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Standard Practice for Analysis of Strain Gage Rosette Data[1]

This standard is issued under the fixed designation E 1561; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

INTRODUCTION

There can be considerable confusion in interpreting and reporting the results of calculations involving strain gage rosettes, particularly when data are exchanged between different laboratories. Thus, it is necessary that users adopt a common convention for identifying the positions of the gages and for analyzing the data.

1. Scope

1.1 The two primary uses of three-element strain gage rosettes are (*a*) to determine the directions and magnitudes of the principal surface strains and (*b*) to determine residual stresses. Residual stresses are treated in a separate ASTM standard, Test Method E 837. This practice defines a reference axis for each of the two principal types of rosette configurations used and presents equations for data analysis. This is important for consistency in reporting results and for avoiding ambiguity in data analysis—especially when computers are used. There are several possible sets of equations, but the set presented here is perhaps the most common.

2. Referenced Documents

2.1 *ASTM Standards:*

E 6 Terminology Relating to Methods of Mechanical Testing[2]

E 837 Test Method for Determining Residual Stresses by the Hole-Drilling Strain-Gage Method[2]

3. Terminology

3.1 The terms in Terminology E 6 apply.

3.2 Additional terms and notation are as follows:

3.2.1 *reference line*—the axis of the *a* gage.

3.2.2 *a, b, c*—the three-strain gages making up the rosette. For the 0° – 45° – 90° rosette (Fig. 1) the axis of the *b* gage is located 45° counterclockwise from the *a* (reference line) axis and the *c* gage is located 90° counterclockwise from the *a* axis. For the 0° – 60° – 120° rosette (Fig. 2) the axis of the *b* gage is located 60° counterclockwise from the *a* axis and the *c* axis is located 120° counterclockwise from the *a* axis.

3.2.3 ϵ_a, ϵ_b, ϵ_c—the strains measured by gages *a*, *b*, and *c*, respectively, positive in tension and negative in compression. After corrections for thermal effects and transverse sensitivity have been made, the measured strains represent the surface strains at the site of the rosette. It is assumed here that the elastic modulus and thickness of the test specimen are such that mechanical reinforcement by the rosette are negligible. For test objects subjected to unknown combinations of bending and direct (membrane) stresses, the separate bending and membrane stresses can be obtained as shown in 4.4.

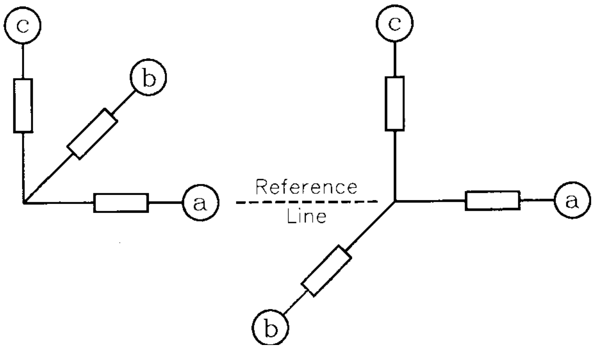

FIG. 1 0° – 45° – 90° Rosette

3.2.4 ϵ'_a, ϵ'_b, ϵ'_c—reduced membrane strain components (4.4).

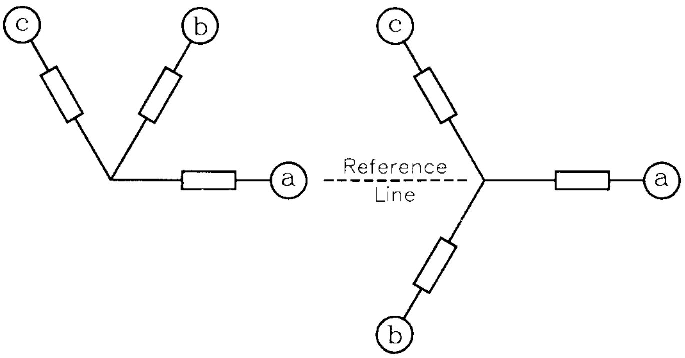

FIG. 2 0° – 60° – 120° Rosette

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[1] This practice is under the jurisdiction of ASTM Committee E-28 on Mechanical Testing and is the direct responsibility of Subcommittee E28.14 on Strain Gages.

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[2] *Annual Book of ASTM Standards*, Vol 03.01.

3.2.5 $\epsilon''_a, \epsilon''_b, \epsilon''_c$—reduced bending strain components (4.4).

3.2.6 ϵ_1—the calculated maximum (more tensile or less compressive) principal strain.

3.2.7 ϵ_2—the calculated minimum (less tensile or more compressive) principal strain.

3.2.8 γ_M—the calculated maximum shear strain.

3.2.9 θ_1—the angle from the reference line to the direction of ϵ_1. This angle is less than or equal to 180° in magnitude.

3.2.10 *C, R*—values used in the calculations. *C* is the location, along the ϵ-axis, of the center of the Mohr's circle for strain and *R* is the radius of that circle.

4. Procedure

4.1 Fig. 3 shows a typical Mohr's circle of strain for a 0° – 45° – 90° rosette. The calculations when ϵ_a, ϵ_b, ϵ_c, are given are:

$$C = \frac{\epsilon_a + \epsilon_c}{2} \qquad (1)$$

$$R = \sqrt{(\epsilon_a - C)^2 + (\epsilon_b - C)^2} \qquad (2)$$

$$\epsilon_1 = C + R \qquad (3)$$

$$\epsilon_2 = C - R$$

$$\gamma_M = 2R$$

$$\tan 2\theta_1 = 2\,(\epsilon_b - C)\,/\,\epsilon_a - \epsilon_c \qquad (4)$$

4.1.1 If $\epsilon_b < C$, then the ϵ_1-axis is clockwise from the reference line.

4.1.2 If $\epsilon_b > C$, then the ϵ_1-axis is counterclockwise from the reference line.

4.2 Fig. 7 shows a typical Mohr's circle of strain for a 0° – 60° – 120° rosette. The calculations when ϵ_a, ϵ_b, ϵ_c, are given are:

$$C = \frac{\epsilon_a + \epsilon_b + \epsilon_c}{3} \qquad (5)$$

$$R = \sqrt{2/3[(\epsilon_a - C)^2 + (\epsilon_b - C)^2 + (\epsilon_c - C)^2]} \qquad (6)$$

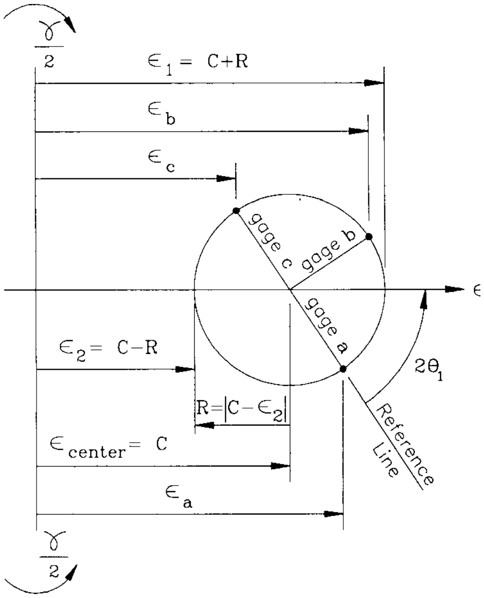

FIG. 3 Typical Mohr's Circle of Strain for a 0° – 45° – 90° Rosette

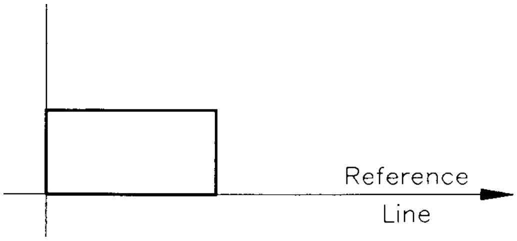

FIG. 4 Differential Element on the Undeformed Surface

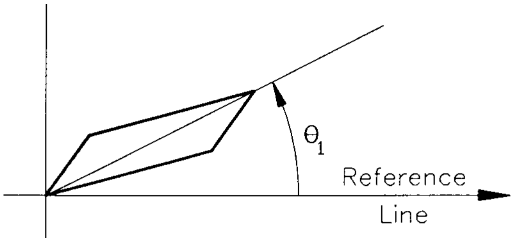

FIG. 5 Deformed Shape of Differential Element

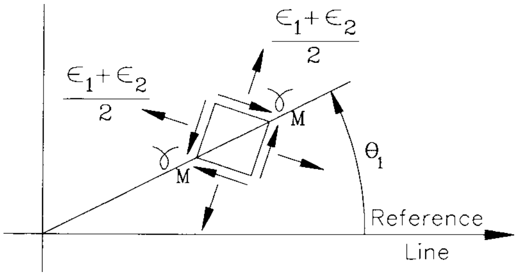

FIG. 6 Planes of Maximum Shear Strain

$$\epsilon_1 = C + R \qquad (7)$$

$$\epsilon_2 = C - R$$

$$\gamma_M = 2R$$

$$\tan 2\theta_1 = \frac{(\epsilon_b - \epsilon_c)}{\sqrt{3}(\epsilon_a - C)} \qquad (8)$$

4.2.1 If $\epsilon_c - \epsilon_b < 0$, then the ϵ_1-axis is counterclockwise from the reference line.

4.2.2 If $\epsilon_c - \epsilon_b = 0$, then $\theta_1 = 0°$.

4.2.3 If $\epsilon_c - \epsilon_b > 0$, then the ϵ_1-axis is clockwise from the reference line (see Note 1).

4.3 *Identification of the Maximum Principal Strain Direction*:

4.3.1 Care must be taken when determining the angle θ_1 using (Eq 4) or (Eq 8) so that the calculated angle refers to the direction of the maximum principal strain ϵ_1 rather than the minimum principal strain ϵ_2. Fig. 10 shows how the double angle $2\theta_1$ can be placed in its correct orientation relative to the reference line shown in Fig. 1 and Fig. 2. The terms "numerator" and "denominator" refer to the numerator and denominator of the right-hand sides of (Eq 4) and (Eq 8). When both numerator and denominator are positive, as shown in Fig. 10, the double angle $2\theta_1$ lies within the range $0° \leq 2\theta_1 \leq 90°$ counterclockwise of the reference line. Therefore, in this particular case, the corresponding angle θ_1 lies within the range $0° \leq \theta_1 \leq 45°$ counterclockwise of the reference line.

4.3.2 Several computer languages have arctangent functions that directly place the angle $2\theta_1$ in its correct orientation in

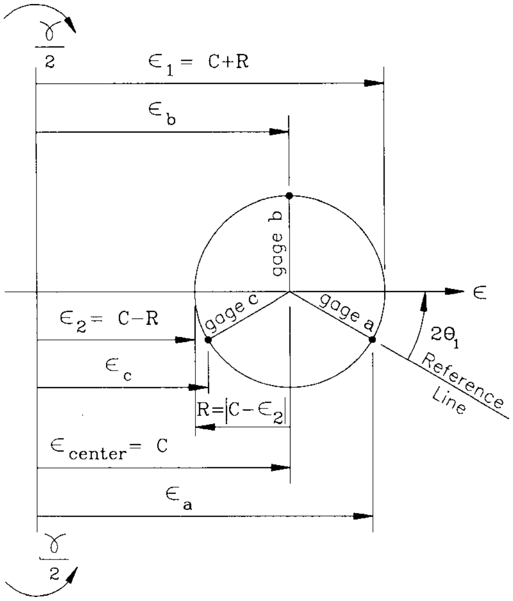

FIG. 7 Typical Mohr's Circle of Strain for a 0° – 60° – 120° Rosette

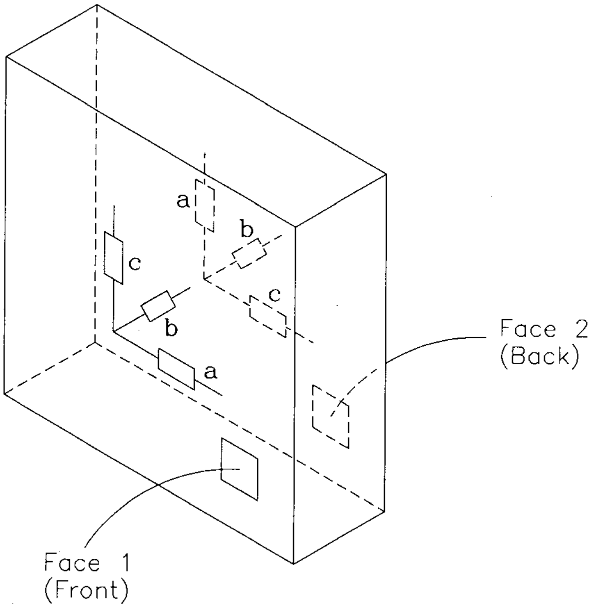

FIG. 8 Gage Labeling for Back-to-Back Rosettes

accordance with the scheme illustrated in Fig. 10. When working in Fortran or C, the two-argument arctangent functions ATAN2 or atan2 can be used for evaluating (Eq 4) and (Eq 8).

4.4 *Interpretation of Maximum Shear Strain*—Ordinarily the sense of the maximum shear strain is not significant when analyzing the behavior of isotropic materials. It can, however, be important for anisotropic materials, such as composites. Mohr's circle for strain can be used for interpretation of the sense of the shear strain. Fig. 3 shows a typical circle for a 0°–45°–90° rosette. A differential element along and perpendicular to the reference line is initially as shown in Fig. 4. Its deformed shape, corresponding to the assumed strains, is shown in Fig. 5. The planes of maximum shear strain are at 45° to the θ_1 direction as in Fig. 6 (see Note 2).

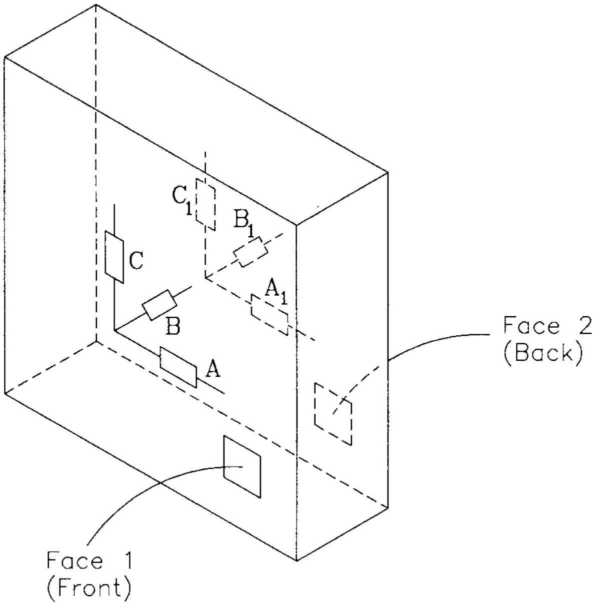

FIG. 9 Gage Labeling for Back-to-Back Rosettes

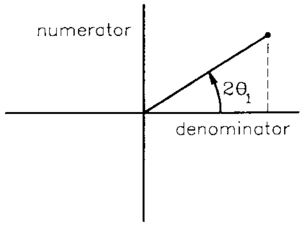

FIG. 10 Correct Placement of the Double Angle 2 θ_1

4.5 *Back-to-Back Rosettes*:

4.5.1 When the loading of a member or structure may introduce bending strains in the surface at the intended site of the rosette, back-to-back rosette installations are commonly employed, as shown in Fig. 8 and Fig. 9, to permit separate determination of the bending and membrane strains.

4.5.2 When rosettes are used on both sides of thin materials, the labeling alternatives are:

4.5.2.1 Label as in Fig. 8, which follows the sign convention of Fig. 1 and Fig. 2 as the observer faces each of the rosettes.

4.5.2.2 Label, for example, the gage on face 1 in the counterclockwise direction and the gage on face 2 in the clockwise direction, both as seen by an observer facing the rosette (see Fig. 9).

4.5.2.3 Labeling (4.5.2.1) requires no sign change in the data reduction equations or in the interpretation of the angles. Results are still interpreted as the observer faces the rosette.

4.5.2.4 Labeling as described in 4.5.2.2, wherein the observer fixes the *a* legs of the rosettes on both sides of the plate or skin to coincide in direction, is particularly convenient for the separation of bending and membrane strains. It also reduces the likelihood of a wiring or computational error which may occur in converting from the labeling in 4.5.2.1 to accomplish the basic purpose of back-to-back rosette installations. The following procedure is limited to test materials which are homogeneous in the thickness direction, or are symmetrically inhomogeneous with respect to the midpoint of the thickness, as in many laminated composite materials.

NOTE 1—The equations in 4.1 and 4.2 are derived from infinitesimal (linear) strain theory. They are very accurate for the low strain levels normally encountered in the stress analysis of typical metal test objects. They start to become detectably inaccurate for strain levels greater than about 1 %. Rosette data reduction for large strains is beyond the scope of this guide.

NOTE 2—The Mohr's circle for strain is constructed in generally the same manner as the Mohr's circle for stress. Normal strains, ϵ, are plotted as abscissae-positive for elongation and negative for contraction. One-half the shear strains, $\gamma/2$, are plotted as ordinates. If the shear strains on opposite sides of an element of area appear to form a clockwise couple, then $\gamma/2$ is plotted on the upper half of the axis. Similarly shear strains which appear to form a counterclockwise couple plot on the lower half. With this convention, angular directions on the circle are the same as angular directions on the specimen. See Fig. 3.

4.6 In those cases where the gages are not wired to automatically cancel the bending components of strain within the Wheatstone bridge circuit, the following relationships can be employed with the rosette labeling in Fig. 9 to separately determine the membrane and bending strain components.

4.6.1 For the membrane components of the strain (that is, the through-the-thickness uniform strains, after removing the superimposed bending strains):

$$\epsilon'_a = (\epsilon_A + \epsilon_{A1})/2 \tag{9}$$

$$\epsilon'_b = (\epsilon_B + \epsilon_{B1})/2 \tag{10}$$

$$\epsilon'_c = (\epsilon_C + \epsilon_{C1})/2 \tag{11}$$

4.6.2 For the bending components of strain, at both surfaces of the test object:

$$\epsilon''_a = \pm(\epsilon_A - \epsilon_{A1})/2 \tag{12}$$

$$\epsilon''_b = \pm(\epsilon_B - \epsilon_{B1})/2 \tag{13}$$

$$\epsilon''_c = \pm(\epsilon_C - \epsilon_{C1})/2 \tag{14}$$

where:

ϵ_a, ϵ_b, ϵ_c = reduced membrane strain components in the directions of the three rosette legs when labeled in accordance with Fig. 9.

ϵ''_a, ϵ''_b, ϵ''_c = reduced bending strain components in the directions of the three rosette legs when labeled in accordance with Fig. 9.

4.6.3 The strain terms in (Eq 9) through (Eq 14) with capitalized subscripts represent the measured strains (after customary corrections) from the corresponding rosette legs as shown in Fig. 9.

5. Report

5.1 The rosette data analysis may be part of the report on a test program. Report the following information:

5.1.1 Description of gages and measuring equipment,

5.1.2 Location and orientation of strain gage rosette,

5.1.3 Measured strains (corrected), and

5.1.4 Calculation of principal strains.

6. Keywords

6.1 bending strain; Mohr's circle for strain; rosette; shear strain; strain; strain gages; tensile strain