A COMPARISON OF PROBABILISTIC MODELS OF DETERIORATION FOR LIFE CYCLE MANAGEMENT OF STRUCTURES

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Abstract

The probabilistic modelling of deterioration in the time-dependent reliability analysis is a necessary step for developing a risk-based approach to the life cycle management of infrastructure systems. The decisions regarding the time and frequency of inspection, maintenance and replacement are confounded by sampling and temporal uncertainties associated with the deterioration of structural resistance. To account for these uncertainties, probabilistic models of deterioration have been developed under two broad categories, namely the random variable model and stochastic process model. The paper presents a conceptual exposition of these two models and highlights their profound implications to the age-based and condition-based preventive maintenances policies. The proposed stochastic gamma process model of deterioration is more versatile than the random rate model commonly used in the structural reliability literature.

Keywords: life cycle management, structural reliability, age-based replacement, condition-based maintenance, gamma process

1 Introduction

Time-dependent reliability analysis is necessary to develop optimum strategies for the life-cycle management of infrastructure systems that include roads, bridges, nuclear plants and transmission lines. The decisions regarding the time and frequency of inspection, maintenance and replacement are confounded by uncertainties associated with the deterioration of structural resistance. In general, the modelling of deterioration is influenced by sampling and temporal uncertainties. The sampling uncertainty refers to the variability of deterioration from sample to sample. The uncertainty inherent with the progression of deterioration over time is referred to as temporal uncertainty. The sampling uncertainty, an epistemic uncertainty, can be reduced by additional inspections. The temporal uncertainty on the other hand is aleatory in nature so that it cannot be eliminated completely by increasing inspections. An adequate consideration of temporal uncertainty is necessary for a credible and effective life cycle management of critical infrastructures.

The probabilistic models of deterioration can be classified in two broad categories, namely, random variable (RV) model and stochastic process model. In the RV model, parameters associated with an empirical deterioration law are randomized to reflect the sampling variability observed in a sample of deterioration data, such as the rate of deterioration (Hong 2000, Pandey 1998). The stochastic process model, such as the Markov chain or Gamma process, incorporates the temporal uncertainty associated with evolution of deterioration (Bogdanoff and Kozin 1985, Nicolai et al. 2004, van Noortwijk and Frangopol 2004). A key distinction between these two models is that a specific sample path is deterministic in RV model, but it remains uncertain in the stochastic process model.

The application of the random variable and stochastic process deterioration models have been hitherto reported, but a clear interpretation of conceptual distinctions between these two models and their impact on maintenance optimization problem have been lacking in the engineering literature. To address this issue, the paper evaluates the random variable and stochastic Gamma process models in a simplified setting of time-dependent structural reliability analysis. The two equivalent versions of the deterioration models are compared in terms of distributions of lifetime, deterioration magnitude and the life cycle cost. The paper presents an original exposition of the implications of deterioration models to the age-based and condition-based preventive maintenances policies.

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2 Models of Deterioration

2.1 Random Variable (RV) Deterioration Model

In case of a deteriorating system, the structural failure is defined as an event when the strength falls short of the applied stress. A corresponding limit state function is defined as

$$R(t) - s = [r_0 - X(t)] - s = \rho - X(t) \le 0 \tag{1}$$

where $R(t)$ denotes the deteriorating resistance at time t, s the load effect, r_0 the initial resistance, $X(t)$ the cumulative deterioration at time t, and $\rho = (r_0 - s) > 0$ is the available design margin or a failure threshold. For the sake of simplicity of the discussion, r_0, s and ρ are assumed as deterministic constants. Thus the failure is the event of cumulative deterioration $X(t)$ exceeding the threshold ρ.

The random variable (RV) model characterizes the randomness of the deterioration by a finite-dimension vector of time invariant random variables Θ as $X(t;\Theta)$. For example, consider a simple linear deterioration model as

$$X(t) = At \tag{2}$$

where A is the deterioration rate, which is typically randomized to reflect the variability in a large population of similar components. Given the probability distribution of random rate, $F_A(a)$, the distribution of the amount of deterioration, $X(t)$, is derived as $F_{X(t)}(x) = F_A(x/t)$. The mean, variance and coefficient of variation (COV) of $X(t)$ are expressed respectively as

$$\mu_{X(t)} = \mu_A t\text{, } \sigma^2_{X(t)} = \sigma^2_A t^2\text{, and } \nu_{X(t)} = \frac{\sigma_{X(t)}}{\mu_{X(t)}} = \nu_A \tag{3}$$

According to the failure definition in Eq. (1), the cumulative probability distribution of the lifetime, T, can be written as

$$F_T(t) = P[T \le t] = P[\rho / A \le t] = P[A \ge (\rho / t)] = 1 - F_A[\rho / t] \tag{4}$$

Depending on the probability distribution of A, the lifetime distribution can be derived analytically or computed numerically.

Suppose the deterioration rate is a gamma distributed random variable with probability density function given as

$$f_A(a) = \frac{(a/\delta)^{\eta-1}}{\delta\,\Gamma(\eta)} e^{-a/\delta}\text{, for } a \ge 0 \tag{5}$$

where η and δ are the shape and scale parameter, respectively. We denote the gamma density function by $ga(a;\eta,\delta)$ and its cumulative distribution function by $GA(a;\eta,\delta)$. From Eq.(2), the deterioration, $X(t)$, is also gamma distributed with density function $ga(x;\eta,\delta\, t)$. From Eq.(4), the lifetime ($T = \rho / A$) follows an inverted gamma distribution with the following density function:

$$f_T(t) = \frac{(\delta/\rho)}{\Gamma(\eta)} \left(\frac{\rho}{\delta\, t}\right)^{\eta+1} e^{-\rho/\delta\, t} \tag{6}$$

and the cumulative distribution function is written as

$$F_T(t) = 1 - \mathrm{GA}(\rho / t; \eta, \delta) \tag{7}$$

It can be shown that the moments of the lifetime are given as follows:

$$\mu_T = \frac{\rho}{\delta(\eta-1)}\,,\ \sigma_T^2 = \frac{\rho^2}{\delta^2(\eta-1)^2(\eta-2)}\,,\ \text{and}\ \nu_T = \frac{\sigma_T}{\mu_T} = \frac{1}{\sqrt{\eta-2}}\,,\ \text{for}\quad (\eta > 2) \tag{8}$$

Since the gamma distributed deterioration rate has mean $\delta\eta$ and COV $\sqrt{1/\eta}$, the moments of lifetime distribution can be related with that of the degradation rate as

$$\mu_T = \frac{\rho}{\mu_A(1-\nu_A^2)}\,,\ \sigma_T^2 = \frac{\rho^2}{\mu_A^2(1-\nu_A^2)^2(1/\nu_A^2-2)}\ \text{and}\ \nu_T = \frac{1}{\sqrt{1/\nu_A^2-2}} \tag{9}$$

2.2 Gamma Process (GP) Deterioration Model

The gamma process is a continuous-time Markov process with independent and gamma distributed increments (Abdel-Hameed 1975). The gamma process is a limit of a compound Poisson process with gamma distributed increments. The limit is reached when the Poisson rate approaches infinity in any finite time interval as the size of the increment tends proportionally to zero (Dufresne et al. 1991). In other words, the gamma process model implies that deterioration progresses with frequent occurrence of very small increment. A physical example such process is the flow accelerated corrosion in nuclear piping system.

In the stationary gamma process (GP) model, the cumulative deterioration $X(t)$ follows a gamma distribution $ga(x;\alpha\, t, \beta)$ with the shape parameter αt and the scale parameter β. The mean, variance and COV of $X(t)$ are given as

$$\mu_{X(t)} = \alpha\beta\, t\,,\ \sigma_{X(t)}^2 = \alpha\beta^2 t\,,\ \text{and}\ \nu_{X(t)} = \frac{1}{\sqrt{\alpha\, t}} \tag{10}$$

In this model, both mean and variance of the deterioration are linear in time. In RV model, the variance is quadratic in time as shown in Eq.(3). From Eq. (1), the cumulative lifetime distribution can be expressed as

$$F_T(t) = P[X(t) \ge \rho] = 1 - GA[\rho;\alpha t,\beta] \tag{11}$$

The probability density function, $f_T(t) = dF_T(t)/dt$, has no closed form expression, but it can be computed numerically. Similarly, the moments of the lifetime have to be evaluated by numerical integration.

3 Comparison of RV and GP Models

3.1 Calibration of the Parameters of RV and GP Models

In order to have a consistent comparison between the RV and GP models, a careful scheme is required for the calibration of the model parameters. In practical situations, typically a sample of lifetime data is available from the past failures records, and the mean (μ_T) and COV (ν_T) of the lifetime can be estimated from such a sample.

If we assume that the random lifetime sample is generated by a RV deterioration model, the parameters of random rate can be derived in terms of the mean and COV of the lifetime using Eq.(8) as

$$\eta = 2 + 1/\nu_T^2\ \text{and}\ \delta = \frac{\rho\nu_T^2}{\mu_T(1+\nu_T^2)} \tag{12}$$

If however the underlying degradation model is assumed to be a stochastic gamma process, the shape and scale parameters can be obtained by solving the following equations of moments:

$$\left.\begin{aligned}\mu_T &= \int_0^\infty P\{T \geq t\}dt = \int_0^\infty GA(\rho;\alpha\, t, \beta)dt \\ \mu_T^2(1+\nu_T^2) &= 2\int_0^\infty tP\{T \geq t\}dt = 2\int_0^\infty tGA(\rho;\alpha\, t, \beta)dt\end{aligned}\right\} \quad (13)$$

The two simultaneous equations must be solved numerically to compute the parameters α and β.

In summary, in the present calibration scheme, both RV and GP models are equivalent in a sense that they have identical mean and variance of the lifetime. The parameters of these two models are given in Tables 1 and 2.

Table 1: Parameters used in the model calibration

Mean Lifetime μ_T	***COV of Lifetime*** ν_T	***Failure threshold*** ρ
50 units of time (fixed)	Varied from 0.1 to 0.9	100 units (fixed)

Table 2: Examples of RV and GP model parameters

Parameters		$\nu_T = 0.3$	$\nu_T = 0.6$	$\nu_T = 0.9$
RV Model	Shape (η)	13.1111	4.7778	3.2346
	Scale (δ)	0.1651	0.5294	0. 8950
GP Model	Shape (α)	0.2099	0.0408	0. 0078
	Scale (β)	10.01	64.63	1,453.6

3.2 *Comparison of Deterioration Distribution*

The evolution of deterioration is compared in the two equivalent RV and GP models. Figure 1 compares the mean rate of deterioration as a function of COV of the lifetime distribution (ν_T) by fixing the mean lifetime and failure threshold as $\mu_T = 50$ and $\rho = 100$.

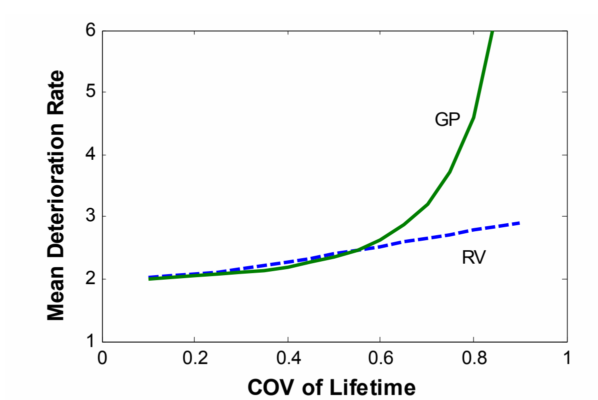

Figure 1: Comparison of the mean deterioration rate in equivalent RV and GP models ($\mu_T = 50$)

The mean deterioration rates in the RV and GP model are given as $\mu_A = \eta\delta$ and $\alpha\beta$, respectively. When $\nu_T < 0.6$, the mean rates in both models are almost identical. However, in cases of $\nu_T > 0.6$, the GP deterioration rate accelerates much faster than that in the RV model.

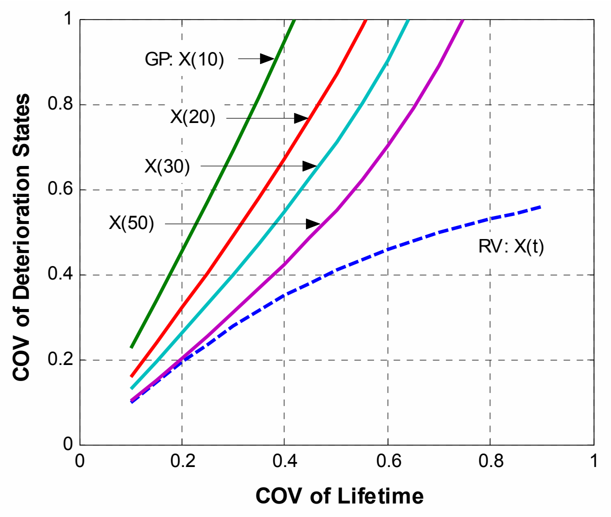

Figure 2: Coefficient of variation (COV) of deterioration states in equivalent RV and GP models

Figure 2 compares the COV of deterioration, $\nu_{X(t)}$, in the two models. It is a time-invariant and nonlinear function (Eq.3 and 9) of ν_T in the RV model. In contrast, $\nu_{X(t)}$ in GP model is a time-dependent parameter (Eq. 10), which is decreasing over time. Nevertheless, $\nu_{X(t)}$ of GP model is always greater than that of an equivalent RV model.

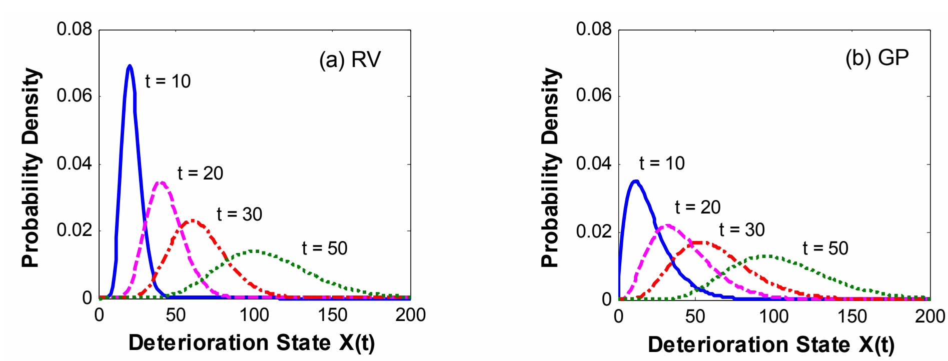

Figure 3: Probability density functions of deterioration *X*(*t*) for $\nu_T = 0.3$

The probability density function (PDF) of *X*(*t*) is given as $ga(x;\eta,\delta\, t)$ and $ga(x;\alpha\, t,\beta)$ in the RV and GP models, respectively. The evolution of the PDF with time is displayed in Figure 3 for $\nu_T = 0.3$. A key observation is that deterioration in GP model has greater variability than that in an equivalent RV model.

3.3 Comparison of Lifetime Distributions

The model calibration scheme is based on the method of moments that yields the parameters of RV and GP models such that the mean and COV of the lifetime distribution are identical in both models. In spite of the identical first two moments, the tails of lifetime distribution in RV and GP models can be remarkably different, as shown in Figure 4. Figure 4(a) shows that the lifetime distribution of the RV model is more skewed that that of the GP model. The difference between distribution tails can be seen more clearly through a comparison of survival functions in Figure 4(b). The GP model is more pessimistic than RV model about the prospect of survival due to uncertainty associated with evolution of deterioration that takes place in form of independent gamma distributed increments. In contrast, the deterioration in the RV model is fully correlated over the lifetime, which results in an overestimation of the survival probability.

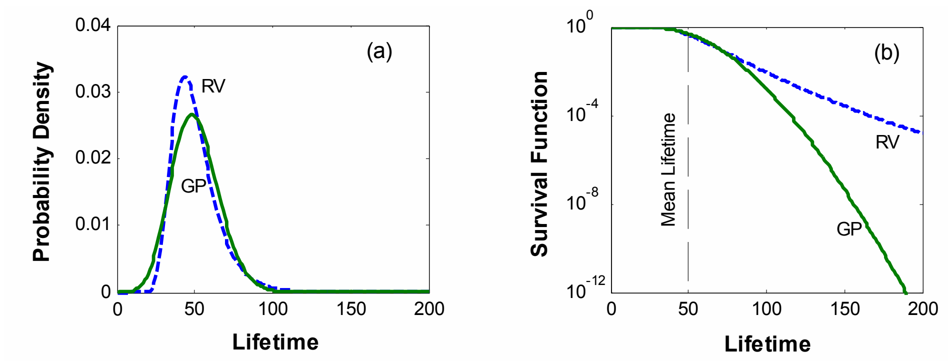

Figure 4: Comparison of lifetime distributions in equivalent RV and GP models for $\nu_T = 0.3$: (a) probability density, and (b) survival function

4 Age-Based Replacement Policy

The age-based replacement is the simplest policy for the renewal of aged fleet of structures and components. In this policy, a component is replaced when it reaches to a specific age (t_0) regardless of its condition. The component is of course replaced, if failure occurs before the replacement time, t_0.

Denote the total cost associated with all the consequences of a structural failure as C_F, and the cost of a preventive replacement as C_P. According to the renewal theory, the average cost per unit time in long term, also known as the mean cost rate K, can be computed as a function of the replacement age (Barlow and Proschan 1965):

$$K(t_0) = \frac{F_T(t_0)C_F + [1 - F_T(t_0)]C_P}{\int_0^{t_0} [1 - F_T(t)]dt} \tag{14}$$

It is easy to check that $K(t_o) \rightarrow C_F / \mu_T$ as $t_0 \rightarrow \infty$. Using Eq.(14), an optimal age of preventive replacement (t_0) can be found that would minimize the mean cost rate.

Since the calculation of the cost rate is sensitive to the lifetime distribution $F_T(t)$, it would be of interest to examine the impact of RV and GP model on the replacement policy. For an illustration, the cost data are assumed as $C_P = 10$ and $C_F = 50$. The basic data given in Tables 1 and 2 are used in the calculation of lifetime distribution. Figure 5 shows the variation of mean cost rate with the replacement age when the lifetime COV is $\nu_T = 0.3$. The optimal replacement age for both models is about the same, 29 units, but corresponding mean cost rate for the GP model ($K(29) = 0.45$ units) is greater than that of the RV model (0.38). The reason is that the GP model involves higher uncertainty than the RV model, as discussed in Section 3.

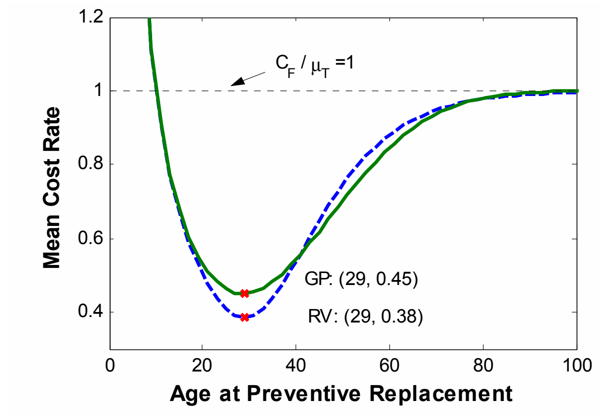

Figure 5: Mean cost rate as a function of replacement age in equivalent RV and GP models ($\nu_T = 0.3$)

A comprehensive comparison of optimal cost rate and replacement age results for other values of ν_T in the RV and GP models are shown in **Figure 6**. The results of optimal replacement age obtained from the two models are qualitatively different. In the RV model, the optimal replacement age decreases continuously as the lifetime COV ν_T increases. This observation is somewhat intuitive in the sense that when faced with increased uncertainty it is prudent to reduce the replacement age. The results of GP model exhibit two distinct trends. Initially, with increase in lifetime COV the replacement age decreases. However, as ν_T increases beyond 0.5, the trend reverses and the replacement age begins to increase. It means that in case of large uncertainty associated with component lifetime, the life-cycle cost can be optimized by extending the replacement age.

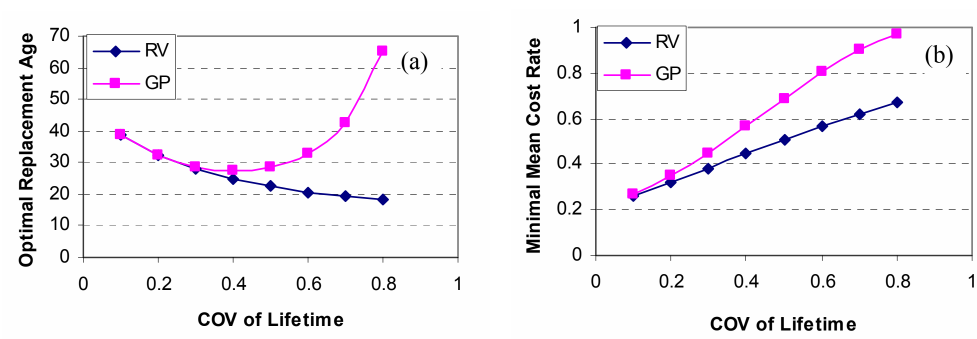

Figure 6: Comparison of age replacement policy in equivalent RV and GP models: (a) replacement age, and (b) mean cost rate

The comparison of the cost rate in Figure 6(b) shows that mean cost rate obtained from GP model is always higher than that of the RV model as a result of additional temporal uncertainty associated with GP model. The difference between the optimum cost rates increases with increase in lifetime uncertainty.

5 Condition Based Maintenance (CBM) Policy

5.1 The Strategy

The deterioration along a specific sample path is deterministic in the RV model, whereas it varies probabilistically in the GP model. In a linear RV model, one inspection determines the deterioration rate and it fixes the future deterioration path. An inspection in GP model reveals only its current state from which we can infer only the probability distribution of future deterioration. This distinction has profound implications to the optimization of condition-based maintenance strategies.

The condition based maintenance (CBM) strategy involves the periodic inspection of a structure at a fixed time interval t_I and cost C_I. We assume that the inspection is accurate such that the deterioration $X(t)$ can be measured with negligible error. The threshold for the preventive maintenance, $c\rho$ $(0 < c < 1)$, is a fraction of the failure threshold. The preventive maintenance (PM) results in complete renewal (as good as new) of the component. If $X(t_I) < c\rho$, no action is taken until the next inspection. A component is renewed with cost C_P when $c\rho < X(t_I) < \rho$. The structure would be immediately replaced upon failure at any time when $X(t) > \rho$, incurring a failure cost, C_F. Typically PM cost is much lower than the failure cost $(C_P < C_F)$.

The optimization of the condition-based maintenance means finding the inspection interval (t_I) and the PM ratio (c) that would minimize the mean cost rate. This in principle involves a two-dimensional optimization problem. In practical situations, however, the PM threshold $c\rho$ is known from experience or prescribed by operation standards or regulation. In such cases, the inspection interval is the only optimization variable. For illustration purpose, we fix the PM ratio as $c = 0.8$ and determine the optimal inspection interval and the associated mean cost rate for the RV and GP models.

5.2 RV Model for Condition Based Maintenance

The three possibilities that arise at any time of inspection are: do nothing, PM or renewal, as shown in the maintenance decision tree in Figure 7.

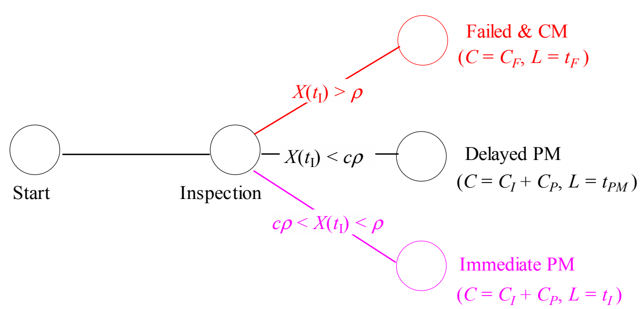

Figure 7: CBM decision tree for the RV model

At the time of first inspection t_I, if we observe $X(t_I) < c\rho$ we do nothing and the time of the future preventive maintenance can be predicted as $t_{PM} = c\rho t_I / X(t_I)$, since one inspection suffices to determine the sample path in the linear RV deterioration. This is referred to as delayed PM and note that no additional inspection is required before the time of replacement. The other two situations are straightforward. The PM is immediately conducted when $c\rho < X(t_I) < \rho$, and it is replaced when $X(t_I) > \rho$. A renewal is thus a delayed PM, immediate PM, or a CM.

The most important point is that the under assumption of RV deterioration model, only one inspection is required for the implementation of CBM strategy.

Using the maintenance decision tree and associated costs shown in Figure 7, it is easy to set up expressions for the mean renewal cycle cost, C, and length, L in terms of the time of first inspection interval t_I and PM ratio c. The renewal cycle cost is simply evaluated as

$$E[C(t_I,c)] = (C_I + C_P)P\{X(t_I) \le \rho\} + C_F P\{X(t_I) > \rho\} = (C_I + C_P - C_F)F_A(\rho / t_I) + C_F \quad (15)$$

Note that the mean cycle cost is independent of the PM ratio, c. The renewal cycle length is evaluated as

$$E[L(t_I,c)] = \int_0^{t_I} P\{X(t) < \rho\}dt + \int_{t_I}^{\infty} P\{X(t) < c\rho\}dt = \int_0^{t_I} F_A(\rho / t)dt + \int_{t_I}^{\infty} F_A(c\rho / t)dt \quad (16)$$

The derivation of Eq. (16) uses the mathematical identity for the expectation of a non-negative random variable, i.e., $E[T] = \int_0^{\infty} [1 - F_T(t)]dt$.

Eq. (15) and (16) can be easily evaluated given the parameters of the gamma distribution of the random rate A. According to renewal theory, the mean cost rate is given as

$$K(t_I,c) = \frac{E[C(t_I,c)]}{E[L(t_I,c)]} \quad (17)$$

When c = 1, the renewal cycle length equals the mean lifetime for any inspection interval. As the inspection interval $t_I \to \infty$, the CBM policy becomes equivalent to the age-based replacement policy.

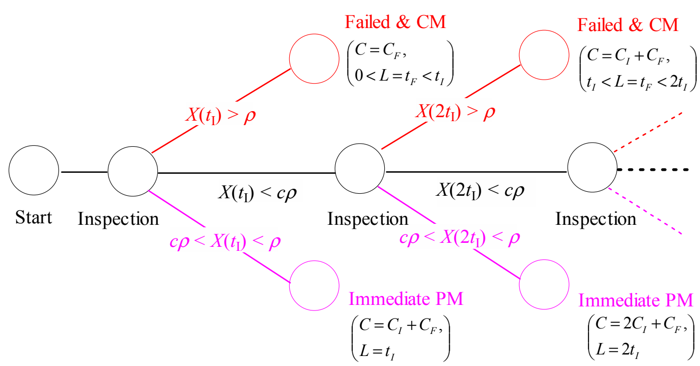

Figure 8: CBM decision tree for the GP model

5.3 *GP Model for Condition Based Maintenance*

The inspection and replacement scenarios in the GP model are more involved due to temporal uncertainty. The three possible situations arise at every inspection interval, namely, inspection and do nothing, preventive maintenance, or failure, as shown in the maintenance decision tree in Figure 8. In this case, a renewal is either an immediate PM or a corrected maintenance after unchecked failure. The renewal cycle cost is derived as (Park 1988, van Noortwijk et al. 1996)

$$E[C(t_I,c)] = \sum_{n=1}^{\infty} \left((nC_I + C_P)P\{\text{PM at } nt_I\} + [(n-1)C_I + C_F]P\{\text{Failed in } ((n-1)t_I, nt_I]\}\right) \quad (18)$$

The mean cycle length can be obtained as

$$E[L(t_I,c)] = \sum_{n=1}^{\infty} \left(nt_I P\{PM \text{ at } nt_I\} + E[T_F \mid (n-1)t_I < T_F \le nt_I]\right) \quad (19)$$

Expressing each probability above in terms of the probabilities of the deterioration at corresponding times, we calculate numerically the mean cycle cost and mean cycle length using the following expressions (Park 1988):

$$E[C(t_I,c)] = C_P + (C_F - C_P)[1 + S(c\rho)] \\ - (C_F - C_I - C_P)[GA(\rho;\alpha\, t_I,\beta) + \int_0^{c\rho} s(x) GA(\rho - x;\alpha\, t_I,\beta)dx] \quad (20)$$

$$E[L(t_I,c)] = \int_0^{t_I} GA(\rho;\alpha\, t,\beta)dt + \int_0^{c\rho}\int_0^{t_I} s(x) GA(\rho - x;\alpha\, t,\beta)dtdx \quad (21)$$

where $S(x) = \sum_{n=1}^{\infty} GA(x;n\alpha\, t_I,\beta)$ and $s(x) = \sum_{n=1}^{\infty} ga(x;n\alpha\, t_I,\beta)$.

There are two limiting cases of the CBM policy. As the inspection interval t_I becomes large, the mean cost rate converges to C_F / μ_T. Secondly, as the PM ratio c approaches 1, t_I converges to the optimal replacement age similar to that in the age-based maintenance policy.

5.4 Numerical Examples

In the optimization of inspection interval with respect to the cost rate, the following cost data are used: C_I = 1, C_P = 10, C_F = 50 and the PM ratio c = 0.8. The parameters of the RV and GP models are the same as calibrated in Section 3.1.

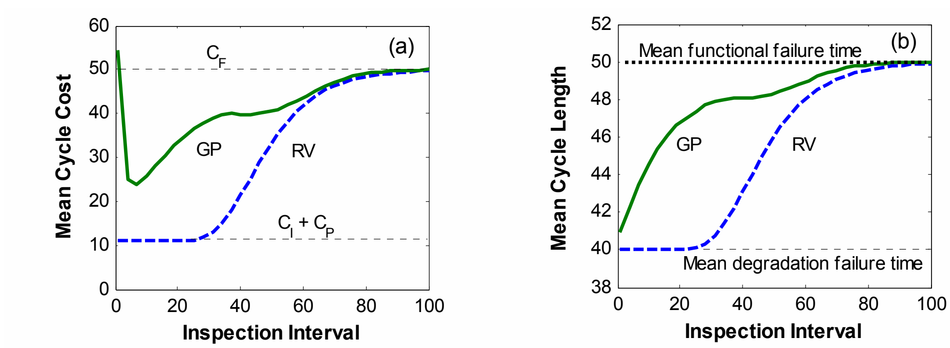

Figure 9: Results for equivalent RV and GP models (ν_T = 0.3): Inspection interval versus (a) Mean cycle cost, and (b) mean cycle length

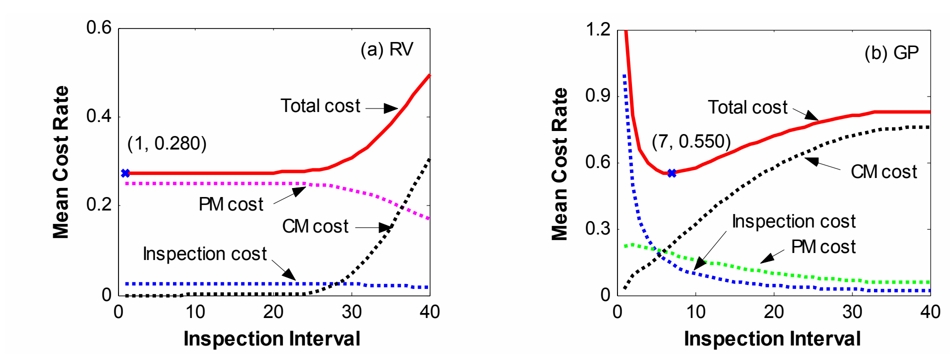

Figure 10: Mean cost rate as a function of inspection interval in equivalent RV and GP models (ν_T = 0.3)

Figure 9 compares the mean cycle cost and mean cycle length obtained from the RV and GP models (for ν_T = 0.3), whereas the mean cost rates, and its different component, are plotted in **Figure 10**. These results show remarkable differences between the two equivalent models of deterioration.

The cost curve for RV model has two plateaus, the lower one extends from t_I =0 – 30 and the corresponding cost is $C_I + C_P$. The upper limit of the cost is C_F. Because of the lack of temporal uncertainty, the RV model favors a short inspection interval. The GP model shows a distinct optimum of inspection interval (t_{opt} = 7 units) in Figure 10(b). For a small inspection interval, the cycle cost is fairly high due to increase in cost associated with frequent inspections. This aspect is qualitatively different from the RV model, which associates a smaller cost with a short inspection interval. For a large interval, the GP cycle cost approaches to the failure cost as expected. In general, the GP cycle cost is higher than that for RV model due to the effect of temporal uncertainty.

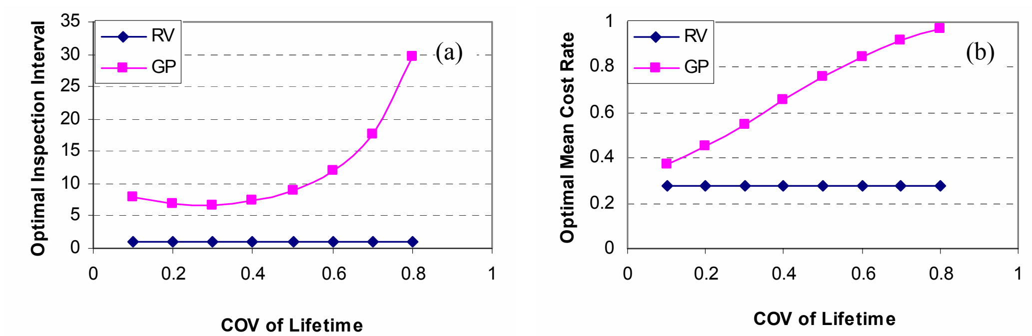

Figure 11: Comparison of DBM policy for RV and GP models: (a) inspection interval age, and (b) mean cost rate

The optimization results for RV and GP models for other values of the lifetime COV are compared in **Figure 11**. It is remarkable in the RV model that both the optimum cost rate and inspection interval are insensitive to the variance of lifetime distribution, which is due to the lack of consideration of temporal uncertainty. In contrast, the minimum cost rate in the GP model increases with lifetime uncertainty. The cost rate increases from 0.38 to 0.97 as the COV of lifetime (ν_T) is increased from 0.1 to 0.8. The optimal inspection interval increases from 7 to 30 units, as ν_T is increased from 0.1 to 0.8.

6 Conclusion

The purpose of infrastructure life cycle management is to mitigate the risk of structural failures caused by unchecked deterioration. The uncertainty associated with deterioration can be incorporated through probabilistic models that can be classified in two broad categories, namely the random variable (RV) model and stochastic process model. The paper introduces a versatile stochastic gamma process (GP) model and compares it with an equivalent random variable (RV) model with respect to distributions of lifetime, deterioration magnitude and the life cycle cost.

The paper underscores the points that a RV model cannot capture temporal variability associated with evolution of degradation. As a consequence, the deterioration along a specific sample path is deterministic in the RV model, whereas it varies probabilistically in the GP model. This distinction has profound implications to the optimization of age-based and condition-based maintenance strategies. The results presented in the paper show that the optimum cost and inspection interval obtained from stochastic process model are qualitatively different than that obtained from the RV model. The RV model appears to underestimate the life cycle cost due to lack of consideration of temporal uncertainty.

In summary, a careful consideration of the nature of uncertainties associated with deterioration is important for a meaningful time-dependent reliability analysis and life-cycle management of structures. If the deterioration process is affected by temporal uncertainty, it is mandatory to model it as a stochastic process.

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