DAMAGE DETECTION USING STATIC RESPONSE DATA AND OPTIMALITY CRITERION

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Introduction

Damage of a structure is caused by many factors, some of which include earthquakes, wind, snow, increased live loads and simply the age of a structure in the form of fatigue or deterioration of its structural elements. Early detection of damage is vital in order to prevent the continuous degradation of the structure, ultimately leading to a catastrophic failure of the system. Traditional methods of detecting this damage are invasive and often require partial destruction of non-structural elements, such as wall panels and finish work. This detection process is executed blindly with little a priori knowledge of the possible location or extent of damage. Furthermore, damage often is not detected until it becomes so extensive that it is visible by building occupants. Structural health monitoring is employed to avoid unnecessary demolition through accurate prediction of damage location.

Background

Even though the area of structural health monitoring has been studied extensively in recent years, the majority of research and implementation has been based on damage detection using dynamic properties of the system. Using changes in vibration characteristics of a structure, damage may be detected and located (Salawu 1997). These vibrational characteristics are dependent on several system parameters such as; mass, stiffness, and damping, all of which require time and effort to obtain. Dynamic structural health monitoring has yielded significant results and therefore extensive research is justified, however requiring multiple system parameters creates inherent challenges in dynamic analysis. Because of the difficulties associated with dynamic analysis, a process using static properties merits exploration. Several methods have been developed using static responses to detect damage. Some methods combine both static and dynamic responses in order to locate damage. One such method predicts the location of damage by attempting to correlate expected and actual damage signs using first-order approximations for changes in static displacements and natural frequencies. Next a separate routine is used to determine the extent of damage at the predicted location (Wang et al.,2001). Although the results of such a procedure have been shown to be accurate for a planar truss and fixed-fixed beam, the effectiveness of the procedure for more complicated structures is not well known. Still other methods have been explored using solely static data. Such methods have investigated the correlation between measured displacements at various degrees of freedom and applied forces at other degrees of freedom in order to assemble the stiffness matrix of the system (Sanayei et al. 1991). Approximation techniques can then be used to obtain unknown displacement values. Such methods have accurately located and quantified damage for 2-D truss and beam element frames. A similar technique has been developed using unconstrained nonlinear optimization (Johnson et al. 2004). Displacements are incorporated into an error function which is dependent on only the cross sectional properties of each structural member. This method successfully detected damage in continuous beams and multi-bay, multi-story frames. Again, the effectiveness of this method is not known for other structural systems. The current theory will also present a method of detecting and quantifying damage using only static measurements in order to assemble the stiffness matrix of the structure. Once the damaged stiffness matrix is known, comparison with the healthy stiffness matrix can locate and quantify damage using the cross sectional properties of each structural element.

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Theory

Damage Detection

In order for a damage detection procedure to be practical to implement it must use easy to obtain information. Static response data, i.e. displacements resulting from static loads, can be obtained using electronic distance meters and can even be automated. Therefore it seems natural that static response data can serve well as a basis for a damage detection procedure. The measured displacements are a function of the loads applied to the structure and also the stiffness matrix of the structure. Damage must be quantified using the variables obtained from static response data.

What is damage?

Structural damage is usually associated with the inability of a structure to support loads. As a result, large displacements occur that normally would have been minimal. It is natural to associate the increased displacements of a structure with a decrease in overall stiffness. Therefore damage is defined as a reduction in stiffness of a structure. For this reason, the change in structural properties that comprise the stiffness matrix must be used as a measure for the location and severity of damage.

Structural Elements

The problem presented in this paper utilizes bending elements to construct the two dimensional frame structures. The stiffness matrix of a bending element is of the form:

$$K = \begin{bmatrix} \frac{12EI}{L^3} & \frac{6EI}{L^2} & \frac{-12EI}{L^3} & \frac{6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{4EI}{L} & \frac{-6EI}{L^2} & \frac{2EI}{L} \\ \frac{-12EI}{L^3} & \frac{-6EI}{L^2} & \frac{12EI}{L^3} & \frac{-6EI}{L^2} \\ \frac{6EI}{L^2} & \frac{2EI}{L} & \frac{-6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}. \qquad \textbf{Eq(1)}$$

By utilizing bending elements, the stiffness matrix is dependent on only the length, modulus of elasticity, and moment of inertia of each element. In reality the length of each element can be determined either from structural drawings or direct measurement of each member and the modulus of elasticity can be taken as the industry standard. Therefore, these values can be considered constant for both the healthy and damaged element. On the other hand, the moment of inertia of each element can change between the healthy and damaged member. By assembling the global stiffness matrix of the damaged structure and extracting the moment of inertia values, comparison with the healthy stiffness matrix values allows damage to be located and quantified.

Frame Assembly

A typical one-bay, one-story frame comprised of 3 bending elements is shown in Figure1(a). As shown, the frame has 3 degrees of freedom (DOF), two rotations and one lateral displacement. The damage detection procedure will require loads be applied at a subset of DOF and displacement measurements be taken at another subset of DOF. Although mathematically feasible, practical applications prevent concentrated moments be applied to a structure and additionally rotational displacements can be difficult to measure. Therefore, the frame shown in Figure1(a) is not a good model to use in actual damage detection routines. As a solution, rather than using a single bending element to connect each column span, two elements are used. This modification creates two additional degrees of freedom at the joint between each element, as shown in Figure1(b). As can be seen, now a vertical degree of freedom exists where loads can be applied and displacements can be measured. To reiterate, the additional DOF are not required for the mathematical formulation, but it does create a damage detection procedure that is more suitable to conversion to actual practice as it is less difficult to obtain vertical displacements than it is to obtain rotational displacements.

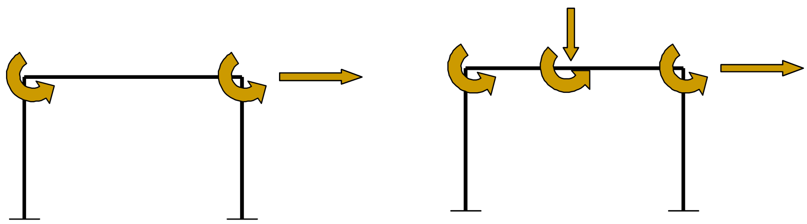

Figure 1(a,b). One-bay, One-story Frame Comprised of Bending Elements

Why use optimization?

As stated, the structures being considered in this paper are formed using bending elements, whose stiffness is a function of the moment of inertia of each structural member. Resulting is an engineering design problem with two main variables; a set of design variables (moment of inertia values) and a set of analysis values (measured displacements). Unfortunately, without an entire set of measured displacements at all DOF, there is not enough information to determine the design variables directly from the analysis values. As such, an optimization algorithm must be utilized to determine all unknown design variables. This can be accomplished by using constrained nonlinear optimization.

Fundamental to the optimization process is the force displacement relationship,

$$P = Ku\ , \qquad \textbf{Eq(2)}$$

where P is the static load vector applied to the structure, K is the global stiffness matrix, and u is the displacement vector of the structure. With knowledge of the healthy stiffness matrix of a structure, a set of displacements for all DOF can be calculated for any set of applied static loads. Next, by applying the same static loads to the present, damaged structure and measuring strategic displacements, insight can be gained as to which structural elements have reduced in stiffness. A general explanation of the optimization routine to gain this insight will be explained next.

Optimization

Because the moment of inertia of a damaged member can take on any positive real value, the structure contains continuous design variables and, in turn, can be formulated to have continuous constraint and objective functions. This engineering problem lends itself well to gradient-based optimization. Several gradient based optimization procedures have been developed over the years, but most follow a similar form. The typical constrained optimization problem has the form:

$$\begin{aligned} &Minimize \rightarrow \Phi = f\left(x_1, x_2, x_3, ..., x_n\right) \\ &s.t. \rightarrow g_i\left(x_1, x_2, x_3, ..., x_n\right) = 0 \end{aligned} \qquad \textbf{Eq(3a-b)}$$

The classical approach to solving constrained optimization is the method of Lagrange multipliers. This approach converts the constrained optimization problem into an unconstrained problem (Kirsch 1993). The objective function and constraint equations can be used to create another scalar valued function, known as the Lagrangian, which takes the form:

$$L = \Phi + \sum_{j=1}^{m} \lambda_j g_j \; . \qquad \textbf{Eq(4)}$$

where,

λ_j = lagrange multiplier for the j[th] constraint

By giving a set of necessary conditions to identify optimal points of equality constrained problems, each stationary point found by the optimization routine can be guaranteed to be a local minimum. These conditions, known as the Karush-Kuhn-Tucker conditions are as follows:

$$\nabla L = \nabla \Phi + \sum_{j=1}^{m} \lambda_j \nabla g_j = 0 \qquad \textbf{Eq(5)}$$

$$\lambda_j g_j = 0$$

The aforementioned conditions provide a powerful means to verify solutions. Next, the gradient of each constraint equation can be written as:

$$g_j(x + \Delta x) - g_j(x) = \sum_{i=1}^{n} \frac{dg_j}{dx_i} \Delta x_i \qquad \textbf{Eq(6)}$$

Note, Eq(5) and Eq(6) provide sufficient equations to solve for all lagrange multipliers.

Side Constraints

A useful concept in constrained optimization is the idea of side constraints. As an example, although mathematically feasible, a negative moment of inertia for a structural element is physically meaningless. As a result, the current problem must contain a subroutine within the optimization process that prevents design variables from taking on negative values. Side constraints restrict the range of values that each design variable can assume. Each iteration of the optimization process can check as to whether a design variable has left the feasible range. When such an event occurs, the algorithm can repeat the current iteration but artificially set the design variable to the maximum or minimum of the feasible range. In turn, the algorithm can adjust the gradient calculations to reflect the restriction on the design variable and the remaining design variables in the feasible range will change accordingly (Cheng et al.). An optimal solution can then be found with each design variable remaining within the feasible range.

Linking

Just as it may be desirable to restrict design variables to a certain range of values, it may also be desirable to force several design variables to take on the same value for every iteration of the optimization process. This is accomplished through linking. For example, the current formulation uses two bending elements to span a single bay. Although two elements are used, in reality this bay would be spanned by a single beam. In turn, the two separate elements should be forced to take on the same design value. Again, within the optimization algorithm it is possible to represent multiple design variables with a single variable yet still account for the change in both structural elements within the gradient calculations (Cheng et al.). Although both beam elements can be represented by a single variable, the change in cross sectional properties of each element must be accounted for as they both have an effect on the convergence to a local minimum of the objective function subject to the constraint equations.

Scale factor

To aid convergence of the optimization algorithm, it is beneficial to include a scale factor that will automatically scale all design variables to produce a structure that will satisfy at least one constraint equation exactly. To illustrate, the current problem will use displacement measurements as constraints. At the beginning of each iteration, the displacements will be calculated using the same load cases used to produce the measured displacements for the damaged structure. The most violated displacement will be used as the basis for the scale factor. For example, say at the beginning of the first iteration, where all design variables start at those of the healthy structure, the **calculated** displacement at the first degree of freedom is half as large as the **measured** displacement at that same degree of freedom. In order for the calculated and measured displacements to match, all design variables must be cut in half, thus reducing the stiffness of the structure by a factor of 2. As a result, the displacement constraint on the first DOF will automatically be satisfied. A scale factor is calculated and applied to the initial design variables of each iteration.

Damage Detection Problem

Objective function

In order to create an optimization routine that can locate and detect damage, an objective function must be found that achieves a minimum value at the damaged state of the structure. Without prior knowledge of the damage state it is difficult to create an objective function that reaches a minimum value at all possible damage states. In order to solve this challenge, it was hypothesized that the damage state of the structure would be that which produces the least damage compared to the healthy structure. Since damage has been defined as a reduction in stiffness of the structure, the objective function is written as to minimize the total reduction in stiffness. Although bending elements are used for each element, it was found that the reduction of the shear stiffness of each element produced the best results. As a result, the objective function is written as:

$$\Phi = \sum_{i=1}^{n} \frac{\Delta I_i}{L_i} . \qquad \textbf{Eq(7)}$$

where,

I_i = moment of inertia of the ith member
L_i = length of the ith member

Since each change in moment of inertia is divided by the length of each respective member, the function inherently weights the change in cross section properties according to the length of each member, thereby taking into account the affect each member has on the total stiffness of the structure.

Constraints

When loads are applied to the structure, naturally displacements occur at each degree of freedom. These displacements are a function of the applied loads, which are known, and the stiffness matrix of the damaged structure, which is not known. Using Eq(1), the displacements can be solved for in terms of the known and unknown values by:

$$u = K^{-1} * P . \qquad \textbf{Eq(8)}$$

If every displacement were measured this equation could easily be solved to find the damaged stiffness matrix. However, as stated earlier, the objective is to develop a damage detection procedure that is easy, accurate and practical. It is impractical to measure every displacement as one, it is time consuming, and two, often impossible due to obstructions. What remains is an expression with more unknowns than equations. Fortunately, strategic displacement measurements coupled with the correct objective function will provide a good basis that will determine the damaged stiffness matrix and therefore the cross-sectional properties of each member.

The number of constraints is equal to the number of measured displacements for all load cases. Using the measured displacements of the structure and Eq(8), equality constraints can be created that must be satisfied by the optimization procedure given by:

$$K^{-1} * P = u_{measured} . \qquad \textbf{Eq(9)}$$

where,

$u_{measured}$ = vector of measured displacements
K = global stiffness matrix
P = Load vector used to produce measured displacements

Next, in order to write the constraints in a form suitable for optimization, i.e. Eq(3b), Eq(9) can be written as:

$$(K^{-1} * P) - u_{measured} = 0 . \qquad \textbf{Eq(10)}$$

Thus, Eq(10) provides a set of constraint equations that must be satisfied at the optimum set of design variables.

Side Constraints

In order to avoid convergence to a physically meaningless point, it was desired to include side constraints on each design variable. First, as explained earlier side constraints were applied to each design variable preventing negative values. Second, it is physically impossible for damage to create additional moment of inertia for each structural element, therefore an upper bound equal to the value of the healthy moment of inertia was placed on each design variable.

Even though complete damage or collapse of a member would reduce its moment of inertia to zero, it was not desirable to set the lower side constraint to zero in this process. This is because if the optimization process follows a path that forces one of the design variables to pass through zero, as will be seen, the recursive formula which predicts the design values for each subsequent iteration will be unable to change any design variable once it has been set to zero. To solve this apparent problem, the lower bound was set to a value that is small enough to be insignificant in reality but large enough to carry mathematical weight in the optimization process. Therefore, the lower side constraint was set to 5in^4 for every member.

Problem Statement

Using the above formulations the engineering problem becomes:

$$\begin{aligned} &\text{Min } \Phi = \sum_{i=1}^{n} \frac{\Delta I_i}{L_i} = \sum_{i=1}^{n} \frac{I_i - I_{i\_initial}}{L_i} \\ &\text{s.t. } \{u\} - \{u_{measured}\} = 0 \\ &\text{s.t. } 5 \le I_i \le I_{i\_healthy} \end{aligned} \qquad \textbf{Eq(11a-c)}$$

As before, the optimality criterion must be developed for the current engineering problem. To do this the Lagrangian must be written using the objective function and constraint equations. With the inclusion of lagrange multipliers for each constraint, the Lagrangian becomes:

$$L = \sum_{i=1}^{n} \frac{\Delta I_i}{L_i} + \sum_{j=1}^{m} \lambda_j (u_j - u_{j\_measured}), \quad \textbf{Eq(12)}$$

where u_j are the calculated displacements of the healthy structure. From this, the necessary condition for optimality can be found by:

$$\nabla L = 0 \quad \textbf{Eq(13)}$$

In turn, a local minimum of Eq(12) occurs when

$$\frac{dL}{dI_i} = -\frac{1}{L_i} + \sum_{j=1}^{m} \lambda_j \frac{du_j}{dI_i} = 0 \quad \text{(for } i\text{=1,2,…,n)} \quad \textbf{Eq(14)}$$

Eq(14) can also be used as a barometer for how close each intermediate point within the optimization routine is to the optimal solution. Thus, a recursive formula can be created to determine the design variables for the next iteration based on the optimality criterion of the previous iteration (Cheng et al.). Manipulation of Eq(14) gives,

$$T_i = L_i \sum_{j=1}^{m} \lambda_j \frac{du_j}{dI_i}. \quad \textbf{Eq(15)}$$

When the optimality criterion is satisfied T_i is equal to 1.

In order to solve for each lagrange multiplier, gradients of each constraint equation must be calculated in order to provide a system of equations sufficient to solve for all unknowns (Haug et al. 1979). From the definition of a derivative, the gradient of the constraint equations can be written as such:

$$\frac{g_j(I + \Delta I) - g_j(I)}{\Delta I} = \sum_{i=1}^{n} \frac{dg_j}{dI_i} \quad \textbf{Eq(16)}$$

Simple manipulation of Eq(16) yields:

$$g_j(I + \Delta I) - g_j(I) = \sum_{i=1}^{n} \frac{dg_j}{dI_i} \Delta I_i \quad \textbf{Eq(17)}$$

The ΔI term that appears in Eq(17) is determined using Eq(15). The change in I can be calculated using the recursive formula:

$$I_i^{k+1} = I_i^k + \frac{1}{r}(T_i - 1) I_i^k \quad \textbf{Eq(18)}$$

Solving for ΔI yields:

$$\Delta I = I_i^{k+1} - I_i = \frac{1}{r}(T_i - 1)I_i^k \quad \textbf{Eq(19)}$$

Substituting Eq(19) in to Eq(17) yields:

$$-g_j(I) = \sum_{i=1}^{n} \frac{dg_j}{dI_i} \frac{1}{r}(T_i - 1)I_i, \quad \textbf{Eq(20)}$$

where r is a convergence control parameter that restricts the step size for each iteration (Cheng et al.). The control parameter is assumed to be 2 for all problems covered in this paper. Manipulation of Eq(20) yields:

$$rg_j - \sum_{i=1}^{m} \frac{dg_j}{dI_i} I_i = \sum_{i=1}^{m} - \frac{dg_j}{dI_i} T_i I_i \quad \textbf{Eq(21)}$$

Eq(21) provides n equations for n unknown Lagrange multipliers, which can be used in Eq(15) and Eq(19) to determine the design values for the next iteration. The cycle is repeated until the optimality criterion is met.

Example

A two-bay frame is pictured in Figure 2. It is used to demonstrate the importance of the location of the applied loads in the damage detection process. Each structural element is assumed to be made of steel with modulus of elasticity 29,000 ksi and dimensions shown in Figure 2.

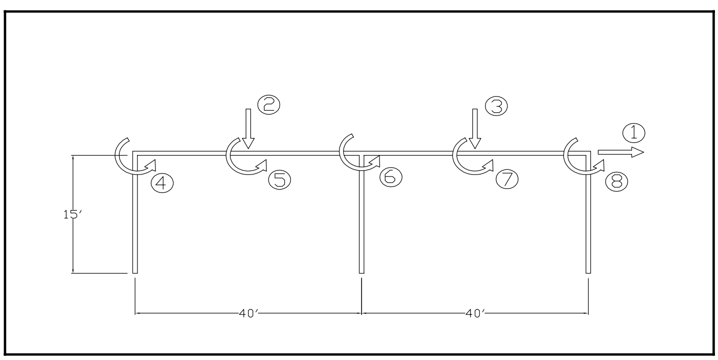

Figure 2. Two-bay Frame

Table 1 shows the initial and final design variables of each structural element. For this example, all elements will experience damage. All initial values are assumed to be known and will serve as the starting point for each member size. The final design values are not known, however, and will be determined by the optimization routine.

Table 1. Initial and Final Design Values

Member	$I_{initial}(in^4)$	$I_{final}(in^4)$
1	3500	1500
2	3500	1000
3	3500	1000
4	3500	1000
5	3500	1000
6	3500	700
7	3500	700

Case 1

For the first instance, 5 different load cases are used to measure displacements of the damaged structure. Loads are applied individually at each of the first 5 DOF and the resulting displacement at the same DOF is measured. The applied load and resulting displacement for each of the 5 DOF is shown in Table 2.

Table 2. Load Cases and Resulting Displacements for Case 1 & 2

Load Case	DOF Loaded	LOAD	DOF Measured	Displacement
1	1	10 k	1	0.10743 in
2	2	10 k	2	0.28251 in
3	3	10 k	3	0.30796 in
4	4	10 k-in	4	0.000017438 in/in
5	5	10 k-in	5	0.000012803 in/in

Each measured displacement is used as a constraint equation given by Eq(10). Table 3 shows the results of the optimization process using 5 displacements as constraint equations. As can be seen, the optimization routine converged to the damage state in 5 iterations. The first 7 columns of the last row are identical to the final design values shown in Table 1. It is also observed that all T_i values converged to 1. From the previous formulation (Eq.14-15) this signifies that the optimality criterion has been met and the current design variables are those that minimize the objective function and satisfy the constraint equations. The results are promising in that only 5 measured displacements were required for convergence to the correct damage state rather than all 8 measured displacements. However, two of the five measured displacements required not only rotation measurements but also concentrated moments to be applied, which can be difficult. In order for the damage detection procedure to be practical, these complications must be eliminated.

Table 3. Optimization Results for Case 1

Iteration	I_1	I_2	I_3	I_4	I_5	I_6	I_7	Obj	λ_1	λ_2	λ_3	λ_4	λ_5	T_1	T_2	T_3	T_4	T_5	T_6	T_7
1	3500	3500	3500	3500	3500	3500	3500	88.83	-45.864	-7.6083	-29.748	-23246	-920240	1.9243	1.2886	1.2886	1.2881	1.2881	0.6796	0.9914
2	1221	955.6	955.6	955.4	955.4	701.3	831.5	85.44	22.267	-6.6294	-27.925	-268290	-1E+06	1.3802	1.0788	1.0788	1.0795	1.0795	0.9998	0.7224
3	1453	993.3	993.3	993.4	993.4	701.2	716.1	84.16	6.2744	-10.394	-27.727	-238190	-1E+06	1.0627	1.0132	1.0132	1.013	1.013	0.9966	0.9562
4	1499	999.8	999.8	999.8	999.8	700	700.4	83.9	1.1466	-11.446	-27.548	-232980	-1E+06	1.0017	1.0004	1.0004	1.0003	1.0003	0.9999	0.9987
5	1500	1000	1000	1000	1000	700	700	83.89	0.9853	-11.478	-27.543	-232820	-1E+06	1	1	1	1	1	1	1

Case 2

If the number of displacement measurements is reduced to 4, the final design values are not close to the actual damage state. Case 2 uses the same initial and final design values and the same loading cases, however only the first 4 DOF measurements are used. The results of reducing the number of measured displacements to 4 are shown in Table 4. The structural members spanning each bay, namely I_2, I_3, I_4, and I_5, converged to values within 8% of their actual values and I_6 to within 2.1%. However, I_1 and I_7 were grossly misjudged with error in excess of 50% for I_1 and 70% for I_7.

Table 4. Optimization Results for Case 2

Iteration	I_1	I_2	I_3	I_4	I_5	I_6	I_7	Obj	λ_1	λ_2	λ_3	λ_4	T_1	T_2	T_3	T_4	T_5	T_6	T_7
1	3500	3500	3500	3500	3500	3500	3500	88.83	-167.66	-24.429	-25.749	224600	1.044	1.3504	1.3504	1.2011	1.2011	0.6952	1.5845
2	853.5	981.4	981.4	919.1	919.1	707.9	1079	86.16	-146.39	-27.873	-21.749	-37047	0.8936	1.085	1.085	1.0261	1.0261	1.0154	1.1011
3	808.1	1023	1023	931.1	931.1	713.3	1134	85.631	-147.38	-27.955	-21.3	-41909	0.9229	1.0127	1.0127	0.995	0.995	1.0014	1.0453
4	776.9	1030	1030	928.7	928.7	713.8	1159	85.625	-147.44	-28.131	-21.118	-42553	0.9534	1.0059	1.0059	0.9967	0.9967	1.0008	1.0259
5	758.9	1033	1033	927.2	927.2	714.1	1174	85.627	-147.39	-28.253	-21.013	-42938	0.9724	1.0035	1.0035	0.9981	0.9981	1.0005	1.0149
6	748.4	1034	1034	926.3	926.3	714.2	1183	85.628	-147.35	-28.325	-20.952	-43182	0.9839	1.002	1.002	0.9989	0.9989	1.0003	1.0086
7	742.3	1036	1036	925.8	925.8	714.3	1188	85.629	-147.32	-28.367	-20.917	-43329	0.9906	1.0012	1.0012	0.9994	0.9994	1.0002	1.0049
8	738.9	1036	1036	925.5	925.5	714.4	1191	85.629	-147.3	-28.392	-20.896	-43416	0.9946	1.0007	1.0007	0.9996	0.9996	1.0001	1.0028
9	736.9	1036	1036	925.3	925.3	714.4	1193	85.629	-147.29	-28.406	-20.885	-43467	0.9969	1.0004	1.0004	0.9998	0.9998	1.0001	1.0016
10	735.7	1037	1037	925.2	925.2	714.4	1194	85.629	-147.28	-28.414	-20.878	-43496	0.9982	1.0002	1.0002	0.9999	0.9999	1	1.0009

Case 3

At this point several alterations to the method can be made, two of which are to find an alternate objective function to be minimized or to try alternate loading cases from which to measure displacements. The former has been explored, using the minimization of strain energy, and minimizing the reduction in bending stiffness. However results have not shown to improve beyond Case 2. The latter alternative has proven to produce favorable results with a similar number of measured displacements. Table 5 shows an alternate loading condition with the resulting displacements using the final design values in Table 1.

Table 5. Load Cases and Resulting Displacements for Case 3

Load Case	DOF Loaded	LOAD	DOF Measured	Displacement
1	2	10 k	1	-0.024894 in
2	2	10 k	2	0.28251 in
3	2	10 k	3	-0.055233 in
4	2	10 k	4	0.00025174 in/in

For Case 3, a 10 kip load is applied at only degree of freedom 2 and displacements are measured at the first 4 DOF. Again the optimization routine is executed in an identical manner as the previous two cases and the results are shown in Table 6.

Table 6. Optimization Results for Case 3

Iteration	I_1	I_2	I_3	I_4	I_5	I_6	I_7	Obj	λ_1	λ_2	λ_3	λ_4	T_1	T_2	T_3	T_4	T_5	T_6	T_7
1	3500	3500	3500	3500	3500	3500	3500	100.54	-345.81	-19.46	207.68	-18825	3.1223	1.9186	1.9186	1.067	1.067	0.8183	1.8332
2	997.1	705.9	705.9	499.9	499.9	439.8	685.3	94.828	-924.66	-27.458	524.62	-36304	2.2241	1.5161	1.5161	1.4795	1.4795	1.2379	1.1959
3	1607	888.1	888.1	619.8	619.8	492.1	752.4	91.626	-1708.4	-16.893	790.29	-85194	1.3868	1.4052	1.4052	1.9392	1.9392	1.7721	1.1523
4	1691	941.4	941.4	802.9	802.9	601.2	713.7	85.433	-2055.6	-21.505	734.75	-144990	0.8756	1.1042	1.1042	1.3687	1.3687	1.2473	0.978
5	1586	990.5	990.5	950.9	950.9	675.6	705.8	84.161	-1889.3	-28.551	572.51	-159140	0.9451	1.0229	1.0229	1.0792	1.0792	1.0499	0.9987
6	1535	997	997	983.8	983.8	689.1	702	83.92	-1829.8	-30.415	533.78	-159880	0.988	1.0028	1.0028	1.0119	1.0119	1.0105	0.999
7	1525	998	998	989.2	989.2	692.5	701.3	83.893	-1818.7	-30.74	527.84	-159730	0.9977	1.0003	1.0003	1.0016	1.0016	1.0016	0.9998
8	1523	998.1	998.1	990	990	693	701.2	83.892	-1816.7	-30.793	526.99	-159660	0.9997	1	1	1.0002	1.0002	1.0002	1
9	1523	998.2	998.2	990.1	990.1	693.1	701.2	83.892	-1816.4	-30.799	526.9	-159650	1	1	1	1	1	1	1

The optimization routine did not identically match the actual damage state given in Table 1, however the found values are all within 2% of the actual damaged values. By adjusting the loading condition when measurements were taken, results improved compared to Case 2 and one rotational degree of freedom measurement and both concentrated moment loads were eliminated compared to Case 1. Utilizing a loading condition with only vertical loads also provides a process better suited for implementation on real structures.

Conclusion

As shown from examples presented, the damage detection process is highly dependent on the load cases used to produce the measured displacements utilized by the optimization routine. In order for the damage detection process to be easily implemented it is desired that only vertical loads and vertical displacements be required for the damage detection procedure. The example presented is just one example of the possible load cases that can be used to measure displacements. Future work will include the exploration of additional load cases coupled with strategic displacements measurements in order to determine the effectiveness with regards to damage detection.

The optimization routine lends itself well to such engineering problems as all design variables are continuous. Furthermore, the results show quick convergence to a local minimum as signified by the satisfaction of the optimality criterion. Since so few iterations are required computational effort is kept to a minimum. Minimizing computing effort will prove beneficial for damage detection in larger structures.

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