OPTIMIZATION METHOD OF PILE FOUNDATIONS

Anthony J. Hurd and Kevin Z. Truman
Department of Civil Engineering, Washington University in St. Louis, St. Louis, Missouri 63130, USA

Abstract

A computer-automated design and optimization process for pile foundations with rigid concrete slabs is presented. Optimality Criteria methodology is used to provide optimal pile designs. A three-dimensional optimization computer program has been developed that designs a foundation system with an optimal number of piles, geometric layout, pile orientation, batter, and size for a given structure subjected to multiple load cases. The optimization procedure controls displacements while reducing the overall weight of the pile foundation design. A new method for optimizing weightless variables, such as batter, was also created. Thus, the challenges of optimizing variables that indirectly affect the weight of the pile foundation can still be designed to create weight savings. In one example, the total volume of the steel piles is reduced from 61,920 in^3 to 49,570 in^3 by optimizing only the pile sizes. Furthermore, the weight is reduced again by simultaneously optimizing each pile group's size coupled with the weightless variable, batter.

1. Introduction

The purpose of this research is to create a computer-automated optimization process for large pile foundations. The U.S. Army Corps of Engineers (USACE) designs large-scale locks and dams that can easily contain thousands of piles, costing millions of dollars. The USACE currently uses pile analysis computer programs but none with optimal design. Therefore, the process for reducing the number and size of the piles is very time-consuming and uncertain, involving the tedious process of manual design, computer analysis, and redesign. This can easily take months and still result in a non-optimal final design. An automated computer optimization process would find an optimal design and take only minutes rather than months. The designer only enters an initial design, load cases, soil conditions, and constraints while the program alters the many design variables to create an optimal foundation design.

Work in pile foundation optimization was originally developed for the USACE by Hill in 1981, using a trial and error approach (Hill, 1981). His method involved first optimizing the batter and then finding an optimal pile spacing. The process finished by iteratively deleting the most and/or least stressed piles. However, this method is not numerically based, and it does not simultaneously optimize all the pile variables, so a true optimum solution is never found. Hoback and Truman used a numerical method to optimize pile designs of both rigid-slab foundations (Hoback et al., 1991) and flexible-slab foundations (Hoback et al., 1993). Their method utilized an Optimality Criteria method employed earlier by Cheng and Truman for structural frames (Cheng et al., 1983).

Optimality Criteria was chosen for this optimization for the following reasons: first, the Optimality Criteria method converges quickly with most examples converging in less than ten iterations. Second, Hoback and Truman were able to simultaneously optimize pile size, layout, batter, orientation, and number while still controlling all stress, strain, or displacement constraints.

The Optimality Criteria method works very well when optimizing a variable that *directly* affects the objective function, whether the objective function to minimize is weight, volume of steel, or cost. For

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example, pile size directly affects the weight of a design. However, some variables such as batter or geometric layout do not have direct effects on the weight of a design, but altering these weightless variables can clearly create large reductions in weight. Thus, a novel approach to optimizing these weightless variables was developed. Using a weightless scaling factor that acts as a pseudo-weight gradient, the Optimality Criteria method can still optimize these weightless variables. The challenges in choosing an appropriate weightless scaling factor were overcome by creating a unique way of gradually decreasing the scaling factor from iteration to iteration, approaching a more optimal design.

2. Optimality Criteria Method

The Optimality Criteria method requires an objective function to be minimized. Because the goal of this research is to minimize the cost of the pile foundation while still satisfying the constraints, the objective function is the total weight of the steel, W_T, in the piles which is directly related to cost:

$$W_T = \sum_{i=1}^{n} \rho L_i A_i \tag{2.1}$$

where:	i	=	Pile number
	n	=	Number of piles
	ρ	=	Density of Steel
	L_i	=	Length of element i
	A_i	=	Area of element i

An unconstrained minimum weight of Equation 2.1 does not exist. Once displacement, stress, and/or strain constraints, h_j, are introduced to the problem, the Lagrangian function, L, can be written as

$$L = W_T + \sum_{j=1}^{m} \lambda_j h_j \tag{2.2}$$

where:	j	=	Constraint number
	m	=	Number of constraints
	λ_j	=	Lagrange multiplier for constraint j

and when the constraint is displacement, u_j, with a maximum displacement in the j^{th} direction of $\bar{u}_j$,

$$h_j = u_j - \bar{u}_j \leq 0 \tag{2.3}$$

A minimum of the Lagrangian will be located where the derivative with respect to each design variable, d, is equal to zero. Thus, a pile foundation design can be a local minimum weight if all the constraints, $h_j \leq 0$, are satisfied and there exist λ_j such that

$$\frac{\partial L}{\partial d_i} = \frac{\partial W_T}{\partial d_i} + \sum_{j=1}^{m} \lambda_j \frac{\partial h_j}{\partial d_i} = 0 \qquad i = 1, 2, \ldots n \tag{2.4}$$

where:	d	=	Design variable (pile size, batter, etc.)
	i	=	Design variable number
	n	=	Number of design variables

It is important to note that the Lagrange multipliers, λ_j, cannot be negative because negative λ_j values would still allow the constraints to be satisfied while the weight of the piles increases. Thus,

$$\lambda_j \geq 0 \tag{2.5}$$

Equations (2.4) and (2.5) are the *Kuhn-Tucker conditions* (Kirsch, 1993). Equation (2.4) is rewritten to provide the *optimality criteria:*

$$T_i = \frac{-\left(\sum_{j=1}^{m} \lambda_j \frac{\partial h_j}{\partial d_i}\right)}{\frac{\partial W_T}{\partial d_i}} = 1 \qquad i = 1, 2, \ldots n \tag{2.6}$$

If the optimality criteria, T_i, is less than one and the weight gradient, $\partial W_T/\partial d_i$, is positive, then the i^{th} variable can be decreased. This is because the weight gradient is larger than the constraint gradient (the numerator), indicating a benefit in reducing the i^{th} variable. The opposite is true if the weight gradient is negative. *Linear recurrence equations* are used to change the design variables, pushing the design toward a local minimum. The recurrence formula is based on the expanded power law, and T_i is used as the efficiency of each variable:

$$d_i^{k+1} = d_i^k + \frac{1}{r}\left(T_i - 1\right)d_i^k \qquad i = 1, 2, \ldots n \tag{2.7}$$

where:	k	=	Index of the iteration number
	r	=	Convergence control parameter

The convergence control parameter, r, assures that the prediction of the design variable, d_i, for the next iteration does not go beyond the optimum. A reasonable value for r is 2, but $r > 1$.

Now, the only unknowns in the recurrence relation are the Lagrange multipliers, λ_j. The change in the active constraints will give us these values:

$$\Delta h_j = h_j^{k+1} - h_j^k = \sum_{i=1}^{n} \frac{\partial h_j}{\partial d_i} \underbrace{\left(d_i^{k+1} - d_i^k\right)}_{\Delta d_i} \qquad j = 1, 2, \ldots m \tag{2.8}$$

Substituting Equation (2.7) for Δd and letting h_j^{k+1} go to zero as expected,

$$-h_j^k = \sum_{i=1}^{n} \frac{\partial h_j}{\partial d_i}\left[\frac{1}{r}\left(T_i - 1\right)d_i^k\right] \qquad j = 1, 2, \ldots m \tag{2.9}$$

gives *m* linear equations with *m* unknown λ values. Any negative Lagrange multipliers, λ, require Equation (2.9) to be reevaluated with the corresponding constraint removed and recalculated to get the remaining λ values. If negative λ values still appear after recalculating, then this process will have to be repeated until all values are positive. After substituting the remaining positive Lagrange multipliers into Equation (2.7), the new design variables can be calculated, and the process is repeated to further improve the design variables.

Getting the partial derivative of the constraint with respect to the design variable, $\partial h_j/\partial d_i$, is rather simple when the constraint is displacement, u_j, as in Equation (2.3). Thus, with the goal to find $\partial u_j/\partial d_i$, begin with the pile stiffness equation,

$$\left[K\right]\left\{u\right\} = \left\{P\right\} \tag{2.10}$$

where:	K	=	Global stiffness matrix (6x6)
	u	=	Displacement vector (6x1)
	P	=	Load vector (6x1)

The global stiffness is calculated by transforming and summing the local stiffness of each individual pile at one global point where we want to know and control the displacements. The local pile stiffness is calculated in the same manner used by the USACE and their rigid-pile analysis program, CPGA (CASE Task Group on Pile Foundations, 1983). Taking the derivative with respect to the design variable, d_i, we get:

$$\frac{\partial[K]}{\partial d_i}\{u\}+[K]\frac{\partial\{u\}}{\partial d_i}=\frac{\partial\{P\}}{\partial d_i}=0 \tag{2.11}$$

and rearranging terms gives:

$$\frac{\partial\{u\}}{\partial d_i}=-[K]^{-1}\frac{\partial[K]}{\partial d_i}\{u\} \tag{2.12}$$

3. Optimization of Weightless Design Variables

As seen in Equation (2.1), the weight of the pile foundation is only governed by the length and area of the piles. Thus, optimizing the size of the piles, and assuming an infinite selection of pile sizes, is rather straightforward because $\partial W_T/\partial d_i = \partial W_T/\partial A_i = \rho L_i$. The weight gradient, located in the denominator of the optimality criteria, Equation (2.6), is non-zero. However, for other design variables, such as batter (when the length is constant) or pile spacing, $\partial W_T/\partial d_i = 0$.

Even though the weight gradient of these topological design variables is zero, they still have an indirect effect on the weight. Unfortunately, this effect is immeasurable. To solve this problem, Hoback and Truman created a new optimality criterion that can model the behavior of topological variables (Hoback et al., 1993). First the optimality criteria for the weightless gradient is rewritten from Equation (2.4) with $\partial W_T/\partial d_i = 0$:

$$\frac{\partial L}{\partial d_i}=\sum_{j=1}^{m}\lambda_j\frac{\partial h_j}{\partial d_i}=0 \qquad i=1,\,2,\,\ldots\,n \tag{3.1}$$

Because the efficiency is one when the design variable is optimal, the weightless optimality criteria from Equation (2.6) is rewritten as:

$$T_i=\frac{-\sum_{j=1}^{m}\lambda_j\dfrac{\partial h_j}{\partial d_i}}{w}+1=0 \qquad i=1,\,2,\,\ldots\,n \tag{3.2}$$

The weightless scaling factor, w, replaces the weight gradient in the optimality criteria, effectively acting as a pseudo-weight gradient. Because the weightless optimality criteria is formulated to approach one at optimum, T_i can still be used in the same recurrence relationship, Equation (2.7).

The difficult step is finding an acceptable value for w. Hoback's method involved two steps. First, he scaled w so that the coefficients of the weightless Lagrange multipliers, λ_j, along the main diagonal of the linear equations were at least as large as the corresponding weighted coefficients. The second step involved increasing w until an estimated weight change converged. Although Hoback found this method successful, his work showed inconsistencies at finding a better, lower-weight design, and a new method has been developed.

After testing random optimization problems for the effects of the weightless scaling factor, w, on the optimization process, a pattern was discovered. First, a single value for w would not work for several reasons. If w is too large, the weightless variable would remain unchanged, for T_i will always be too close to one. The final solution would still optimize the weighted variables, but the final weight would be less than optimal without the optimization of the weightless variables. If w is too small, T_i will take on very large and very small values, throwing the weightless variable to their extremes, never reaching an optimum. This creates a very unstable optimization process that may never converge to a single design.

Figure 3.1 shows the typical results of simultaneously optimizing weighted variables and weightless variables. Weightless scaling factors of 10^{12}, 10^8, 10^6, and 10^3 are each tried from the same initial design point. When w = 10^{12}, the weightless variables were untouched because w was too high, and when w = 10^3, the optimization became unstable, resulting in sudden large increases and decreases in weight.

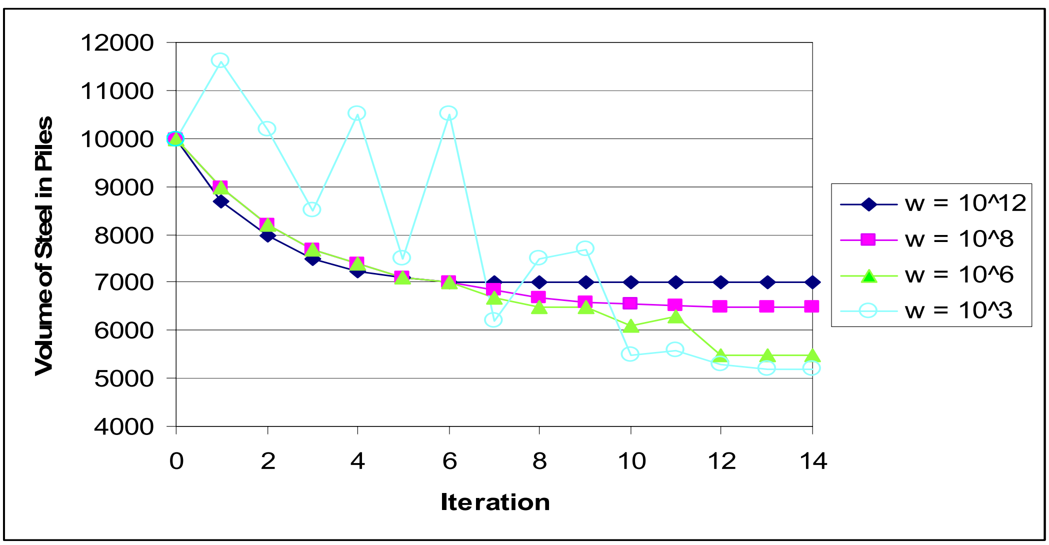

Figure 3.1. A typical optimization, including weightless variables, with different w values tested. Note that a gradually decreasing weightless scaling factor, beginning at 10^{12} and gradually reducing to 10^3, would end with a near-optimal design.

Because several random examples displayed similar results as figure 3.1, a new method for determining w was created. The weightless scaling factor, w, will begin at a very large value (such as 10^{12} in figure 3.1). Optimization will continue at that value of w until the weight converges (at iteration 4 in figure 3.1). For the next iteration, the square root, or another appropriate reduction factor, of w will be taken (thus, reducing w to 10^6). Then, this process will be repeated, continuing to reduce the weightless scaling factor and further decreasing the overall weight of the pile foundation design. Figure 3.1 shows how gradually decreasing w would result in a continuously decreasing weight if w were to decrease from 10^{12} to 10^6 to 10^3. Eventually, the weightless scaling factor will become too small, and the design will become unstable. At this point, the optimization is complete, and the lowest weight design is chosen.

4. Example

The following example illustrates the effortless weight reduction of a rigid-slab pile design, using the optimization methods described. First, the program will be run, optimizing only the pile *sizes*. Because pile size is a weighted variable, the optimization is rather straightforward since it does not require the use of the weightless scaling factor, w. Second, the optimization will include both pile size and the weightless variable, batter (weightless because the length will be held constant). For simplicity, all other variables will not be varied.

4.1 Optimization of a 2-D Pile Foundation, Optimizing Only Pile Size

The problem consists of a rigid slab foundation with 5-piles, arranged into 4 groups, as seen in figure 4.1. Group 1 initially has no batter but is only allowed to batter to the left while groups 2, 3, and 4 are battered to the right, initially at a value of 5. The loading, constraints, and site conditions are listed in figure 4.1.

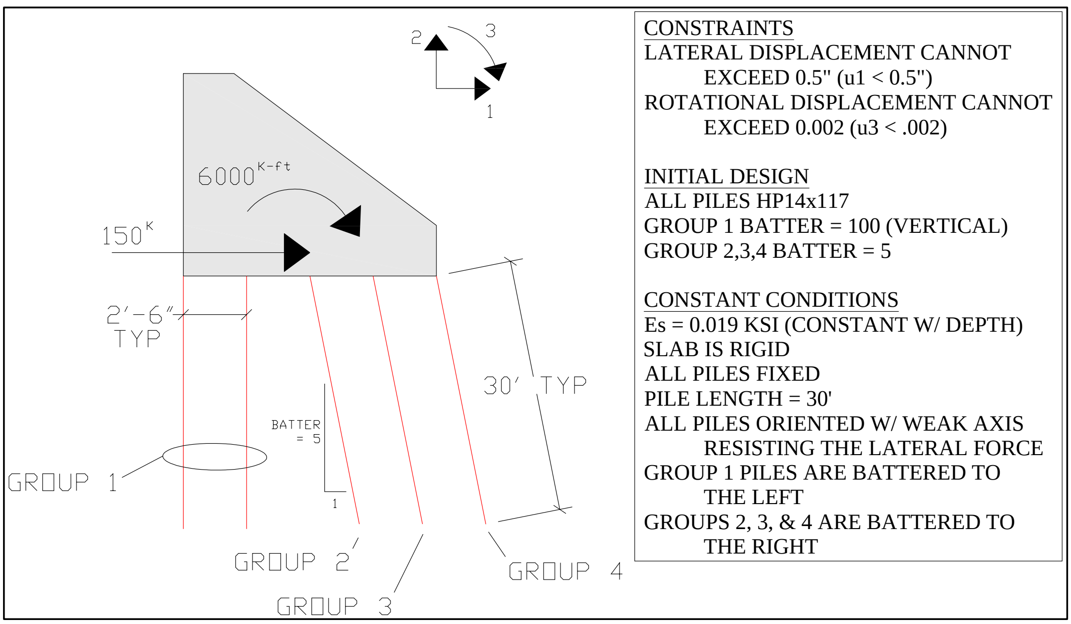

Figure 4.1. Example problem, showing constraints, initial design, and constant site and design conditions.

From analysis of the initial design and the prescribed loading, the displacements are already rather close to the maximum displacements. The lateral displacement is 0.46", and the rotational displacement is 0.0015 rad., both just under the constraints of 0.5" and 0.002 rad., respectively. The weight of the initial volume of steel in the five HP14x117 piles is **61,920 in^3**. With the displacements this close to their maximums, it is difficult to find a significantly lighter design by hand.

For this first optimization, only the pile sizes are optimized, keeping all other variables constant. After only four iterations, the design converged, and the total volume of the piles reduced to **49,570 in^3**. Even though the total weight of the piles decreased, the displacements remained under their constraints. The lateral displacement is now 0.49" and the rotational displacement is 0.0020 rad. The volume decrease is shown in Figure 4.2, and the original and optimized pile foundation design is compared in Table 4.1.

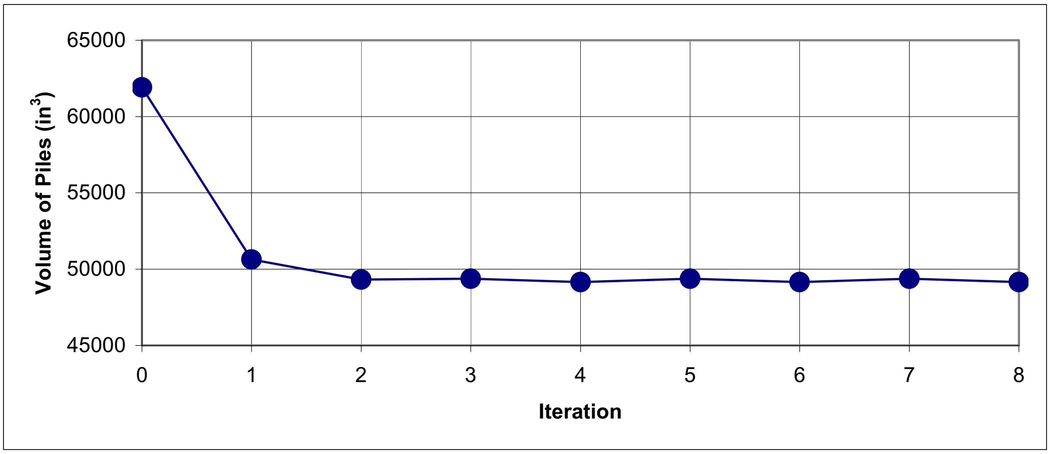

Figure 4.2. Convergence of optimization iterations.

		Original Design	***Optimized Design***
Pile Size:	*Group 1*	HP14x117	HP14x117
	Group 2	HP14x117	HP14x73
	Group 3	HP14x117	HP14x89
	Group 4	HP14x117	HP14x73
	Lateral Displacement	0.46”	0.49”
	Rotational Displacement	0.0015 rad.	0.0020 rad.
	Total Volume of Piles	**61,920 in^3**	**49,570 in^3**

Table 4.1. Original and Optimized Foundation Design

4.2 Introducing the Optimization of Batter

Optimizing the batter involves using the weightless scaling factor when pile length is held constant. The initial design will be the optimized design from example 4.1. From there the optimization is run, simultaneously optimizing pile size and batter. The batter’s weightless scaling factor will begin at 10^{24} and will decrease at iterations that have only a small change in weight. The weight reduction is shown in the figure 4.3, and the corresponding weightless scaling factor, w, in each iteration can be seen in figure 4.4.

The weightless scaling factor begins very high, and is reduced gradually by taking it to the 2/3 power when the weight reduction from the previous iteration is low. Comparing Figures 4.3 and 4.4, w does not take on a weight reducing value until the fourth iteration ($w \approx 10^7$). This value proved to optimize batter while reducing the overall weight of the piles for several iterations. Finally, as the weight began to converge near iteration 11, w was further reduced, but the optimization became unstable at iteration 13. As explained earlier, when w becomes too low, the changes in the weightless variable (batter in this case) become too large, creating an unstable optimization. The final design, reached on iteration 12, changed the batters only slightly but was able to significantly alter the pile sizes to find an overall lighter pile design. The volume of steel was further reduced **from 49,570 in^3 to 46,580 in^3**.

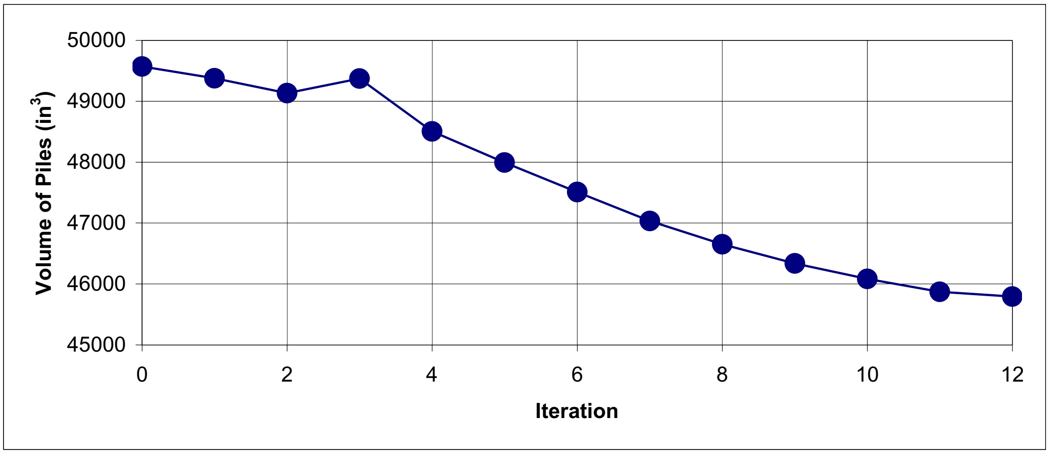

Figure 4.3. Convergence of optimization iterations after introducing the optimization of batter.

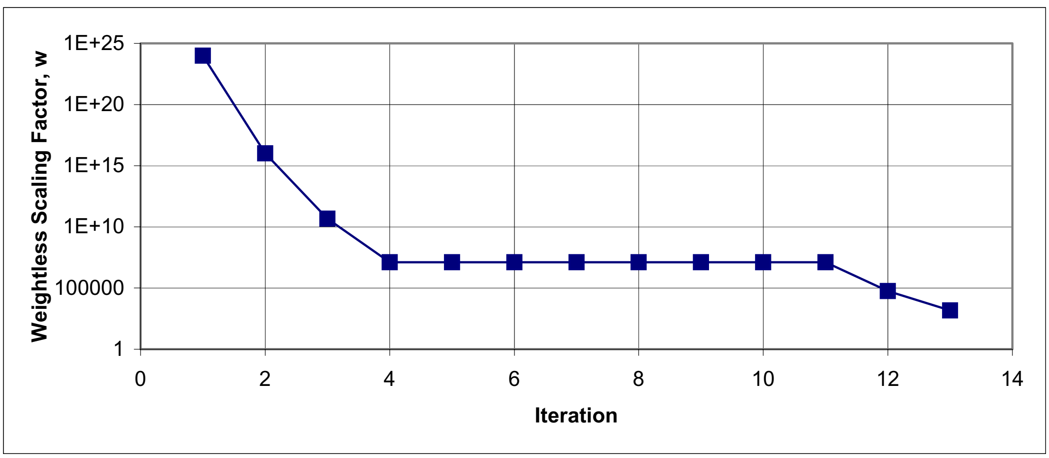

Figure 4.4. Change in the weightless scaling factor, w, with each iteration.

		Original Design	***Optimized Design***
Pile Size:	*Group 1*	HP14x117	HP14x89
	Group 2	HP14x73	HP14x73
	Group 3	HP14x89	HP14x73
	Group 4	HP14x73	HP14x117
Batter:	*Group 1*	100 (Vertical)	86
	Group 2	5	5.3
	Group 3	5	4.8
	Group 4	5	4.4
	Lateral Displacement	0.49”	0.46”
	Rotational Displacement	0.0020 rad.	0.0019 rad.
	Total Volume of Piles	**49,570 in^3**	**46,580 in^3**

Table 4.2. Original and Optimized Foundation Design after simultaneously optimizing pile size and batter

5. Conclusion

Optimality Criteria is an effective method at reducing the weight of steel in piles under rigid, concrete slabs. The method has successfully reduced the weight of piles in many real-life problems, both simple and complicated. Using weightless scaling factors provides a way to optimize zero-weight gradient variables such as batter or spacing. By gradually reducing this factor exponentially when weight reductions become sufficiently small, a near-optimal final design can be reached. However, it is important to note that the final design reached is only a *local* minimum, not a global minimum. Thus, the initial design plays an important role in determining the final pile layout. Because of the ease of use of this optimality criteria program, varying the initial design and rerunning the program with the same loads and constraints can result in several low-weight final designs, allowing the engineer to choose between several near-optimal designs.

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