# RELIABILITY OF BILINEAR SDOF SYSTEMS SUBJECTED TO EARTHQUAKE LOADING

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**Abstract:** Probabilistic assessment of the ductility demand and reliability analysis were carried out for bilinear hysteretic SDOF systems. The assessment considered two sets of strong ground motion records, and was focused on the evaluation of the mean and the coefficient of variation of the ductility demand for a given value of the normalized yield strength. The results indicate that the ductility demand could be modeled as a Frechet (Extreme value type II) variate. Based on the obtained results, empirical equations were provided to predict the mean of the ductility demand for bilinear SDOF systems of different natural vibration periods, damping ratios, and ratios of the post yield stiffness to the initial stiffness. The numerical results show that the coefficient of variation (cov) of the ductility demand can go as high as to about 1.0 depending on the characteristics of the structure. Also, a simple approach was given to estimate the probability of incipient damage and the probability of incipient collapse using the developed probabilistic characterization of the ductility demand. The approach, which could be suitable for carrying out design code calibration analysis, is illustrated numerically.

## Introduction

For a given strong ground motion, the peak responses of a linear or nonlinear single-degree-of-freedom (SDOF) system with and without strength degradation can be carried out using time-step integration methods. The obtained peak responses of SDOF systems can be employed in defining the linear elastic response spectrum and yield response spectra, and/or the ratios between peak linear elastic response and inelastic responses (Chopra 2000). These quantities are relevant for designing and assessing the safety of structures. Its use for the so-called displacement-based design has been discussed by many including Chopra and Geol (2000) and Borzi et al. (2001).

Let $F_E$ denote the minimum strength required for a SDOF system to remain linear elastic during a ground motion, and $D_E(T_n,\xi)$ denote the peak linear elastic displacement where $T_n$ and $\xi$ are the natural vibration period and the damping ratio, respectively. If the strength of the structure is less than $F_E$, the system responds inelastically with yield displacement represented by $D_y(T_n,\xi,\mu)$ and peak inelastic displacement represented by $D_I(T_n,\xi,\mu)$, where $\mu$ represents the displacement ductility factor. Given a set of strong ground motion records, the yield reduction factor $R_y$, $R_y = D_E(T_n,\xi)/D_y(T_n,\xi,\mu)$, and the ratio $R_\mu$, $R_\mu = D_I(T_n,\xi,\mu)/D_E(T_n,\xi)$, can be calculated. Note that $R_\mu = \mu/R_y$ which can be written as $R_\mu = \mu\phi$ where $\phi$ is defined as $1/R_y$ and is known as the normalized yield strength or the de-amplification factor. Note also that $\mu$ does not always increase monotonically as $R_y$ decreases and more than one value of $R_y$ could lead to the same ductility demand $\mu$. By considering that for a given value of $\mu$ it is the largest yield strength, hence the largest $\phi$ (or smallest $R_y$) that is relevant for design, an iterative procedure that is described in detail in Chopra (2000) can be employed to evaluate the required $\phi$ (or $R_y$) for a given ductility factor $\mu$. Note that the above is equivalent to say that given a value of $\phi$, it is the maximum ductility demand, for all the normalized yield strength less than or equal to the specified value of $\phi$, that is relevant for design. This view is adopted though out this study.



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Samples of $R_y$ or $R_\mu$ obtained to meet specified target ductility level are employed to find statistics of the ratios $R_y$ and/or $R_\mu$ and to develop empirical equations to predict the mean of the $R_y$ and/or $R_\mu$ as functions of ductility demand $\mu$ (Veletsos and Newmark 1960, Krawinkler and Nassar 1992, Vidic et al. 1994, Miranda 2000 and Riddell et al. 2002, Hong and Jiang 2004). The means of $R_y$ and/or $R_\mu$ are employed to scale the design response spectrum or peak linear elastic responses to obtain the design yield strength or the yield responses.

The evaluation of $R_y$ or $R_\mu$ to meet specified target ductility factor is computationally intensive because the iteration mentioned previously. It is much more efficient to evaluate the ductility demand for a given value of the normalized yield strength because the latter does not require the iteration over the ductility factor. Also, it is noted that rather than develop empirical equations to predict the values of $R_y$, $R_\mu$, or the normalized yield reduction factor $\phi$ for given values of $\mu$, one may instead develop empirical equations to predict the ductility demand $\mu$ based on regression analysis conditioned on $\phi$. A regression equation developed to predict the expected normalized yield strength is likely to differ from the one developed to predict the expected ductility (factor) demand. Perhaps, the former may be interpreted as a designer knows the ductility capacity of the structure to be designed and is interested in finding the minimum required yield strength; and the latter may be considered as a designer's task is to check a new design or evaluate an existing structure with a known yield strength level and is interested in finding what would be the ductility demand due to strong ground motions. Therefore, the latter that seems lacking in the literature is equally relevant as the former. The need for empirical equations to predict the ductility demand may be further justified based on that the uncertainty or variability associated with the ductility capacity is much greater than the yield strength (Nakashima 1997), hence, a designer could have better control on the yield strength level than on the ductility capacity, and a codification should be focused on incorporating the uncertainty in ductility capacity and ductility demand. Note that systematic assessment of the impact of uncertainty in ductility capacity on the structural reliability is not often investigated

In the following, statistics of the ductility demand are evaluated using two sets of strong ground motion records. The evaluation of samples of the ductility demand is carried out for given values of $\phi$. This largely reduces the computing time since iterations over $\phi$ to find the ductility factor that matches a specified ductility level are not required. Also, empirical equations for the statistics of $\mu$ conditioned on the normalized yield strength are presented. In a few cases, comparison of these results to the ones obtained to meet specific ductility level is also given. The evaluation of the ductility demand presented in this study considers several damping ratios and the elastoplastic as well as bilinear hysteretic systems. A very simple method for assessing the reliability of bilinear system by using the developed empirical equations is presented. The method can be used to evaluate the probability of incipient collapse as well as the incipient of damage. Its use is illustrated by numerical examples.

## Statistics, ratios and inelastic displacement

### *Records*

Two sets of records considered in this study are for California earthquakes. The first set includes 230 components of records that were used by Miranda (2000) and were found in the database prepared by Silva (2001). This set was adopted by Hong and Jiang (2004) as well.

The second set of records adopted in the present study is the one used by Riddell et al. (2002) to represent earthquakes occurred in California. This set contains 44 records obtained for only two earthquakes, Northridge earthquake and Loma Prieta earthquake. 22 of these 44 components of records are common to the first set. The use of this set of records is aimed at gauging how sensitive are the estimated ratios mentioned previously to the selected records.

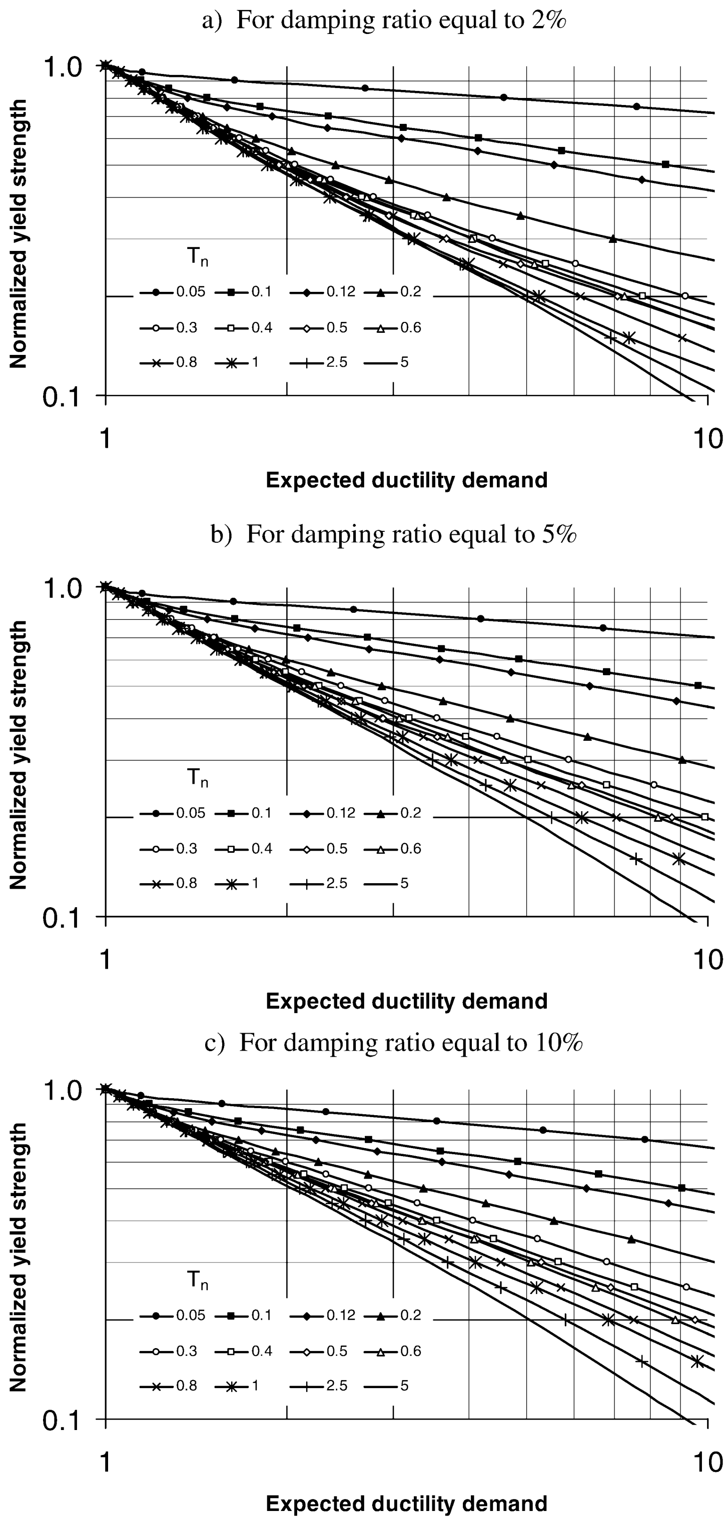



**Figure 1.** Expected ductility demand for elastic-perfectly-plastic SDOF system.

***Results for elastocplastic systems***

By using the first set of records and carrying out nonlinear dynamic analysis of elastic-perfectly plastic single-degree-of-freedoom (SDOF) system, the obtained mean of the ductility demand is presented in Figures 1a to 1c for several values of the natural vibration period and $\xi$ = 2%, 5%, and 10%.

The results shown in the figure suggest that one may consider that the logarithmic of the expected ductility demand, $m_\mu$, is a power function of ln($\phi$). This leads to

$$m_\mu = \exp\left((-\alpha_1 \ln\phi)^\beta\right), \tag{1}$$

where $\alpha_1$ and $\beta$ are parameters to be determined. The parameters for the models given in Eq. (1) may depend on the natural vibration period $T_n$ and the damping ratio $\xi$. By minimizing the error $\varepsilon$ defined by,

$$\varepsilon = \sum\left(m_{\mu i} - m_\mu(\phi)\right)^2, \tag{2}$$

the estimates of $\alpha_1$ and $\beta$ can be obtained. In Eq. (2), $m_{\mu i}$ is the mean of $\mu$ obtained from the samples such as those shown in Figure 4, $m_\mu(\phi)$ represents $m_\mu$ predicted using Eq. (1) for each given set of values of $T_n$ and $\xi$.

If one is interested in obtaining simple empirical equations for predicting $\alpha_1$ and $\beta$, the following fitted equations may be employed,

$$\alpha_1 = a_1 \exp\left(a_2 / T_n^{a_3}\right), \tag{3}$$

and,

$$\beta = \begin{cases} b_1 - b_2 \ln(T_n/0.2)/\ln(0.01/0.2) & 0.05 \le T_n < 0.2 \\ b_1 + (b_3 - b_1)\ln(T_n/0.2)/\ln(15/0.2) & 0.2 \le T_n < 5 \end{cases}, \tag{4}$$

where values of the parameters $a_i$ and $b_i$, $i$ = 1,2,3, are given in Table 1. An example of the predicted $m_\mu$ obtained by using the model given in Eq. (1) with $\alpha_1$ and $\beta$ calculated from Eqs. (3) and (4) is illustrated in Figure 2 for $\xi$ equal to 5%. Comparison of the results shown in this figure and those presented in Figure 1b suggests that the empirical predicting model provides a good approximation to those given in Figure 1b.

**Table 1.** Parameters for Eqs. (3) and (4)

| $\xi$ | For Eq. (3) | | | | | For Eq. (4) | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $\gamma = 0$ | 0.01 | 0.05 | 0.1 | | $\gamma = 0$ | 0.01 | 0.05 | 0.1 |
| 2% | $a_1$ | 0.893 | 0.866 | 0.813 | 0.786 | $b_1$ | 1.22 | 1.14 | 1.14 | 1.14 |
| | $a_2$ | 0.105 | 0.109 | 0.105 | 0.097 | $b_2$ | 0.30 | 0.30 | 0.40 | 0.60 |
| | $a_3$ | 0.972 | 0.952 | 0.941 | 0.928 | $b_3$ | 0.82 | 0.82 | 0.82 | 0.82 |
| 5% | $a_1$ | 0.927 | 0.907 | 0.857 | 0.828 | $b_1$ | 1.32 | 1.28 | 1.28 | 1.28 |
| | $a_2$ | 0.162 | 0.158 | 0.148 | 0.139 | $b_2$ | 0.30 | 0.40 | 0.70 | 1.00 |
| | $a_3$ | 0.815 | 0.817 | 0.818 | 0.810 | $b_3$ | 0.82 | 0.82 | 0.82 | 0.82 |
| 10% | $a_1$ | 0.973 | 0.952 | 0.876 | 0.858 | $b_1$ | 1.42 | 1.38 | 1.38 | 1.38 |
| | $a_2$ | 0.182 | 0.180 | 0.199 | 0.176 | $b_2$ | 0.30 | 0.50 | 0.90 | 1.10 |
| | $a_3$ | 0.752 | 0.755 | 0.697 | 0.711 | $b_3$ | 0.82 | 0.82 | 0.82 | 0.82 |

The obtained cov of the ductility demand is illustrated in Figure 3 for $\xi$ equal to 0.05. The results shown in the figure suggest that for the mean ductility demand less than about 10 (see Figure 1) the cov of $\mu$ increases as $\phi$ decreases. The cov of $\mu$ for relatively rigid structures is larger than that for the flexible structures, and decreases as the damping ratio increases. In almost all cases with a mean ductility demand less than 10, the cov of $\mu$ can be considered to be less than

1.0. Similar trends of the cov values were observed for the results obtained for ξ equal to 0.02 and 0.10.

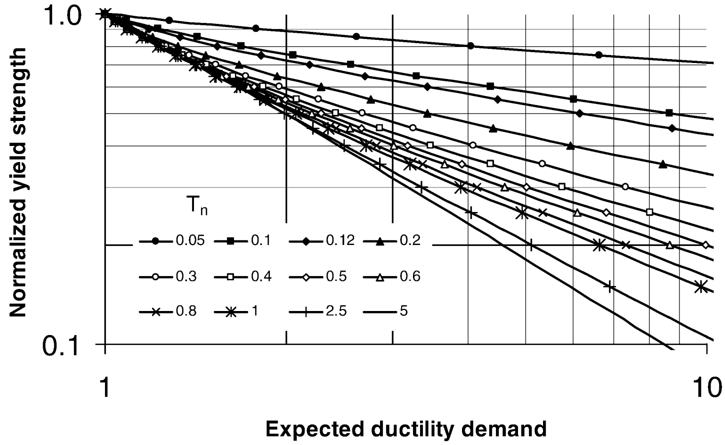



**Figure 2.** Predicted expected ductility demand using the model given in Eq. (1) for $\xi$ = 5%.

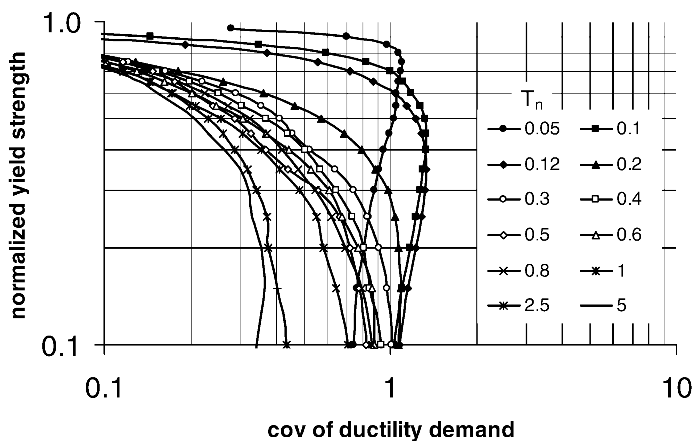



**Figure 3.** Coefficient of variation of the ductility demand for elastic-perfectly-plastic SDOF system with $\xi$ = 5%.

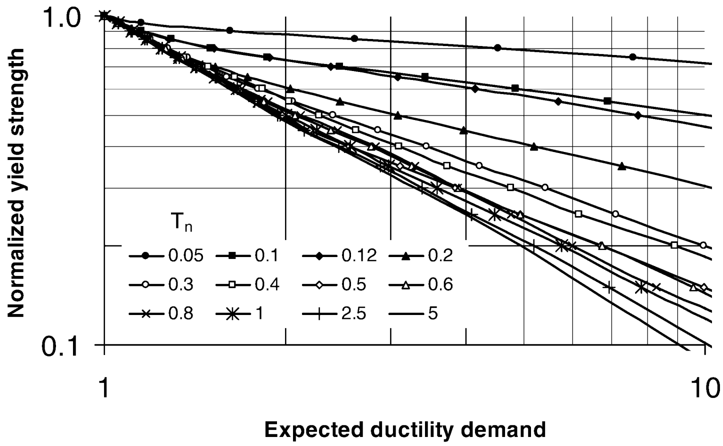
Now if the second set of records mentioned in the previous section is employed, the obtained mean and cov of the ductility demand for $\xi$ = 0.05 are shown in Figures 4a and 4b. Comparison of the results shown in Figure 1b and Figure 4a and the results shown in Figure 3 and Figure 4b suggest that:

1) the difference between the predicted ductility demand obtained by using the first set of records and the second set of records is not very significant; and

2) The values of the cov of the ductility demand depend somewhat on the set of records used; however, the conclusion, that the cov of $\mu$ is less than about 1.0 for the mean of $\mu$ less than 10, is still adequate.

Note that no detailed analysis of the cov of μ was presented. This is because that the uncertainty in peak elastic displacement rather than that in the ductility demand is likely to play a dominant role in characterizing the uncertainty in the peak inelastic displacement since the cov of the peal elastic displacement usually ranges from 0.8 to higher than 10 for different sites in Canada.

a) Mean ductility demand




b) Coefficient of variation of ductility demand

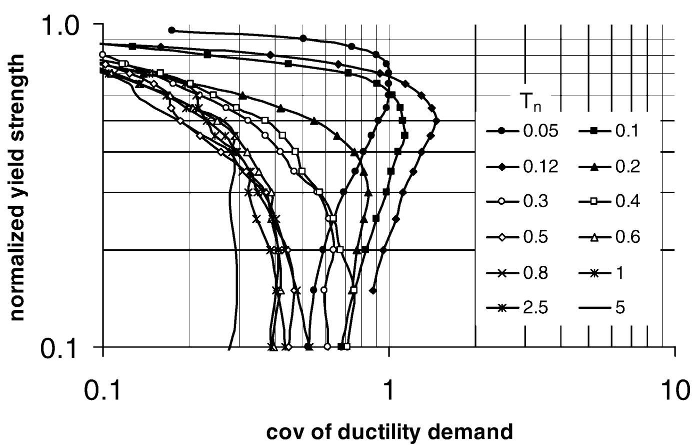



**Figure 4.** Statistics of ductility demand obtained using the second set of records for elastic-perfectly-plastic SDOF system with $\xi = 5\%$.

To investigate possible probabilistic distribution models for the ductility demand, we plot the samples of the ductility demand in the lognormal probability paper for a given value of $\phi$. It was found that the samples slightly curved, therefore, the assumption that the ductility demand is lognormally distributed can be very convenient but may not be very adequate. However, if the samples are presented in the Frechet probability paper as illustrated in Figure 5, the ductility demand samples could be approximate by straight liners for each given values of $\phi$. Therefore, the lognormal variate could be adopted to model the ductility demand but the use of Frechet distribution is preferred.

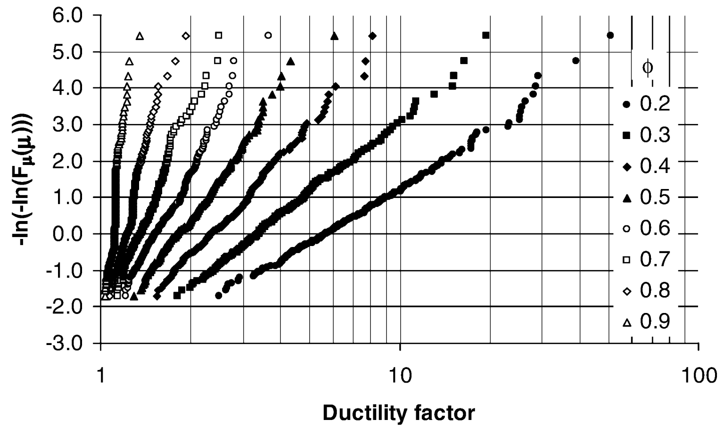



**Figure 5.** Frechet probability paper plot for the ductility demand samples

***Results for Bilinear systems***

To investigate the effect of the strain-hardening on the statistics of the ductility demand, bilinear hysteretic systems are considered in this section. Let $\gamma$ denote the ratio of the post yield stiffness to the initial stiffness.

The analysis carried out for the elasto-perfectly-plastic SDOF systems is repeated for the bilinear system for the combinations of $\gamma$ (= 0.01, 0.05 and 0.1) and $\xi$ (= 0.01, 0.05 and 0.1) values. The obtained results are employed to find the values of $\alpha_1$ and $\beta$ for the model given in Eq. (1) as was done for elasto-perfectly-plastic SDOF systems. Eqs. (3) and (4) are then employed to fit these values leading to the parameters presented in Table 1.

**Use of empirical equations in estimating reliabilities**

***Limit state functions and reliability analysis***

The developed empirical equations can be employed for the probabilistic evaluation of the performance of a designed or an existing structural system, and for design code calibration. For the evaluation, only two levels, one for the incipient damage (incipient inelastic deformation) and the other for the incipient collapse (Wen 2001), will be considered. Let $D_R(T_n,\xi)$ denote the yield displacement capacity of the structural system, and $\zeta$ denote the ratio of $D_R(T_n,\xi)$ to $D_E(T_n,\xi)$. In this section, $D_E(T_n,\xi)$ represents the annual peak linear elastic displacement. Since the yield displacement capacity $D_R(T_n,\xi)$ less than the seismic demand $D_E(T_n,\xi)$ implies that the structure will at least sustain damage, and the maximum inelastic displacement (capacity) $\mu_R D_R(T_n,\xi)$ less than the maximum inelastic ductility demand implies that the structure will collapse, the limit state functions for these two performance levels are,

$$g_D = D_R(T_n,\xi)/D_E(T_n,\xi) - 1 = \zeta - 1, \tag{5}$$

and,

$$g_C = \left(\mu_R D_R(T_n,\xi)\right)/\left(\phi\mu(\phi)D_E(T_n,\xi)\right) - 1, \tag{6}$$

where $g_D$ represents the limit state function of the incipient damage; $g_C$ represents the limit state function of the incipient collapse, respectively; $\zeta = D_R(T_n,\xi)/D_E(T_n,\xi)$; and $\mu_R$ denotes the ductility capacity of the structural system. To emphasize that the ductility demand $\mu$ is a function of the normalized yield strength $\phi$, the notation for the ductility demand $\mu$ in Eq.(6) is replaced by $\mu(\phi)$. Damage occurs if $g_D$ is less than zero and, collapse occurs if $g_C$ is less than zero.

Note that if $\zeta$ is greater than unity (i.e., $g_D > 0$) implying that the yield displacement capacity of the structural system is greater than the linear elastic demand caused by the strong ground motions. Note also that since by definition $\zeta = D_R(T_n,\xi)/D_E(T_n,\xi)$, $\zeta$ represents the normalized yield strength (i.e., $\zeta = \phi$), Eq. (6) can be re-written as

$$g_C = \mu_R / \mu(\zeta) - 1. \tag{7}$$

The yield strength is usually considered to be normal or lognormally distributed with a cov of about 0.1 to 0.15 (Ellingwood et al. (1980), Nakashima (1997)). Therefore, the yield displacement capacity of the structure can be considered to be lognormally distributed with a cov of 0.15. The uncertainty associated with the ductility capacity $\mu_R$ is much more significant than that associated with the yield strength. According to Nakashima (1997), the cov of $\mu_R$ can vary from about 0.5 to 1.0. In this section it is considered that $\mu_R$ can be modeled as a lognormal variate with a cov with of 0.5. Further, it is considered that the peak linear elastic displacement $D_E(T_n,\xi)$ is lognormally distributed as well with a cov within 1 to 10. This can be justified based on the seismic hazard studies given by Adams and Halchuk (2003).

Based on these adopted probabilistic models, the evaluation of the probability of the incipient damage, $P_D$, is straight forward (Madsen et al. (1986)). It can be calculated from,

$$P_D = \text{Prob}(\zeta < 1) = F_\zeta(1), \tag{8a}$$

where

$$F_\zeta(\zeta) = \Phi\left(\left(\ln\zeta - \ln\left(m_\zeta / \sqrt{1+v_\zeta^2}\right)\right) / \sqrt{\ln(1+v_\zeta^2)}\right) \tag{8b}$$

where $m_\zeta = \left(1+v_E^2\right) m_R / m_E$, $v_\zeta^2 = (1+v_R^2)(1+v_E^2) - 1$, $m_R$ and $m_E$ denote the means of $D_R(T_n,\xi)$ and $D_E(T_n,\xi)$, respectively; $v_R$ and $v_E$ denote the cov of $D_R(T_n,\xi)$ and $D_E(T_n,\xi)$, and $\Phi(\bullet)$ is the standard normal probability distribution function. Note that $\zeta$ is lognromally distributed with mean $m_\zeta$ and cov of $v_\zeta$ since $D_R(T_n,\xi)$ and $D_E(T_n,\xi)$ are lognormally distributed.

The probability of the incipient collapse, $P_C$, $P_C = \text{Prob}(g_C \le 0)$, can be evaluated by recursively using the first-order reliability method (FORM) or the simulation techniques. In this study, the simulation technique is employed for the analysis. Note that $P_C = \text{Prob}(g_C \le 0)$ can be expressed as

$$P_C = \text{Prob}\left(\mu_R/\mu(\zeta) \le 1 \middle| \zeta < 1\right)\text{Prob}(\zeta < 1) = \text{Prob}\left(\mu_R/\mu(\zeta) \le 1 \middle| \zeta < 1\right) P_D, \tag{9}$$

since $\text{Prob}\left(\mu_R/\mu(\zeta) \le 1 \middle| \zeta \ge 1\right) = 0$. The basic steps for evaluating $\text{Prob}\left(\mu_R/\mu(\zeta) \le 1 \middle| \zeta < 1\right)$ by using simulation technique are:

1) Generate a sample of $\zeta$ according to the updated (or truncated) probability distribution function of $\zeta$, $F_\zeta(\zeta)/P_D$
2) Find the mean of $\mu(\zeta)$ using Eq. (1) with parameters defined in Eqs. (3) and (4), and the cov of $\mu(\zeta)$ from figures similar to Figure 3;
3) Using the obtained value in Step 2) define the probability distribution of $\mu(\zeta)$, which is considered to be Frechet distributed;
4) Generate samples of $\mu_R$ and $\mu(\zeta)$ according to their probability distributions; and check if $\mu_R/\mu(\zeta)$ is less than or larger than unity;
5) Repeat Steps 1) to 5) to generate enough samples of $\mu_R/\mu(\zeta)$ and to count number of times that $\mu_R/\mu(\zeta)$ is less than one for estimating $\text{Prob}\left(\mu_R/\mu(\zeta) \le 1 \middle| \zeta < 1\right)$.

***Numerical examples***

The formulations given in previous section are illustrated by simple numerical examples. For the analysis, it is assumed that the structure can be modeled as a SDOF system with natural vibration period equal to 0.5 (sec). Consider that the ratio between the yield displacement capacity of a bilinear hysteretic system $m_R$ and the displacement corresponding to the design earthquake load for a linear elastic SDOF system, $D_{EN}(T_n,\xi)$, $\phi_D$, is known. $\phi_D$ takes into account factors such as the ductility-related force reduction and the mean resistance is greater than the factored resistance. The yield displacement capacity is considered to be lognormally distributed with cov of 0.15. Further, consider that $\phi_D$ equals 0.5; the mean of the ductility capacity, $m_{\mu R}$, equals 4; the ratio of the post yield stiffness to the initial stiffness $\gamma$ takes the value of 0, 0.01 or 0.05; and $D_{EN}(T_n,\xi)$ equals 475-year return period value of the peak linear elastic displacement demand $D_E(T_n,\xi)$ that is considered to be lognormally distributed. The cov of $D_E(T_n,\xi)$, $v_E$, that equals 0.8, 2 and 10 are considered. The small and large values of $v_E$ were used to represent approximately the seismic hazard conditions for, respectively, the west and the east of Canada Adams and Halchuck (2003).

Based on these considerations, it can be shown that $m_\zeta$ is given by,

$$m_\zeta = \left(1+v_E^2\right)\frac{m_R}{m_E} = \left(1+v_E^2\right)\frac{\phi_D D_{EN}(T_n,\xi)}{m_E} = \phi_D\sqrt{1+v_E^2}\,\exp\left(\sqrt{\ln(1+v_E^2)}\Phi^{-1}\left(1-\frac{1}{475}\right)\right), \tag{10}$$

which is independent of $m_E$. In Eq. (10), $\Phi^{-1}(\bullet)$ denotes the inverse of the normal probability distribution function. Substituting Eq. (10) into Eq. (8) gives $P_D$ equal to $3.44\times10^{-2}$, $1.10\times10^{-2}$, and $5.74\times10^{-3}$ for $v_E$ equal to 0.8, 2 and 10, respectively.

To simplify the evaluation of $P_C$, the numerical analysis was carried out by considering that the cov of $\mu(\zeta)$ is independent of $\zeta$. The effect of this assumption on the estimated probability will be investigated by comparing the results obtained for the cov of $\mu(\zeta)$ equal to 0.4 and 0.8, which are shown in Table 2. The results shown in the table suggest that the probability of incipient collapse is not very sensitive to the assumed cov of the seismic demand (i.e., cov of $D_E(T_n,\xi)$, $v_E$) nor to the assumed cov of $\mu(\zeta)$. Also, it is noted that that the obtained probability of incipient collapse is insensitive to the considered post-yield stiffness. This may be explained by noting that a structure with $T_n$ = 0.5 and for a normalized yield strength (i.e., $\phi_D$) around 0.5 the expected ductility demand for $\gamma = 0$ does not differ significantly from that for $\gamma = 5\%$.

It should be noted that no attempt is made in this study to carry out a design code calibration excise (Madsen et al. (1986)). However, it is noteworthy that given the mean ductility capacity of structure $m_{\mu R}$ and the nominal or factored design earthquake demand $D_{EN}(T_n,\xi)$, the formulation and procedure given in this study can be used to calibrate the required resistance factor for the yield displacement (or strength) $\phi_D$ such that use of the factor in design will leads to the designed structures to meet a specified target reliability level.

**Table 2.** Estimated probability of incipient collapse

| cov of $\mu(\zeta)$ | $\gamma$ | $v_E$ | | |
|---|---|---|---|---|
| | | 0.8 | 2 | 10 |
| 0.4 | 0 | 1.54E-03 | 1.15E-03 | 1.40E-03 |
| | 1% | 1.52E-03 | 1.15E-03 | 1.33E-03 |
| | 5% | 1.25E-03 | 1.02E-03 | 1.16E-03 |
| 0.8 | 0 | 2.41E-03 | 1.61E-03 | 1.54E-03 |
| | 1% | 2.54E-03 | 1.50E-03 | 1.44E-03 |
| | 5% | 2.17E-03 | 1.38E-03 | 1.34E-03 |

**Conclusions**

A statistical analysis of the ductility demand was carried out for bilinear hysteretic SDOF systems. The analysis results indicate that the ductility demand can be modeled as a Frechet (Extreme value type II) variate for given values of the normalized yield strength.

The mean of the ductility demand and the normalized yield strength when plotted in a logarithmic paper, follow approximately a straight line for SDOF systems having the same initial natural vibration period. This observation leads to a simple empirical equation in predicting the expected ductility demand. Model parameters for the proposed empirical predicting models were obtained for different natural vibration periods, damping ratios, and ratios of the post yield stiffness to the initial stiffness. The coefficient of variation (cov) of the ductility demand can go as high as to about 1 depending on the characteristics of the structure.

Using the developed probabilistic characterization of the ductility demand, a simple approach to estimate the probability of incipient damage or incipient collapse was given. Numerical results suggest that an accurate empirical predicting model for the cov of the ductility demand may not be necessary since sensitivity analysis results indicate that the variation of this cov on the probability of incipient collapse is not very significant.

**Acknowledgements**

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