SUMMARY OF DEVELOPMENT AND USE OF CSA 2004 SHEAR DESIGN PROVISIONS

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Introduction

In the design of concrete structures, the engineer is tasked with selecting structural geometry, member sizes and reinforcement details to ensure that the applied loads to which the structure is subjected can be safely carried to the foundation. For design against axial load and moment there is essentially global unanimity concerning the relationships that should be used to achieve this design. Specifically, the well-known rules of engineering beam theory or "plane sections remain plane" as originally explained by Robert Hooke are used; see Figure 1 (Hooke, 1678). This simple, general and accurate theory allows the engineer to design a member with confidence even when confronted with unusual geometry or new materials.

In contrast to the agreement that exists for flexural design, the mechanisms of shear resistance and provisions based on these mechanisms have not yet reached international consensus. In some jurisdictions purely empirical methods are used while others use more theoretically derived methods of various complexity and understandability. While the use of different methods in different places is not of direct concern, that these different methods produce transverse reinforcement requirements that vary by as much as a factor of six suggests that engineers' understanding of shear is far behind their understanding of flexure.

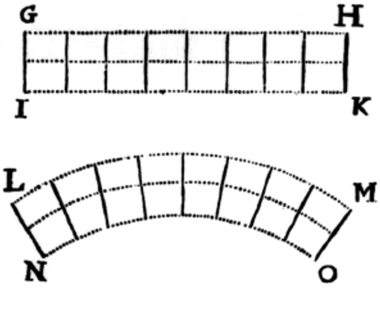

Fig. 1. Robert Hooke's figure demonstrating that plane sections remain plane published in 1678.

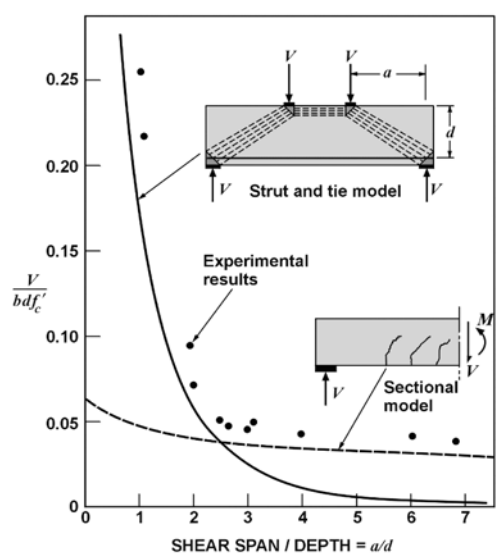

Fig. 2. Different modes of shear failure modelled in Canadian code.

M. Pandey et al. (eds), Advances in Engineering Structures, Mechanics & Construction, 67–80.

As shown in Figure 2, shear forces in beams are modelled in the Canadian code (CSA, 2004) with either a strut-and-tie model, suitable for short shear spans or a sectional model suitable for longer shear spans. As the strut-and-tie provisions of the 2004 concrete design code are largely unchanged from the 1994 standard, they will not be described further in this document. The sectional provisions of the 2004 concrete design code, on the other hand, have changed significantly and this paper will summarize the development and use of these provisions. In the author's opinion, the new shear provisions of the 2004 standard represent a breakthrough in the modelling of shear behaviour and should help foster international discussion towards the generation of a consensus on shear behaviour.

Historical Considerations

Up to the 1980s, the Canadian shear design provisions directly paralleled the US provisions of the American Concrete Institute (ACI) Building Code Requirements (ACI Committee 318, 2005). These provisions, which are still in use in the United States today, were empirically based on test results from a large number of shear experiments, generally of small size and with a large amount of longitudinal reinforcement.

Staring in 1984, the Canadian provisions (CSA, 1984) began to diverge from the ACI rules. Specifically, the Canadian provisions became divided into a "simplified method" and a "general method". The simplified method was essentially the same as the ACI provisions but the general method was a new set of design rules based on the Compression Field Theory (CFT) (Collins, 1978; Collins & Mitchell, 1980). This general method required the explicit evaluation and satisfaction of constitutive and compatibility equations. While the method was significantly more complex than the ACI provisions, it did apply to general loading conditions and would usually produce more economical designs. Unlike the ACI provisions, but like the then equally new CSA strut-and-tie provisions, the engineer needed to explicitly select the angle of diagonal compression in the web of a member. Because of this, different engineers would produce different designs for the same applied loading. While some engineers appreciated the freedom that this allowed, others were uncomfortable in that it became more difficult to check each others work; more than one solution to the same problem could be equally correct. An additional limit to the method was that it could not determine the strength of members without transverse reinforcement such as slabs.

In the 1994 CSA shear design provisions (CSA, 1994), a number of changes were made. For the simplified method provisions, a size effect was added to the shear strength of members without transverse reinforcement. This resulted from the experimental observation that members of a larger overall depth tended to fail at lower shear stresses, which was relevant for thick slabs for example. The 1994 general method of shear design was based on the Modified Compression Field Theory (MCFT) (Vecchio and Collins, 1986) and now provided a single design solution for a given set of applied loads. The use of the MCFT instead of the CFT also extended the general method to the design of members without transverse reinforcement, including the size effect. To use the new general method, however, the engineer needed to consult tables of values of β and θ, the two main variables for the shear strength of members. Partly due to the difficulty of using table-lookups with spreadsheets, the general method was still significantly more difficult to use than the simplified method, particularly for the analysis of existing designs. Despite this, the method has allowed many impressive structures to be built that exceed the limits of the ACI code. As just one example, Figure 3 shows a 176 metre tall tower in Toronto designed by Yolles Partnership which has high demands on the lateral load resisting system due to the narrow nature of the tower (a height to width ratio of 11:1). Were this tower designed according to the ACI shear provisions, it likely could not have been built as economically since significantly larger coupling beams would have been required by that standard. The necessary increase in floor-to-floor height would have lowered the sellable number of condominium units for the same height of building. Similar shear provisions to the CSA general method were also adopted by the Ontario and Canadian Highway Bridge design codes (OHBDC, 1993; CHBDC, 2000), and the US AASHTO LRFD Bridge Design Specifications (AASHTO, 2004).

Despite the advantages of the CSA 1994 general method, there remained difficulties with its practical use. As noted above, the difficulty of using tables of values with spreadsheets made the method difficult to automate for day-to-day design work. In addition, it was difficult for engineers to develop confidence in the values given by the tables of β and θ as it was not clear where these values came from or why they varied the way that they did.

For the generation of the 2004 shear design provisions, a conscious decision was made to make the provisions as general as the 1994 general method, but as easy to apply as the 1994 simplified method. The intention was to make design with the general method non-iterative and to replace the tables of behaviour with simple equations. As an intentional side effect of doing this, it became possible for the simplified method to be a special case of the general method rather than being based on the ACI provisions. The 2004 provisions remain based on the MCFT but are now simple enough to explain and use that they represent a significant advance in research on shear.

Fig. 3. CSA 1994 general method allowed this tower to be more economical.

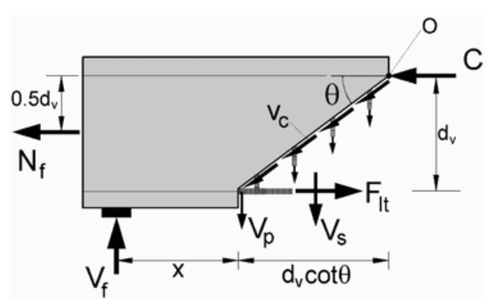

Fig. 4. Basic shear resisting mechanism assumed in 2004 code.

The Strength of Members Subjected to Shear

The 2004 CSA shear provisions are based on the shear resisting mechanism shown in Figure 4. This free body diagram shows the end portion of a beam, cuts through the longitudinal reinforcement and stirrups, follows the diagonal shear crack and then cuts the top flexural compression region. Flexural moments (M_f) and axial tension (N_f) are resisted by the force couple between the flexural compressive force C and the tension in the reinforcement F_{lt}, here calculated at the crack. Shear forces (V_f) are resisted by three mechanisms. Shear stresses (v_c) on the crack surface itself are resisted by aggregate interlock, which is the primary method of strength resistance observed for members without stirrups (Fenwick, 1968; Taylor, 1970; Kani, 1979). Stirrups that cross the diagonal crack provide a steel contribution, V_s, and any vertical component to prestressing forces, V_p, will also resist shear forces. Even from such a simple diagram as that in Figure 4, useful deductions may still be made. In this case, note that the shear on the crack, v_c, has a vertical component that resists shear, but also has a horizontal component that must be balanced by additional strain in the longitudinal reinforcement. This is one of the causes of shear-moment interaction in that shear force cause stress in the flexural reinforcement.

If the contribution due to V_p is ignored, the following equations are the basic shear strength terms used in the CSA code for both the general and simplified methods:

$$\begin{aligned} V_r &= V_c + V_s \leq 0.25\phi_c f_c' b_w d_v, \\ V_r &= \phi_c \lambda \beta \sqrt{f_c'} \cdot b_w d_v + \phi_s \frac{A_v}{s} f_y d_v \cot\theta, \end{aligned} \tag{1}$$

where V_r is the factored shear strength, $b_w d_v$ is the shear area taken as the web width multiplied by the shear depth shown in Figure 4 and taken simply as $0.9d$. The term λ relates to the use of lightweight concrete. The equation is written in metric notation (MPa, mm, Newtons), with $\phi_c = 0.65$ and $\phi_s = 0.85$ as the material reduction factors for concrete and steel. The ability of the member to resist aggregate interlock stresses is represented by the variable β, and the angle of the crack, θ, indicates how many stirrup legs will cross the crack, and is necessary to determine V_s.

Development of shear provisions can therefore be boiled down to finding a method to quantify the parameters β and θ. The 1994 provisions used tables to estimate these values, but the 2004 code uses simple equations to do this. As these two variables are conceptually independent, they will be treated in separate sections.

Modelling Aggregate Interlock to Determine β

Aggregate interlock is assumed to carry all shear forces for members without stirrups, and a potentially significant proportion of the total shear force for members with stirrups. The ability of a crack to resist these stresses is predicted by the MCFT to decrease with decreasing concrete strength; decreasing maximum specified coarse aggregate size, representing crack roughness; and increasing absolute crack width. The suggestion that wider cracks are less able to resist sliding shear stresses is hopefully intuitive. To estimate a crack width, w, the following relationship can be used:

$$w = \varepsilon \cdot s, \tag{2}$$

where s is the crack spacing and ε is the average strain perpendicular to the crack. As wider cracks will be associated with lower aggregate interlock strength, anything that increases the value of w can be expected to result in decreased shear capacity. Thus if the crack spacing increases due to, say, the construction of a larger member, it can be expected that the shear strength will decrease. This is called the size effect and is an important part of the behavior of members without stirrups. Secondly, if the average strain in the concrete increases due to, say, applied tension then the shear strength is also predicted to drop. This is called the strain effect in shear and is less well known than the size effect, though it is of comparable importance.

Fig. 5. Crack pattern in member without stirrups. s_z is spacing at mid-depth.

Overall, then, a size effect and a strain effect are predicted to be important aspects of the concrete component of shear strength of members. Experiments show that these are indeed the two most important aspects influencing shear stress at failure and should be included in any state of the art shear provisions.

The full derivation of the proposed method is given elsewhere (Bentz, 2006; Bentz et al., 2006), but the important concepts are explained here. Determining shear strength will depend on the terms s and ε in Equation (2). The value of the crack spacing will depend largely on the size of the member. The crack spacing in the longitudinal direction, s_z, is taken as $s_z = jd = 0.9d$ if no stirrups are provided. If the member is constructed with an aggregate size, a_g, different from 20 mm, the aggregate interlock capacity will be affected, and this is accounted for by using an effective crack spacing, s_{ze}, given by:

$$s_{ze} = \frac{35d_v}{15 + a_g} \geq 0.85d_v. \tag{3}$$

For high strength concrete, the aggregate fractures and does not contribute to crack roughness. To account for this, take $a_g = 0$ for $f'_c > 70$ MPa. To avoid a discontinuity, linearly interpolate a_g from the specified value at $f'_c = 60$ MPa down to zero at $f'_c = 70$ MPa. For members with stirrups, the stirrups will control the crack spacing and the term s_{ze} may be simply taken as 300 mm.

Figure 5 shows the test of a 300 mm wide strip taken from a 1500 mm (5 foot) thick slab. It can be seen that the spacing of the cracks at the mid-depth of the member is much greater than the spacing of the cracks at the flexural tension face. The parameter s_z refers to the longitudinal spacing of the cracks at mid-depth of the member where the shear stress is generally critical.

The value of ε is slightly more complex to determine compared to the effective crack spacing as it depends on the currently applied load level, amount of prestress, material properties of the flexural reinforcement, etc. Consider that a given amount of applied load will be associated with a given strain in the longitudinal reinforcement based on a free body diagram such as that in Figure 4. This value of strain will be present in the reinforcement, whereas Equation (2) requires the strain at 90° to the diagonal crack. As such, the equations of the MCFT are employed to derive a relationship between the width of a diagonal crack, for a given crack spacing, at shear failure given that the longitudinal strain is a known quantity. This involves the simultaneous solution of 15 nonlinear equations and is described elsewhere (Bentz, 2006).

Figure 6 shows the results of the calculation of diagonal crack width for various longitudinal strains by the MCFT. As the longitudinal strain increases, the critical crack width also increases. The analysis results are nonlinear, but a simplified equation is also shown that conservatively approximates the nonlinear behaviour. This equation is intentionally selected to provide a good match to the MCFT for mid-depth strains that are expected for a member reinforced with 400 MPa flexural

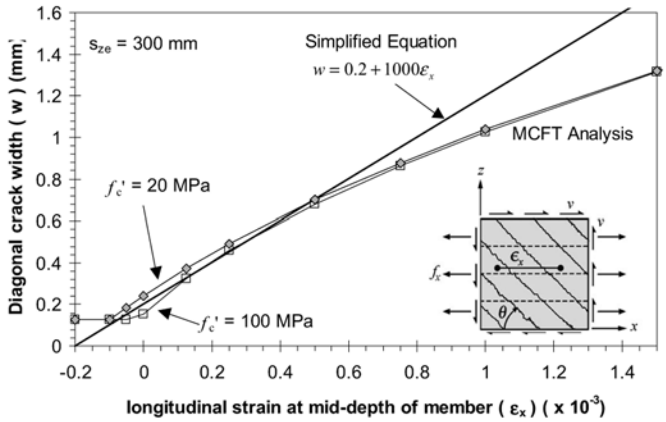

Fig. 6. Diagonal crack widths at shear failure for members without stirrups.

reinforcement. If a member is to be subjected to larger longitudinal strains, say with FRP reinforcement, these provisions should be conservative as the crack width will be overestimated by the simplified equation.

When the simplified equation in the figure is substituted into the MCFT equation for aggregate interlock, and a size effect terms is also added (Bentz, 2006), the following equation is obtained for the value of β which defines the concrete contribution based on the MCFT:

$$\beta = \frac{0.40}{(1 + 1500\varepsilon_x)} \cdot \frac{1300}{(1000 + s_{ze})}. \tag{4}$$

The first term in this equation accounts for the strain effect whereby members with smaller longitudinal strains are stronger in shear. The second term accounts for the size effect which cancels out for members with stirrups ($s_{ze} = 300$ mm). The estimation of the longitudinal strain in the member, ε_x, will be discussed below.

Modelling of Diagonal Crushing of Concrete to Determine θ

Members with at least a minimum quantity of well-anchored stirrups are predicted not to fail by sliding on the crack, but by yielding of the stirrups and eventual crushing of the concrete in the web. In the 1984 shear provisions which allowed the engineer to select the value of θ, equations were provided to ensure that the concrete did not crush before reaching the design shear strength, which provided a lower limit on θ, and additional rules were provided to ensure that the stirrups would yield at design shear failure, which provided an upper limit on θ. At low applied shear forces, the range over which the value of θ could be selected was large, but as the shear stress increased, the range became more restrictive. For the 2004 shear provisions, it was decided to maintain the maximum shear limits that were present in the 1994 standard, so this high shear loading would control the selection of θ for all applied loading levels.

Figure 7 shows the limits on allowable angle of principal compression, θ, based on the MCFT for members heavily loaded in shear and for different strains in the member at mid-depth (ε_x). As can be seen, the range of allowable angles to select from at this high shear loading is rather narrow. Members designed based on angles in the upper shaded region would be expected to fail in shear before yielding of the transverse steel making the use of Equation (1) unconservative. Members designed based on angles from the lower shaded region would also be unconservative as here the

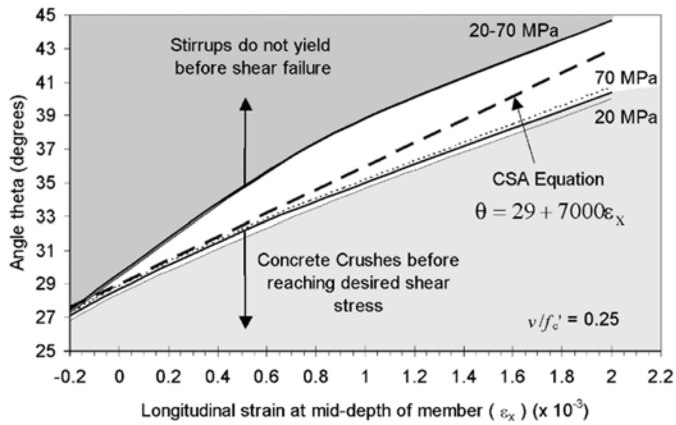

Fig. 7. Selection of equation for θ.

member is predicted to fail by crushing of the concrete in diagonal compression before achieving the design shear strength. Only within the unshaded region would a member as heavily loaded as this be predicted to be able to resist the applied shear force. Shown in Figure 7 is a simple equation that lies within the allowable range as:

$$\theta = 29^\circ + 7000\varepsilon_x. \tag{5}$$

Longitudinal Strain in Member

Embedded in Equations (4) and (5) is the strain term ε_x which represents the average longitudinal strain in the member at the mid-depth. Figure 5 shows that the crack spacing, s_z, is calculated in the longitudinal direction of the member at mid-depth as well as it is appropriate to calculate behaviour based on coexisting strains and crack spacings. For reinforced concrete members subjected only to moment and shear, the equation to determine ε_x can be generated from a simple free body diagram such as that shown in Figure 4 and can be taken as:

$$\varepsilon_x = \frac{M_f/d_v + V_f}{2E_s A_s}. \tag{6}$$

In the use of this equation, the values of moment and shear are always taken as positive. The numerator of the equation estimates the force that must be resisted in the flexural reinforcement of a member including the shear-moment interaction mentioned above. The denominator converts this force to a strain in the tension reinforcement by dividing by the quantity and stiffness of flexural reinforcement. As a final simplification, the strain at mid-depth of the member, ε_x, is simply taken as one half of the value in the flexural tension reinforcement. For prestressed concrete or members subjected to axial loads, additional terms are added to the numerator of Equation (6) which are not shown here.

Checking for Flexure-Shear Failures

As a final check that a given structural member is safe, it is necessary to ensure that the flexural reinforcement does not yield under the combined effect of applied loading including the interaction of shear and moment. The member will have been designed against flexural forces, but the addition of the shear term in the numerator of Equation (6) means that it may still be dominated by yield of the longitudinal reinforcement. To ensure that this does not occur, the following equation is derived from Figure 4 by taking moments about point O and solving for the necessary force in the reinforcement:

$$F_{lt} = \frac{M_f}{0.9d} + 0.5N_f + (V_f - 0.5V_s - V_p)\cot\theta. \tag{7}$$

The reinforcement that is provided at the location where this equation is being checked must be sufficiently developed to resist the force calculated by Equation (7). This force need not be taken as greater than the reinforcement force calculated under maximum moment alone at the maximum moment location of the same member. The reason for this limit is that near applied loads, where the moment is maximum, the shear is carried by a “compression fan”, similar to a strut-and-tie model rather than by the sectional behaviour assumed in Figure 4.

If Equation (7) is not satisfied, then the applied loads (M_f, N_f, V_f), can be proportionately reduced until it is satisfied, and these new values are the final estimate of the strength of the member. A member which is controlled by this equation is predicted to fail in a flexure-shear mode rather than simply in flexure or shear alone.

Solving for the Strength of Members by the CSA General Method

Overall, then, solving for the shear strength of a member by the 2004 CSA shear equations require the solution of Equation (1) using the value of β from Equation (4) and θ from Equation (5). Equation (7) must also be checked at the end of the process to ensure that a flexure-shear case does not control. These calculations are all performed at the critical shear location, which is generally taken as d_v back from the edge of the column, pedestal or wall that applies the load. For uniformly loaded members, multiple sections along the span should be checked to determine where the critical location is. To use either Equation (4) or Equation (5), however, the value of ε_x must be determined. A number of different cases exist for performing the estimate of ε_x which differ depending on if they represent a design or analysis case:

Design by General Method

For the design of members to resist a known combination of M_f, and V_f, it is appropriate to substitute these values directly into Equation (6) and explicitly determine the value of ε_x. This value is then substituted into Equation (5) to determine θ. If it is not clear whether stirrups are required, the shear strength may then be determined without stirrups first by substitution of ε_x into Equation (4) with the term $s_{ze} = d_v$ to see whether the calculated value of $V_r = V_c$ is greater than or equal to the factored load V_f. If the calculated V_c is insufficient, then the engineer may conservatively use this with Equation (1) and solve for the quantity of stirrups (A_v/s) which is necessary so that $V_r \geq V_f$.

For more accurate results when stirrups are required, the engineer should recalculate V_c as the presence of at least minimum stirrups mitigates the size effect. Equation (4) should therefore be reevaluated with $s_{ze} = 300$ mm to determine the value of V_c for members with stirrups. Then Equation (1) is again used to determine the required quantity of stirrups (A_v/s) so that $V_r = V_f$.

Equation (7) will generally only control near the ends of members where it is necessary to check the anchorage of flexural reinforcement. If the reinforcement detailing is sufficient this check will usually not control here or elsewhere in the member.

Overall, the process above is non-iterative and easy to perform either by hand or with a spreadsheet. The steps required are identical to those used with the 1994 general method, but now there is no iteration required for design and the use of equations for β and θ makes the design much easier to perform, particularly with a spreadsheet.

Design by Simplified Method

As was noted above, the general method and the simplified method in the 2004 CSA code are in fact now the same thing. The key to the simplified method is the appreciation that the value of the longitudinal strain at the mid-depth of the beam, ε_x, is in fact a "constrained" parameter for shear failures. Consider that for 400 MPa steel the flexural reinforcement will yield and cause a flexural failure rather than a shear failure when the strain in this steel equals $f_y/E_s = 400/200{,}000 = 0.002$. With the strain at mid-depth taken as one half of the strain in the bottom chord of the member for simplicity, the ε_x strain associated with flexural yield will equal 0.001, or 1×10^{-3} for this reinforcement. Thus for 400 MPa steel, if the strain ε_x exceeds 1×10^{-3}, the member will not fail in shear, but will fail in flexure and none of the equations in this paper will govern.

The simplified method is derived by assuming that the strain term, ε_x, is equal to 0.85×10^{-3}, or slightly less than the value associated with the yield of 400 MPa reinforcement. Substituting this value into Equation (5) produces a constant angle $\theta = 35°$. Substituting ε_x into Equation (4) will produce a constant value of β for members with stirrups, and a size effect equation for members without stirrups. Thus the shear strength of members with stirrups can be determined by the simplified method as:

$$V_r = 0.18 \cdot \phi_c \lambda \sqrt{f_c'} \cdot b_w d_v + 1.43 \phi_s \frac{A_v}{s} f_y d_v, \tag{8}$$

which is no more complicated than the traditional ACI shear strength provisions. The major difference from the ACI provisions is that the V_c term is slightly lower and the beneficial effect of adding stirrups is estimated to be higher in Canada. As the Canadian provisions allow the design of members without stirrups up to the full value of V_c whereas the ACI code requires stirrups for members loaded in excess of $V_c/2$, these differences seem acceptable.

For members without stirrups, the same assumptions about the value of ε_x are made and the final version of Equation (1) with all simplifications reduces to:

$$V_r = \frac{230}{1000 + d_v} \cdot \phi_c \lambda \sqrt{f_c'} \cdot b_w d_v. \tag{9}$$

This equation indicates that when stirrups are not provided that a size effect remains which must be taken into consideration at design time. At the same time, similar to the 1994 code, this equation is not iterative as all the terms in this equation will be known once the flexural design has been completed.

A significant benefit of basing the simplified method on the general method is that it becomes clear what to do as the limits of the simplified method are exceeded. As an example, Equations (8) and (9) are only appropriate when used with 400 MPa reinforcement. For designs based on, for example, 500 MPa reinforcement, the code specifies that Equations (8) and (9) may not be used. In this situation, however, it is clear how to generate new simplified equations for use with this higher strength steel. For this case, the strain at mid-depth associated with flexural yield will be $f_y/2E_s = 1.25 \times 10^{-3}$. Taking the same factor of 85% of this yield strain produces a value of $\varepsilon_x = 1.06 \times 10^{-3}$ for use with 500 MPa reinforcement. This too can be substituted into the previous equations to produce safe equations for use with this steel. The value of θ, for example, would be taken as 36.4°, and the numerator in Equation (9) would be replaced by the value of 200 rather than 230. Design by this newly derived simplified method could then proceed. Note that the use of this technique implies that members with high strength reinforcement will be weaker in shear than members with normal strength reinforcement. This is not necessarily the case, however. What is predicted is that when higher strength reinforcement is used, that the shear strength will be lower at flexural failure and so this lower, safe value must be used for all designs if a simplified type equation is desired.

Analysis by the General Method

The concepts of design and analysis can be thought of as opposite circumstances. In the design case the geometry of a member is selected to make it strong enough which means the engineer solves for many of the terms on the right side of Equation (1). For analysis, the engineer solves for the one term on the left side of Equation (1) instead. The general method equations have been intentionally arranged so that they produce design results, as above, with no iteration. That is, the equations have been selected so that they make design easy, but analysis slightly more difficult. This was a philosophical decision as the design code is intended for the construction of new structures.

To solve for the strength of a member by the CSA general method, two options are available. Firstly, for members without stirrups Equations (6), (4), and (1) may be solved simultaneously to produce a closed form solution for the shear strength of a member without stirrups. This derivation is reasonably easy and requires the solution of a quadratic equation.

Alternatively, an iterative process, suitable for spreadsheet calculations can be easily implemented which works for all types of applied loading and all types of members. The process can be explained as follows. The spreadsheet will allow the determination of the shear strength of one beam/slab/column per row of the spreadsheet. The left side of the spreadsheet would include the various material, geometric, and loading properties, again, one beam per row. On the right side of the spreadsheet would be the iterative determination of the shear strength.

Considering the small sample spreadsheet below, the iteration begins with an initial estimate of the strain at the mid-depth of the member, say $\varepsilon_{x-1} = 1 \times 10^{-3}$. For convenience, it is helpful to determine the values of strains in the units of parts per thousand to make mistakes easier to visually identify. Directly to the right of this value, Equations (1), (4), and (5) would be evaluated within a single spreadsheet cell to determine the shear strength for that assumed value of ε_x, listed as V_{r-1} below. This is listed as Equation type B and as the value ε_x is known, it is a direct substitution of values from the left side of the spreadsheet. To the right of this value, Equation (6) would then be evaluated to determine the new estimate of ε_x from the given applied loading, shown as ε_{x-2}. For the sake of numerical stability, it is appropriate to take the new estimate of ε_x as the average of the previous estimate and the newly calculated value, or $\varepsilon_{x-2} = (\varepsilon_{x-1} + \text{Equation (6)})/2$. To avoid any manual iteration actions by the user, the cells which contains the estimates of the shear strength and ε_x terms (Equations B, C) can be copied perhaps 10 times to the right to provide 10 iterations which is usually more than sufficient to converge. The final spreadsheet would look something like this:

b_w	d	rho	etc.	ε_{x-1}	V_{r-1}	ε_{x-2}	V_{r-2}	ε_{x-3}	V_{r-3}
300	925	0.83	etc.	1	200	1.2	183	1.24	177
Equation type				A	B	C	B	C	B

where Equation A is the constant initial guess of 1.0 part per thousand; Equation B is the direct substitution into Equations (1), (4), and (5); and Equation C is Equation (6) in this paper. The final value of the shear strength, perhaps V_{r-10}, can be copied back near the left side of the spreadsheet for easy access. Using this technique, the spreadsheet is iterative but is fully "live" in that no macros or special routines need to be called. When a value is changed on the constitutive properties on the left of the spreadsheet, the iterative equations will automatically update the predicted shear strength. That the spreadsheet requires no macros is curiously powerful as well in that nothing is hidden from the engineer with this spreadsheet: no "black-box" calculations or interpolation routes are present. From experience, engineers quickly become much more confident with the new general method partly due to this transparency.

The method above is specified in such a way that the equations (A, B, C, etc.) can be copied down over multiple rows of the spreadsheet to determine the strength of multiple members in a building, multiple experimental results, or to determine sensitivity plots by systematically varying the various beam properties on the left side of the spreadsheet.

Discussion is provided below concerning the predictions of the general method compared to experimental test results.

Analysis by the Simplified Method

An alternative to estimating beam strengths with the general method is to use the simplified method. This alternative is not recommended, however, as it will ignore the important consequences of the strain effect. Simple strength predictions can be made with the simplified method and these will generally be safe, but as the strain effect is ignored and only the size effect is considered, poor results should be expected, particularly in comparison to statistical databases.

Experimental Support for the CSA Shear Provisions

Extensive comparisons to experimental results have been made for the CSA shear design provisions and only a very small subset of these will be shown here. For members without transverse reinforcement, two main variables are predicted to control behaviour: the size effect and the strain effect. To demonstrate the quality of the predictions on a graph with only a single independent variable, Equation (4) will be used with β as the experimentally measured strength and both sides of the equation divided by either the strain effect term or the size effect term. Thus a 2D plot can be made showing the influence of one variable at a time. When results are normalized by the strain effect, the strain used is that predicted by the CSA provisions rather than that calculated from the experimental results.

Figure 8 shows 411 members of various depths and aggregate sizes without stirrups to demonstrate the size effect and the quality of Equation (4) at predicting this effect. The vertical axis shows the value of β in psi units which are 12 times that given by Equation (4). As seen, the CSA equations do an excellent job and generally provide a conservative estimate of shear strength across the range of depths. Recall that these provisions were not based on curve fits to experimental beam test results, but were based on the MCFT which itself resulted from more fundamental shear and material tests. The line on the plot is therefore a true prediction and not a "post-diction" or curve fit result. In addition, the experiments upon which the MCFT is based had values of the s_{ze} parameter that were generally in the range of 50 to 100 mm. The plot therefore shows a significant extrapolation from the data originally used to calibrate the MCFT. That the prediction is as good as the figure shows confirms both that the MCFT is a good model for the behaviour of cracked reinforced concrete and that the assumptions used in generating Equation (4) are appropriate.

Figure 9 shows the same data but normalized by the size term to highlight the strain effect. Again, to calculate this, the experimentally observed value of β was divided by the size effect term so that the predicted behaviour would only vary by one independent variable and a 2D plot could be generated. As with Figure 8, the overall behaviour of the strain effect is seen to be well modelled by Equation (4) for members without stirrups. As expected from the derivation in Figure 6, as the strain is increased beyond a value of about 1.0×10^{-3}, the method begins to become more conservative as the crack width at failure is overestimated. The strain effect explains why FRP reinforced members have lower shear strengths than members reinforced with steel. Such members generally have a

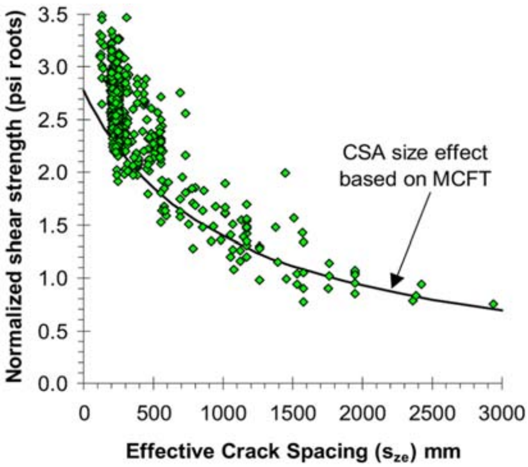

Fig. 8. The size effect in shear.

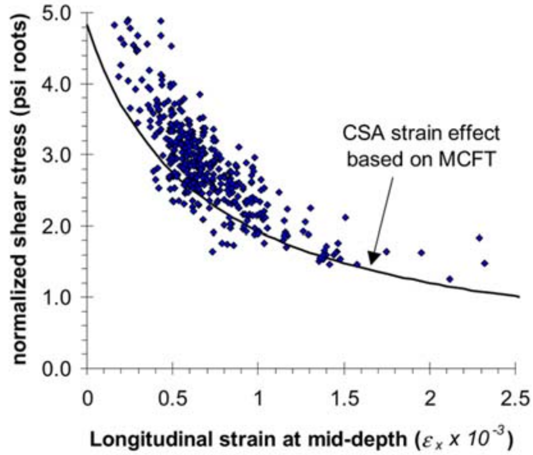

Fig. 9. The strain effect in shear.

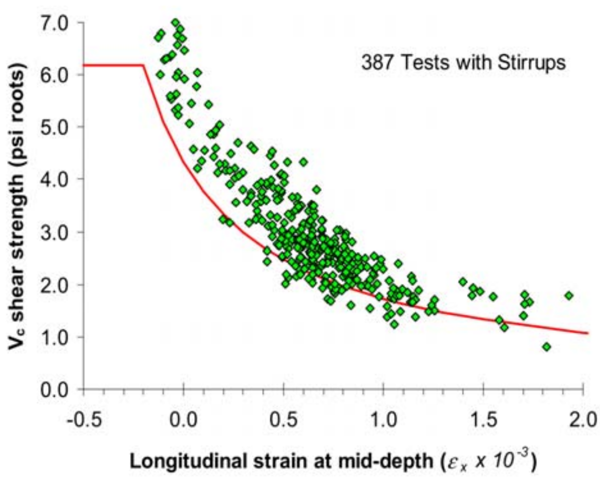

Fig. 10. The strain effect for members with stirrups .

lower stiffness of flexural reinforcement and are therefore subjected to higher strains at shear failure. Due to the strain effect, these members will therefore be weaker in shear. Members reinforced with cast in place FRP are modelled conservatively by the general method shear equations of the 2004 A23.3 concrete code.

As a final comparison, Figure 10 shows a plot of the strain effect for members with transverse reinforcement. This plot includes members of various depths, with and without axial loads and with or without prestressing. The effect of axial compression or prestress is to lower the value of the strain term ε_x and in this figure it can be seen that shear strength is modelled well even for negative values of ε_x. It may not be intuitively obvious that the strain effect should work just as well for members with stirrups, which can undergo redistribution of shear stresses as it does for the members in Figure 9. As seen in Figure 10, however, the effect applies equally well to all members when a realistic estimate of V_s is made as with Equation (5). This figure also shows why the simplified method (which assumes $\varepsilon_x = 0.85 \times 10^{-3}$) will produce poor statistical fits to databases: the systematic effect of strain will be ignored when using the simplified method to predict member strengths.

Conclusions

This paper has provided a brief description of the derivation of the CSA shear design provisions of the 2004 CSA Design of Concrete Structures code. The code continues to employ a simplified and a general method, but now the general method is based on simple equations derived from the MCFT rather than tables derived from the MCFT. Because of this, the simplified method is now a clear derivation from the general method. The use of equations means that the general method is now much easier to apply, particularly with spreadsheets. Design with the general method is now a non-iterative process. Analysis with the general method requires iteration for one variable, and a process for implementing this into a spreadsheet with no programming or macros is provided. The new equations are shown to work well for experimental predictions.

In the generation of the 2004 general method, it was expected that the simplifications necessary to make practical equations would come at the cost of a significant reduction in accuracy or generality. It has been a pleasant surprise to find that the resulting simplified MCFT equations, while much easier to apply, have essentially the same ability to accurately predict shear strength for a very wide range of different parameters. It is believed that the new equations presented in this paper represent a breakthrough in the understanding and modelling of shear. It is hoped that these equations can encourage the necessary discussion to move towards international consensus on shear behaviour just as it currently exists for flexural behaviour.

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