conventional DBD in which the ductility is assumed uniform over the height, the effective stiffness would not change with the ductility. If the ductility is distributed according to the ductile design of braces over the height for example based on the elastic modal vibration of the structure, the brace characteristics will interfere with the lateral displacement. The increase in the resulted ductility in comparison with the uniform distribution, will cause reduction in the effective mass and results in an increase in the effective period due to the rise in effective displacement and thus reduce the resulted effective stiffness of the substitute SDOF structure. In the presented DBD method, the lateral displaced shape of the structure is modified using multi-modal, polynomial and exponential distributions of ductility over the height of the structure in order to take into account higher mode effects and combined shear and flexural lateral deformations. Higher mode effect cause considerable changes in the dynamic response of large period or flexible structures such as tall buildings. Besides, the ductile behavior of the building also results in considerable increases in the system period. This issue has been discussed in the parametric study. In low rise buildings the shear behavior often governs the response and in medium rise buildings a combined shear and flexural deformation is normally expected.

For modal distribution of ductility it is assumed that the distribution is approximately conformed to the some first mode shapes which have a more than 98 percents of the system mass. For most of the structures, the first three modes of a cantilever with known equations may be assumed or alternatively an elastic modal analysis of the structure can be performed and then a mode combination procedure followed. However in order to avoid high strain demand in members in the structural design of the braced or wall buildings, the participation of higher modes must be limited. This issue is generally considered in the capacity design of the structure so that the mass portion of the first mode does not decrease to values less than 70 percents or alternatively reduce the number of modes that own the 98 percents of the system mass. In the presented study, as an alternative approach and in order to conform to the capacity design criteria, the effect of higher modes on the ductility demand distribution in the stories below the effective height have been neglected,

$$H_f = \frac{\sum_{i=1}^{n} f_i . H_i}{V_b} \tag{20}$$

The second selected pattern is an exponential function with parameter a as follows,

$$\phi_{\mu[EXP]} = \mu_{\max} . \frac{1 - EXP(-a\, h/H)}{1 - EXP(-a)} \tag{21}$$

The third function is a simple polynomial function with parameter b as,

$$\phi_{\mu[P]} = \mu_{\max} . \left(\frac{h}{H}\right)^{b} \tag{22}$$

The comparison of various ductility patterns using the mentioned functions has been plotted in fig. 3. The final displaced shape is obtained by multiplying the initial profile obtained from the mechanism models, introduced in the previous section, by ductility demand distribution. The maximum ductility capacity of the each story (defined as the ratio of maximum displacement capacity and yield displacement of the story) must also be determined. This parameter defines the limit state or the performance point. For concentric and chevron braced frames this is governed by the capacity of the brace to beam connection and for eccentric braces the plastic rotation capacity of the link beam defines the ductility. Some modifications for the effects of strain hardening and cumulative damage can also be assessed in the DBD method that has been discussed in the next section.

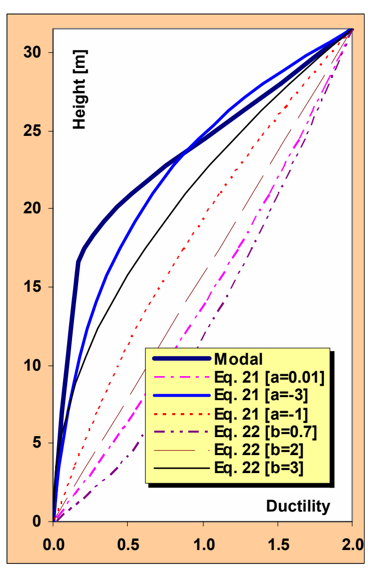


Figure 3: Various ductility distribution patterns considered in the study

Numerical Analyses

In this section, the presented modifications have been verified through some numerical examples including three, nine and twenty story braced frames. The geometric data of the models are presented in table 1 and fig. 5. The steel properties are, yield stress $\sigma_y = 245MPa$, initial elastic modulus $E = 2.10e5MPa$ with a bilinear nonlinear behavior with five percent strain hardening or second modulus $E_s = \alpha.E, \alpha = 0.05$. The dead load is assumed to be 3.9 kPa. and the reduced live load 1.4 kPa at floor levels. At roof level these values are assumed to be 3.2 kPa. and 1.0 kPa. respectively. The assumed data may be sufficient for DBD, but for the nonlinear push over and time history analyses the detail design of the members must also be available. This has been performed using the capacity design procedures and was performed using SAP2000 [21] commercial program. The modified strength reduction factors for MDOF structures taking into account the reductions due to structural over strength have been calculated based on the equations proposed in [23]. The effects of cumulative damage may also be considered using the idea presented in [5] that has been discussed subsequently. Nonlinear dynamic analyses have been performed using DRAIN2DX [22] program using three selected earthquake record which were compatible with the obtained response spectrum shown in fig. 1.

Table 1: Geometric data for numerical examples (Dimensions in meter)

n story	H_1	$(n-1)*H_i$	L_{s1}	$No*L_{span1}$	L_{s2}	$No*L_{span2}$	l_l
3 story	3.5	2*3.5	2(4.0)	2*4	2(3.5)	2*4	1.5
9 story	4	8*3.5	2(4.2)	2*4.2	2(3.5)	2*4	1.5
20 story	4	19*3.5	2(4.2)	3*4.2	2(3.5)	3*4	1.5

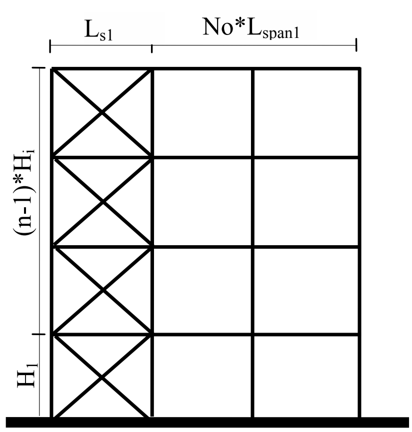


Figure 5: Geometric data for all numerical examples

Figure 6 shows the effect of the ductility demand distribution on story force and story drift using modal ductility distribution and equations 21 and 22 for a nine story building with eccentric bracing. The maximum ductility is assumed to be 2 according to fig. 8. It has been shown that the lateral story force has not been so sensitive to the ductility pattern but the displacement and drift directly change with the ductility in the DBD and equation 21 with a=-3 and equation 22 with b=3 may be acceptable comparing to the dynamic analysis results. In table 2 the effect of ductility pattern on DBD parameters (Effective parameters on the equivalent SDOF structure) has also been presented. As shown the effective damping, total base shear, mass and effective height ratio which may be assumed as the representative for lateral force distribution are not sensitive to the ductility distribution over height and just depend on the maximum ductility value. In figure 7 the period of the first mode from eigen value analysis has been compared to the periods obtained from linear and nonlinear time history analyses showing the effects of vertical loads and nonlinear behavior and also the effective periods from DBD method taking into account the effect of column deformation for the last story based ductility model.

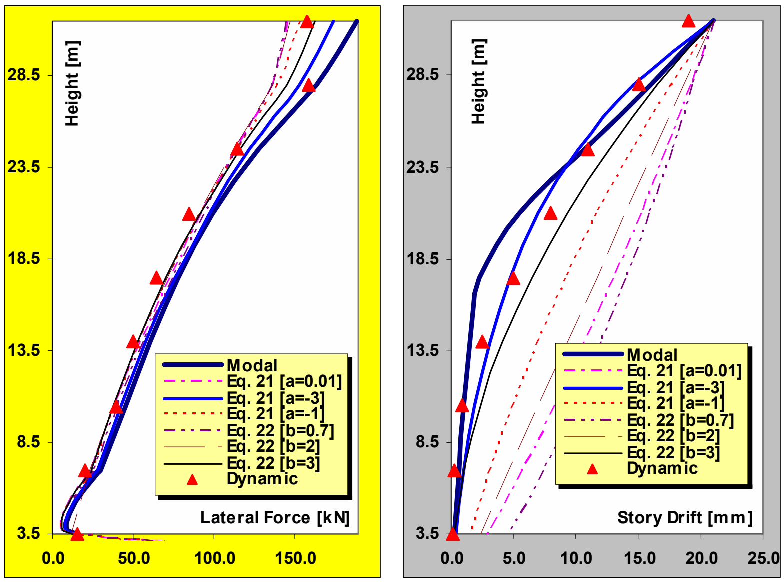


Figure 6: Effect of ductility demand distribution on story force and drift

Table 2: Effect of ductility demand distribution on DBD parameters

Effective Parameters	Units	Modal Pattern	Exponential (Equation 21)			Polynomial (Equation 22)		
			a=0.01	a=-3	a=-1	b=0.7	b=2	b=3
Effective Period	sec	0.85	1.30	0.92	1.14	1.35	1.25	1.01
Effective Damping	%	4.36	4.79	4.42	4.65	4.81	4.74	4.53
Mass Ratio	_	0.71	0.73	0.71	0.72	0.74	0.72	0.70
Effective Stiffness	N/mm	13601	5986	11783	7622	5658	6427	9550
Effective Displacement	mm	92	134	97	116	140	128	104
Effective Height Ratio	_	0.73	0.73	0.73	0.73	0.72	0.73	0.74
Total Base Shear	kN	798	674	756	683	685	674	705

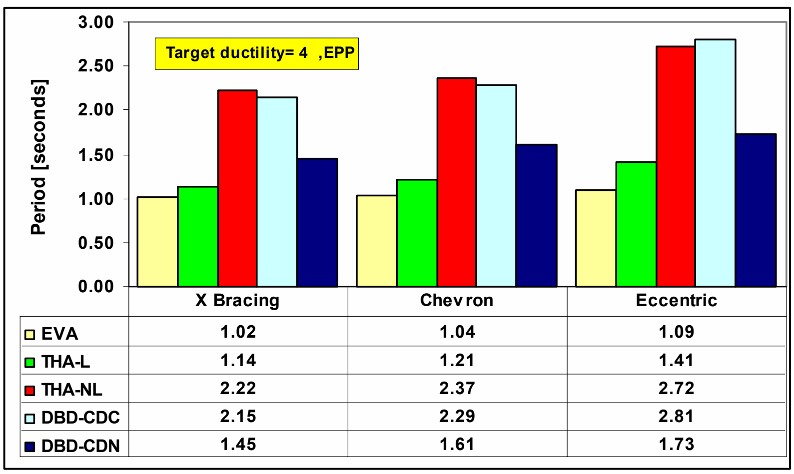
Concluding Remarks

Displacement based procedures can directly lead the designer to the key design parameters such as interstory drifts and displacements. If the ductility is distributed according to the ductile design of braces over the height for example based on the elastic modal vibration of the structure, the brace characteristics will interfere with the lateral displacement. The results obtained from these studies have been summarized as follows,

- The modal ductility demand distribution and the resulted force distributions give acceptable results for tall buildings compared to the dynamic analyses. However in order to avoid high strain demand in members in the structural design of the braced buildings, the participation of higher modes must be limited. This issue is generally considered in the capacity design of the structure so that the mass portion of the first mode does not decrease to values less than 70 percents or alternatively reduce the number of modes that own the 98 percents of the system mass.

- As expected the ductility demand in eccentric systems had the highest values having other specifications constant. This ductility was highly dependent to the plastic rotations in the mid length link beams. It has also been shown that the lateral story force has not been so sensitive to the ductility pattern but the displacement and drift directly change with the ductility. The effective damping, total base shear, mass and effective height ratio which may be assumed as the representative for lateral force distribution are not sensitive to the ductility distribution over height and just depend on the maximum ductility value.
- As the relationship between member strains or member plastic rotations with the inter-story drifts are determined, both local and global performance criteria may be used for such a design method.

As shown by various examples, various effects such as column deformation effects, torsional effects, higher mode effects, $P-\Delta$ effects and low cycle fatigue effects may easily be assessed by the use of equivalent procedures in the direct DBD.



	X Bracing	Chevron	Eccentric
EVA	1.02	1.04	1.09
THA-L	1.14	1.21	1.41
THA-NL	2.22	2.37	2.72
DBD-CDC	2.15	2.29	2.81
DBD-CDN	1.45	1.61	1.73

Figure 7: Effect of column deformation, vertical load and nonlinearity on natural and effective periods for (a) nine and (b) twenty story buildings [Symbols: EVA: Eigen value analysis, THA: Time history analysis, L: Linear, NL: Nonlinear, CDC: Column deformation considered, CDN: Column deformation neglected]

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