Standard Test Method for Shear Modulus at Room Temperature[1]

This standard is issued under the fixed designation E 143; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

This standard has been approved for use by agencies of the Department of Defense.

1. Scope

1.1 This test method covers the determination of shear modulus of structural materials. This test method is limited to materials in which, and to stresses at which, creep is negligible compared to the strain produced immediately upon loading. Elastic properties such as shear modulus, Young's modulus, and Poisson's ratio are not determined routinely and are generally not specified in materials specifications. Precision and bias statements for these test methods are therefore not available.

1.2 Values stated in inch-pound units are to be regarded as the standard. SI units are provided for information only.

1.3 *This standard may involve hazardous materials, operations, and equipment. This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

E 6 Terminology Relating to Methods of Mechanical Testing[2]

E 8 Test Methods of Tension Testing of Metallic Materials[2]

E 111 Test Method for Young's Modulus, Tangent Modulus, and Chord Modulus[2]

E 1012 Practice for Specimen Alignment Under Tensile Loading [2]

3. Terminology

3.1 *Definitions*:

3.1.1 *shear modulus* [FL^{-2}]—the ratio of shear stress to corresponding shear strain below the proportional limit, also called torsional modulus and modulus of rigidity. (See Fig. 1.)

NOTE 1—The value of shear modulus may depend on the direction in which it is measured if the material is not isotropic. Wood, many plastics and certain metals are markedly anisotropic. Deviations from isotropy should be suspected if the shear modulus, G, differs from that determined by substituting independently measured values of Young's modulus, E, and Poisson's ratio, μ in the relation

$$G = \frac{E}{2(1 + \mu)} \tag{1}$$

NOTE 2—In general, it is advisable, in reporting values of shear modulus to state the stress range over which it is measured.

3.1.2 *torque*, [FL]—a moment (of forces) that produces or tends to produce rotation or torsion.

3.1.3 *torsional stress* [FL^{-2}]—the shear stress in a body, in a plane normal to the axis or rotation, resulting from the application of torque.

3.1.4 *angle of twist (torsion test)*— the angle of relative rotation measured in a plane normal to the torsion specimen's longitudinal axis over the gage length.

3.1.5 For definitions of other terms used in this test method, refer to Terminology E 6.

4. Summary of Test Method

4.1 The cylindrical or tubular test specimen is loaded either incrementally or continuously by applying an external torque so as to cause a uniform twist within the gage length.

4.1.1 Changes in torque and the corresponding changes in angle of twist are determined either incrementally or continuously. The appropriate slope is then calculated from the shear stress-strain curve, which may be derived under conditions of either increasing or decreasing torque (increasing from pretorque to maximum torque or decreasing from maximum torque to pretorque).

5. Significance and Use

5.1 Shear modulus is a material property useful in calculating compliance of structural materials in torsion provided they follow Hooke's law, that is, the angle of twist is proportional to the applied torque. Examples of the use of shear modulus are in the design of rotating shafts and helical compression springs.

NOTE 3—For materials that follow nonlinear elastic stress-strain behavior, the value of tangent or chord shear modulus is useful for estimating

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[2] *Annual Book of ASTM Standards*, Vol 03.01.

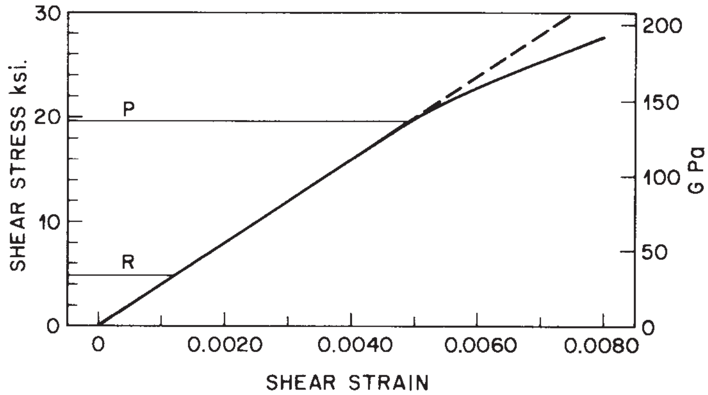

FIG. 1 Shear Stress-Strain Diagram Showing a Straight Line, Corresponding to the Shear Modulus, Between *R* , a Pretorque Stress, and *P* , the Proportional Limit

the change in torsional strain to corresponding stress for a specified stress or stress-range, respectively. Such determinations are, however, outside the scope of this standard. (See for example Ref **(1).**)[3]

5.2 The procedural steps and precision of the apparatus and the test specimens should be appropriate to the shape and the material type, since the method applies to a wide variety of materials and sizes.

5.3 Precise determination of shear modulus depends on the numerous variables that may affect such determinations.

5.3.1 These factors include characteristics of the specimen such as residual stress, concentricity, wall thickness in the case of tubes, deviation from nominal value, previous strain history and specimen dimension.

5.3.2 Testing conditions that influence the results include: axial position of the specimen, temperature and temperature variations, and maintenance of the apparatus.

5.3.3 Interpretation of data also influences results.

6. General Considerations

6.1 Shear modulus for a specimen of circular cross-section is given by the equation[4]

$$G = TL/J\theta \quad (2)$$

where:
G = shear modulus of the specimen,
T = torque,
L = gage length,
J = polar moment of inertia of the section about its center, and
θ = angle of twist, in radians.

6.1.1 *For a solid cylinder*:

$$J = \pi D^4/32 \quad (3)$$

where:
D = diameter.

6.1.2 *For a tube*:

$$J = \frac{\pi}{32}(D_0^4 - D_i^4) \quad (4)$$

where:
D_0 = outside diameter, and
D_i = inside diameter.

7. Apparatus

7.1 *Testing Machine*—The torsion testing machine, which is to be used for applying the required torque to the specimen, shall be calibrated for the range of torques used in the determination. Corrections may be applied for demonstrated systematic errors. The torques should be chosen such as to bring the error ΔG in shear modulus, due to errors in torque ΔT, well within the required accuracy (see 12.3.1).

7.2 *Grips*—The ends of the specimen shall be gripped firmly between the jaws of a testing machine which have been designed to produce a state of uniform twist within the gage length. In the case of tubes, closely fitting rigid plugs, such as are shown in Fig. 11 (Metal Plugs for Testing Tubular Specimens) of Test Methods E 8 may be inserted in the ends to permit tightening the grips without crushing the specimen. The grips shall be such that axial alignment can be obtained and maintained in order to prevent the application of bending moments. One grip shall be free to move axially to prevent the application of axial forces.

7.3 *Twist Gages*—The angle of twist may be measured by two pairs of lightweight but rigid arms, each pair fastened diametrically to a ring attached at three points to the section at an end of the gage length and at least one diameter removed from the grips. The relative rotational displacement of the two sections may be measured by mechanical, optical, or electrical means; for example, the displacement of a pointer on one arm relative to a scale on the other **(2),** or the reflection of a light beam from mirrors or prisms attached to the arms **(3).** Readings should be taken for both sets of arms and averaged to eliminate errors due to bending of the specimen (see 12.3.2).

8. Test Specimens

8.1 *Selection and Preparation of Specimens*:

8.1.1 Specimens shall be chosen from sound, clean material. Slight imperfections near the surface, such as fissures which would have negligible effect in determining Young's modulus, may cause appreciable errors in shear modulus. In the case of machined specimens care shall be taken to prevent changing the properties of the material at the surface of the specimen.

[3] The boldface numbers in parentheses refer to a list of references at the end of this standard.

[4] See any standard text in Mechanics of Materials.

8.1.1.1 Specimens in the form of solid cylinders should be straight and of uniform diameter for a length equal to the gage length plus two to four diameters (see 12.2.1).

8.1.1.2 In the case of tubes, the specimen should be straight and of uniform diameter and wall thickness for a length equal to the gage length plus at least four outside diameters (see 12.2.1 and 12.3.2).

8.2 *Length*—The gage length should be at least four diameters. The length of the specimen shall be sufficient for a free length between grips equal to the gage length plus two to four diameters, unless otherwise prescribed in the product specification. However, the ratio of free length to diameter shall not be so large that helical twisting of the axis of the specimen takes place before the determination is completed.

9. Procedure

9.1 *Measurement of Specimens*—Measure diameter to give an accurate determination of average polar moment of inertia, *J*, for the gage length. In addition, in the case of tubular specimens, determine the average wall thickness at each end to± 0.0001 in. ± (0.0025 mm).

9.1.1 In the case of thin-walled tubes, a survey of thickness variation by more sensitive devices, such as a pneumatic or electric gage, may be needed to determine thicknesses with the required accuracy.

9.2 *Alignment*—Take care to ensure axial alignment of the specimen. Procedures for alignment are described in detail in Practice E 1012. Although E 1012 is for a specimen under tensile loading, it provides guidance for machine setup and fixturing for other loading regimes.

9.3 *Torque and Angle of Twist*—Make simultaneous measurements of torque and angle of twist and record the data.

9.4 *Speed of Testing*—Maintain the speed of testing high enough to make creep negligible.

9.5 *Temperature*—Record the temperature. Avoid changes in temperature during the test.

10. Interpretation of Results

10.1 For the determination of shear modulus it is often helpful to use a variation of the strain deviation method **(4–6),** frequently used for determining Young's modulus. For this purpose, a graph (Fig. 2) may be plotted of torque *versus* deviation from the following equation:

$$\delta = L(\theta - T/K) \tag{5}$$

where:
δ = deviation,
L = gage length,
θ = angle of twist, in radians per unit length,
T = torque, and
K = a constant chosen so that $\theta - T/K$ is nearly constant below the proportional limit.

The range for which data are used for obtaining the shear modulus may be determined by applying some suitable criterion of departure from a straight line, for example, the least count of the twist gage, and examining the deviation graph with the aid of a sheet of transparent paper on which three parallel lines are drawn with the spacing between them equivalent to the least count of the twist gage.

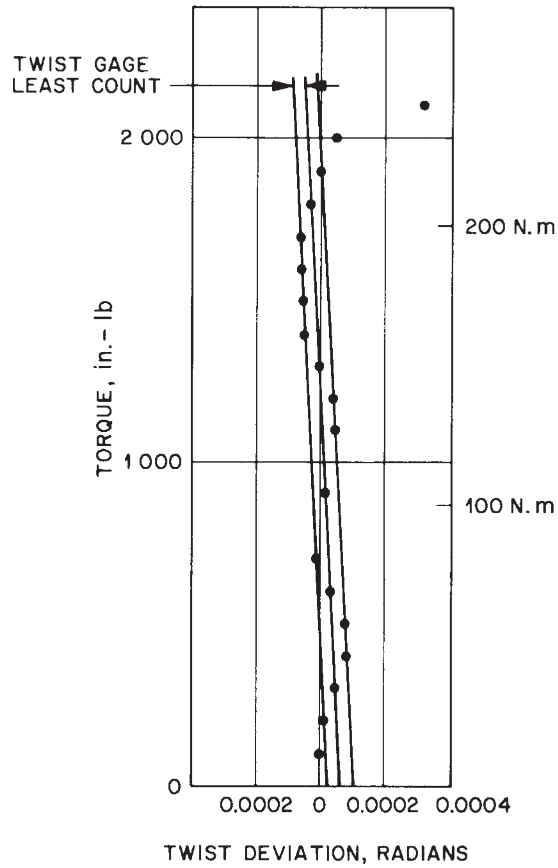

FIG. 2 Torque-Deviation Graph

10.2 The shear modulus may be determined by means of the deviation graph by fitting graphically a straight line to the appropriate points. From this line the deviation increment corresponding to a given torque increment can be read and substituted in the following equation (from Eq 2 and Eq 5):

$$G = \Delta T/J\Delta\theta = \Delta T/(\Delta T/K + \Delta\delta/L)J \tag{6}$$

where:
$\Delta\delta$ = deviation increment,
ΔT = torque increment, and
$\Delta\theta$ = increment in angle of twist, in radians per unit length.

10.3 The best fit of a straight line for the initial linear portion of the curve can be obtained by the method of least squares **(7–9).** For this test method, random variations in the data are considered as variations in the angle of twist θ. In determining the torque-range for which data should be used in the calculations it is helpful to examine the data using the deviation graph described in 10.1. Due to possible small offsets at zero torque and small variations in establishing the load path in the specimen during the first small increment of torque, the readings at zero torque and the first small increment of torque are typically not included in the calculations and the line is not constrained to pass through zero.

11. Report

11.1 *Test Specimen Material*—describe the specimen material, alloy, heat treatment, mill batch, number, grain direction, as applicable, and any relevant information regarding the sample that may have an influenced on its mechanical properties.

11.2 *Test Specimen Configuration*— Include a sketch of the test specimen configuration of reference to the specimen drawing.

11.3 *Test Specimen Dimensions*— State the actual measured dimensions for each test specimen.

11.4 *Test Fixture*— Describe the test fixture or refer to fixture drawings.

11.5 *Testing Machine and Twist Gages*— Include the manufacturer, make, model, serial number and load range of the testing machine and twist gages.

11.6 *Speed of Testing*— Record the test rate and mode of control.

11.7 *Temperature*— Record the temperature.

11.8 *Stress-Strain Diagram—Torque-Twist Deviation Diagram*— Include either the stress-strain diagram showing both shear stress and shear strain or the torque-twist deviation diagram showing both torque and twist deviation, with scales, specimen number, test data, rate and other pertinent information.

11.9 *Shear Modulus*—report the value as described in Section 8 or 10.

12. Precision and Bias

12.1 No interlaboratory test program is currently being conducted and there is presently no indication of what precision (repeatability or reproducibility) to expect. Furthermore there are no reference standards. Therefore no estimate of bias can be obtained.

12.2 Many parameters may be expected to influence the accuracy of this test method. Some of these parameters pertain to the uniformity of the specimen, for example, its straightness and eccentricity, the uniformity of its diameter, and, in the case of tubes, the uniformity of its wall thickness.

12.2.1 According to Eq 2 and Eq 3 (see 6.1 and 6.1.1), the variation in shear modulus ΔG due to variations in diameter ΔD are given by:

$$\frac{\Delta G}{G} = -4\,\frac{\Delta D}{D} \tag{7}$$

12.2.2 According to Eqs 2 and Eqs 4 (see 6.1 and 6.1.2) the variations in shear modulus ΔG due to variations in wall thickness Δt are given by:

$$\frac{\Delta G}{G} = -\,\frac{\Delta t}{t} \tag{8}$$

for a thin-walled tube for which t/D is small compared with unity where $t = (D_o - D_i)/2$.

12.3 Other parameters that may be expected to influence the accuracy of this test method pertain to the testing conditions, for example, alignment of the specimen, speed of testing, temperature, and errors in torque and twist values.

12.3.1 According to Eq 2 (see 6.1), the error in shear modulus ΔG due to errors in torque ΔT are given by:

$$\frac{\Delta G}{G} = \frac{\Delta T}{T} \tag{9}$$

12.3.2 According to Eq 2 (see 6.1), the error in shear modulus ΔG due to errors in angle of twist $\Delta\theta$ are given by:

$$\frac{\Delta G}{G} = -\,\frac{\Delta \theta}{\theta}. \tag{10}$$

The least count of the twist gage should always be smaller than the minimum acceptable value of $\Delta\theta$. In general, the overall precision that is required in twist data for the determination of shear modulus is of a higher order than that required of strain data for determinations of most mechanical properties, such as yield strength. It is of the same order of precision as that required of strain data for the determination of Young's modulus (see Method E 111).

NOTE 4—The committee welcomes task group participation in an interlaboratory study to develop such information if sufficient interest exists.

13. Keywords

13.1 shear modulus; stress-strain diagram; torque-twist diagram

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