Designation: E 561 – 98

AMERICAN SOCIETY FOR TESTING AND MATERIALS
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Standard Practice for *R*-Curve Determination[1]

1. Scope

1.1 This practice covers the determination of resistance to fracturing of metallic materials by *R*-curves using either the center-cracked tension panel M(T), the compact specimen C(T), or the crack-line-wedge-loaded specimen C(W), to deliver applied stress intensity factor, *K*, to the material. An *R*-curve is a continuous record of toughness development in terms of K_R plotted against crack extension in the material as a crack is driven under a continuously increased stress intensity factor, *K*.

1.2 Materials that can be tested for *R*-curve development are not limited by strength, thickness, or toughness, so long as specimens are of sufficient size to remain predominantly elastic throughout the duration of the test.

1.3 Specimens of standard proportions are required, but size is variable, to be adjusted for yield strength and toughness of the materials.

1.4 Only three of the many possible specimen types that could be used to develop *R*-curves are covered in this practice.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

E 399 Test Method for Plane-Strain Fracture Toughness of Metallic Materials[2]

E 616 Terminology Relating to Fracture Testing[2]

E 647 Test Method for Measurement of Fatigue Crack Growth Rates[2]

3. Terminology

3.1 *Definitions:*

3.1.1 *crack size, a (*L)—a lineal measure of a principal planar dimension of a crack. This measure is commonly used in the calculation of quantities descriptive of the stress and displacement fields, and is often also termed crack length.

3.1.1.1 *Discussion*—In practice, the value of *a* is obtained from procedures for measurement of physical crack size, a_p, original crack size, a_o, and effective crack size, a_e, as appropriate to the situation being considered.

3.1.2 *physical crack size,* a_p (L)—the distance from a reference position to the observed crack front. This distance may represent an average from several measurements along the crack front. The reference position depends on the specimen form, and it is normally taken to be either the boundary, or a plane containing either the load line or the centerline of a specimen or plate.

3.1.3 *original crack size,* a_o (L)—the physical crack size at the start of testing.

3.1.4 *effective crack size,* a_e (L)—the physical crack size augmented for the effects of crack-tip plastic deformation.

3.1.4.1 *Discussion*—Sometimes the effective crack size, a_e, is calculated from a measured value of a physical crack size, a_p, plus a calculated value of a plastic-zone adjustment, r_Y. A preferred method for calculation of a_e compares compliance from the secant of a load-deflection trace with the elastic compliance from a calibration of the specimen.

3.1.5 *plastic-zone adjustment,* r_Y (L)—an addition to the physical crack size to account for plastic, crack-tip deformation effects on the linear-elastic stress field.

3.1.5.1 *Discussion*—Commonly the plastic-zone adjustment is given by:

$$r_Y = \frac{1}{2\pi}\frac{K^2}{{\sigma_Y}^2}\text{, for plane}-\text{stress mode 1, and}$$

$$r_Y = \frac{\alpha}{2\pi}\frac{K^2}{{\sigma_Y}^2}\text{, for plane}-\text{strain mode I,}$$

where $\alpha \simeq 1/3$ to $1/4$ and σ_Y is the effective yield strength.

In this practice, plane-stress mode 1 is assumed.

3.1.6 *crack extension,* Δa (L)—an increase in crack size.

3.1.6.1 *Discussion*—For example, Δa_p or Δa_e is the difference between the crack size, either a_p (physical crack size) or a_e (effective crack size), and a_o (original crack size).

3.1.7 *stress–intensity factor, K,* K_1, K_2, K_3 ($FL^{-3/2}$)—the magnitude of the ideal-crack-tip stress field (a stress-field singularity) for a particular mode in a homogeneous, linear-elastic body.

3.1.7.1 *Discussion*—Values of *K* for modes 1, 2, and 3 are given by:

$$K_1 = \underset{r \to o}{\text{limit}} \, [\sigma_y \, (2\,\pi\, r)^{1/2}],$$

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[1] This practice is under the jurisdiction of ASTM Committee E-8 on Fatigue and Fracture and is the direct responsibility of Subcommittee E08.07 on Linear–Elastic Fracture.

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[2] *Annual Book of ASTM Standards*, Vol 03.01.

$$K_2 = \lim_{r \to o} [\tau_{xy} (2 \pi r)^{1/2}], \text{ and}$$

$$K_3 = \lim_{r \to o} [\tau_{yz} (2 \pi r)^{1/2}].$$

where r = a distance directly forward from the crack tip to a location where the significant stress is calculated.

3.1.7.2 *Discussion*—In this practice, plane-stress mode 1 is assumed.

3.1.8 *crack-extension resistance,* K_R ($FL^{-3/2}$), and G_R or $J_R(FL^{-1})$—a measure of the resistance to crack extension expressed in the same units as the stress-intensity factor, *K*, the crack-extension force, *G*, or values of *J* derived using the J-integral concept.

3.1.8.1 *Discussion*—See definition of *R-curve.*

3.1.9 *R-curve*—a plot of crack-extension resistance as a function of slow-stable crack extension, Δa_p or Δa_e.

3.1.9.1 *Discussion*—For specimens discussed in Practice E 561, influence of in-plane geometry appears to be negligible, but *R*-curves normally depend upon specimen thickness and, for some materials, upon temperature and strain rate.

3.1.10 *crack displacement (*L)—the separation vector between two points (on the surfaces of a deformed crack) that were coincident on the surfaces of an ideal crack in the undeformed condition.

3.1.10.1 *Discussion*—In this practice, *displacement, v,* is total displacement as measured by clip gages or other devices spanning the crack. Measurement points on C(W) and C(T) specimens are identified as locations V_0, *V1,* and *V*2.

3.2 *Definitions of Terms Specific to This Standard:*

3.2.1 *plane-stress fracture toughness,* K_c—in Practice E 561, the value of K_R at the instability condition determined from the tangency between the *R*-curve and the applied *K* curve of the specimen.

3.2.1.1 *Discussion*—See the discussion of plane-strain fracture toughness in Terminology E 616.

3.2.2 *fixed-load or fixed-displacement applied K curves*—curves obtained from a fracture mechanics analysis for the test specimen configuration. Assume a fixed applied load or displacement, then generate a curve of *K* versus the crack size as the independent variable.

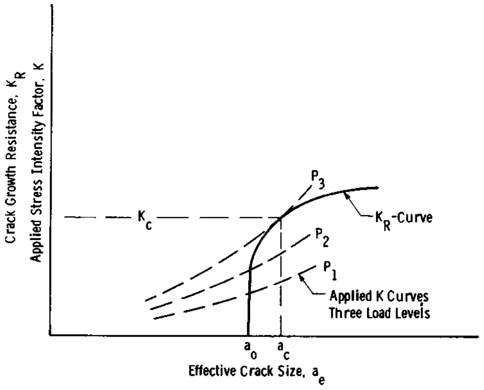

FIG. 1 Schematic Representation of R-Curve and Applied *K* Curves to Predict Instability; K_c, P_3, a_c, Corresponding to an Initial Crack Size, a_o

4. Summary of Practice

4.1 During slow-stable fracturing, the developing crack growth resistance, K_R, is equal to applied *K*. The crack is driven forward by increments of increased load or displacement. Measurements are made at each increment for calculation of *K* values which are individual data points lying on the *R*-curve for the material.

4.2 The crack starter is a low-stress-level fatigue crack.

4.3 Methods of measuring crack growth and of making plastic-zone corrections to the physical crack length are prescribed. Expressions for the calculation of crack-extension force are shown.

5. Significance and Use

5.1 *R*-curves characterize the resistance to fracture of materials during incremental slow-stable crack extension and result from growth of the plastic zone as the crack extends from a sharp notch. They provide a record of the toughness development as a crack is driven stably under increasing applied *K*. They are dependent upon specimen thickness, temperature, and strain rate.

5.2 For an untested geometry, the *R*-curve can be matched with the applied *K* curves to estimate the load necessary to cause unstable crack propagation at K_c **(1)**[3]. (See Fig. 1.) In making this estimate, *R*-curves are regarded as though they are independent of starting crack length, a_0, and the specimen configuration in which they are developed. For a given material, material thickness, and test temperature, they appear to be a function of crack extension, Δa, only (**2**). To predict crack instability in a component, the *R*-curve may be positioned as in Fig. 1 so that the origin coincides with the assumed initial crack length, a_0. Applied *K* curves for a given configuration can be generated by assuming applied loads or stresses and calculating applied *K* as a function of crack length using the appropriate expression for *K* of the configuration. The unique curve that develops tangency with the *R*-curve defines the critical load or stress that will cause onset of unstable fracturing.

5.3 If the *K*-gradient (slope of the applied *K* curve) of the specimen chosen to develop an *R*-curve has negative characteristics (Note 1), as in the crack-line-wedge-loaded specimen of this method, it may be possible to drive the crack until a maximum or plateau toughness level is reached **(3, 4, 5)**. When a specimen with positive *K*-gradient characteristics (Note 2) is used, the extent of the *R*-curve which can be developed is terminated when the crack becomes unstable.

NOTE 1—Fixed displacement in crack-line-loaded specimens result in a decrease of *K* with crack extension.

NOTE 2—With load control, *K* usually increases with crack extension and instability will occur at maximum load.

6. Apparatus

6.1 *Grips and Fixtures for Middle Cracked Tension Specimens,* M(T)—In the center-cracked tension tests, the grip fixtures are designed to develop uniform load distribution on the specimen. To ensure uniform stress entering the crack plane, when single pin grips are used, the length between the

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[3] The boldface numbers in parentheses refer to the list of references appended to this practice.

loading pins shall be at least three specimen widths, 3*W*. For panels wider than 12 in. (305 mm), multiple pin grips are mandatory, and because such fixtures deliver uniform stress, the length requirement (between the innermost row of pins) is relaxed to 1.5*W*. A typical grip arrangement shown in Fig. 2 has proven useful. Pin or gimbal connections are located between the grips and loading machine to aid the symmetry of loading. If extra-heavy-gage ultra-high-strength materials are to be tested, the suitability of the grip arrangement may be checked using the *AISC Steel Construction Manual*.

6.2 *Grips and Fixtures for Compact Specimens*, C(T)—The grips and fixtures described in Test Method E 399 are recommended for *R*-curve testing where C(T)-type specimens are loaded in tension.

6.3 *Fixtures for Crack-Line-Wedge-Loading*, C(W):

6.3.1 Where wedge loading is used, a low-taper-angle wedge with a polished finish and split-pin arrangement shown in Fig. 3 is used. Sketches of a segmented split-pin system which has proved effective for maintaining the load line independent of rotation of the specimen arms are provided in Fig. 4. It has been found convenient to use a wedge whose included angle is 3°. With proper lubrication and system alignment a mechanical advantage of five can be expected. Thus, a loading machine producing ⅕ the maximum expected test load will be adequate. The wedge must be long enough to develop the maximum expected crack-opening displacement. The maximum required stroke can be calculated from the maximum expected displacement *v*, using the *EBv/P* values found in Table 1, the maximum expected *K* level in the test, and the wedge angle.

6.3.2 The wedge-load blocks which drive the load sectors are constrained on top (not shown) and bottom to restrict motion to a plane parallel to the plane of the specimen. This allows the load to be applied or released conveniently without driving the load blocks and sectors out of the hole in the specimen. The wedge-load blocks are designed so that line contact exists between the wedge-load block and the load sector at a point that falls on the load line of the specimen. This enables the load sectors to rotate as the wedge is driven and the original load line is maintained. Any air- or oilhardening tool steel will be suitable for making the wedge and wedge-load blocks. A maraging 300-grade steel should be used for the load sectors. The diameter of the sectors shall be slightly smaller (nominally 1⁄32 in. (0.79 mm)) than the diameter of the drilled hole in the specimen.

6.4 *Face Plates to Prevent Sheet Buckling*—Buckling may develop in unsupported specimens depending upon the sheet thickness, material toughness, crack length, and specimen size. Buckling seriously affects the validity of a *K* analysis and is particularly troublesome when using compliance techniques to determine crack length. It is therefore required that rigid face plates be affixed to the M(T), C(T), and C(W) specimens in critical regions. A procedure for the detection of buckling using autographic records is described in 8.6.

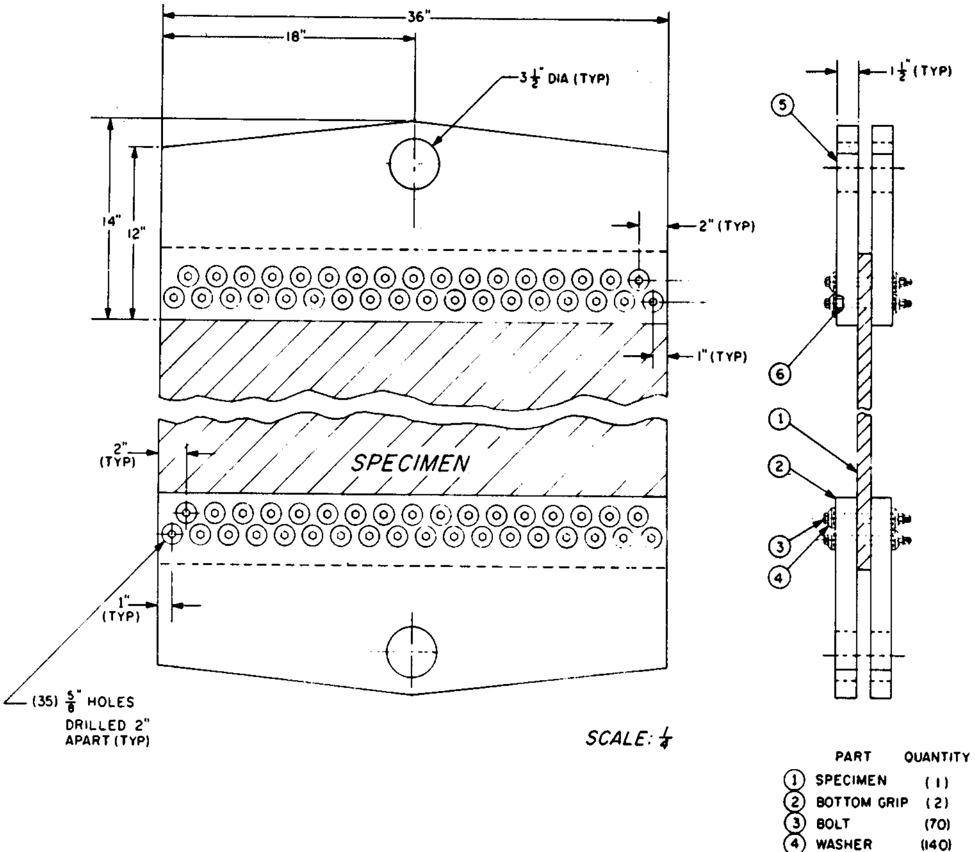


FIG. 2 Middle-Cracked Tension Panel Test Setup

TABLE 1 Dimensionless Stress Intensity Factors and Compliance in Plane Stress for the Recommended C(T) and C(W) Specimens

NOTE 1—$H/W = 0.6$.
V_1 at $0.1576W$ from loading pin centerline; see Fig. 8(a).
V_0 at $0.25W$ from loading pin centerline; see Fig. 8(a).

a/W^A	C(T) or C(W)	C(T) EBv/P^C		C(W) EBv/P^C	a/W^A	C(T) or C(W)	C(T) EBv/P^C		C(W) EBv/P^C
	$KBW^{1/2}/P^B$	at V_0	at V_1	at V_1		$KBW^{1/2}/P^B$	at V_0	at V_1	at V_1
.350	6.392	29.89	25.82	22.83	.480	9.093	50.15	44.31	41.52
.355	6.475	30.44	26.33	23.35	.485	9.230	51.24	45.30	42.52
.360	6.558	31.01	26.85	23.88	.490	9.369	52.36	46.33	43.55
.365	6.644	31.59	27.38	24.43	.495	9.512	53.51	47.38	44.61
.370	6.730	32.20	27.94	24.99	.500	9.659	54.71	48.48	45.70
.375	6.818	32.82	28.50	25.57	.505	9.810	55.93	49.60	46.83
.380	6.906	33.45	29.08	26.16	.510	9.964	57.20	50.76	47.99
.385	6.988	34.10	29.68	26.76	.515	10.123	58.51	51.95	49.18
.390	7.090	34.77	30.29	27.38	.520	10.286	59.86	53.19	50.42
.395	7.183	35.46	30.91	28.02	.525	10.453	61.25	54.47	51.70
.400	7.279	36.16	31.55	28.67	.530	10.625	62.70	55.78	53.02
.405	7.376	36.88	32.21	29.33	.535	10.802	64.18	57.15	54.38
.410	7.475	37.62	32.88	30.01	.540	10.984	65.72	58.56	55.79
.415	7.576	38.37	33.57	30.71	.545	11.172	67.32	60.01	57.24
.420	7.678	39.15	34.27	31.42	.550	11.364	68.96	61.52	58.75
.425	7.783	39.94	34.99	32.15	.555	11.583	70.67	63.08	60.31
.430	7.890	40.75	35.73	32.90	.560	11.767	72.43	64.70	61.92
.435	7.999	41.59	36.49	33.67	.565	11.978	74.25	66.37	63.60
.440	8.110	42.44	37.27	34.45	.570	12.195	76.14	68.10	65.32
.445	8.223	43.31	38.07	35.25	.575	12.420	78.10	69.89	67.12
.450	8.340	44.21	38.89	36.08	.580	12.651	80.12	71.74	68.97
.455	8.458	45.14	39.73	36.93	.585	12.890	82.22	73.66	70.89
.460	8.580	46.08	40.60	37.80	.590	13.136	84.40	75.65	72.88
.465	8.704	47.06	41.49	38.69	.595	13.391	86.64	77.72	74.94
.470	8.830	48.06	42.40	39.61	.600	13.654	88.98	79.85	77.07
.475	8.960	49.09	43.34	40.55					

[A] Inverted form from Ref. **(13)**:

$$a/W = C_0 + C_1(U) + C_2(U)^2 + C_3(U)^3 + C_4(U)^4 + C_5(U)^5$$

$$U = 1/[(EBv/P)^{1/2} + 1]$$

	C_0	C_1	C_2	C_3	C_4	C_5
C(T) at V_1	+1.0008	−4.4473	+15.400	−180.55	+870.92	−1411.3
C(T) at V_0	+1.0010	−4.6695	+18.460	−236.82	+1214.9	−2143.6

Accuracy is a; ±0.0005W

[B] From Refs. **(14, 15)**:

$$KWB^{1/2}/P = \frac{(2 + a/W)[0.886 + 4.64(a/W) - 13.32(a/W)^2 + 14.72(a/W)^3 - 5.6(a/W)^4]}{(1 - a/W)^{3/2}}$$

[C] From Ref. **(12)**:

$$EBv/P = A_0 + A_1(a/W) + A_2(a/W)^2 + A_3(a/W)^3 + A_4(a/W)^4$$

	A_0	A_1	A_2	A_3	A_4
C(T) at V_0	+120.7	−1065.3	+4098.0	−6688.0	+4450.5
C(T) at V_1	+103.8	−930.4	+3610.0	−5930.5	+3979.0
C(W) at V_1	+101.9	−948.9	+3691.5	−6064.0	+4054.0

Accuracy ±0.4 %% $0.35 < a/W < 0.60$

6.4.1 For the M(T) specimen, the buckling restraints shall be attached to the central portion of the specimen. The plates shall be so designed to prevent sheet kinking about the crack plane and sheet wrinkling along the specimen width.

6.4.2 For C(T) and C(W) specimens, the portion of the specimen arms and back edge which are in compression should be restrained from buckling. For sheet specimens it is convenient to use a base plate and cover plate with ports cut in the cover plate at appropriate locations for attaching clip gages and for crack length observations. Friction between buckling restraints and specimen faces is detrimental and should be minimized as much as possible.

6.4.3 Lubrication shall be provided between the face plates and specimen. Care shall be taken to keep lubricants out of the crack to avoid possible crack acceleration due to aggressive attack. Sheet TFE-fluorocarbon or heavy oils or both can be used. The initial clamping forces between opposing plates need not be excessive, but of the order of a few pounds.

6.5 *Displacement Gages*—Displacement gages are used to measure accurately the crack-opening displacement across the crack at a preselected location and span. In testing small C(W) and C(T) specimens, the gage recommended in Test Method E 399 may have a sufficient linear working range to be used. However, in testing larger specimens where W is larger than 5 in. (127 mm), displacements may be of such a magnitude that gages with greater working ranges of the type shown in Fig. 5 are needed. The use of point contacts eliminates error in the readings from the hinge-type rotation of C(T) and C(W) specimens. The precision of all types of gages shall be checked in accordance with the calibration procedure outlined in 6.3.2 of Test Method E 399. In addition, absolute accuracy within 2 % over the working range of the gage is required for use with compliance measurements. The gages shall be recalibrated periodically.

TABLE 2 Double Compliance Elastic Calibration Curve—CT and C(W) Specimens (12)

NOTE 1—Applicable only to the *V*1 and *V*2 locations shown in Fig. 8*a* and Fig. 8*b*.

a/w	$v1/v2^A$		*a/w*	$v1/v2^A$		*a/w*	$v1/v2^A$		*a/w*	$v1/v2^A$	
	C(W)	CT		C(W)	CT		C(W)	CT		C(W)	CT
0.350	4.74	5.56	0.415	3.27	3.67	0.480	2.72	2.96	0.545	2.42	2.56
0.355	4.54	5.25	0.420	3.22	3.59	0.485	2.70	2.92	0.550	2.40	2.53
0.360	4.36	5.00	0.425	3.16	3.53	0.490	2.67	2.88	0.555	2.38	2.50
0.365	4.24	4.78	0.430	3.11	3.46	0.495	2.64	2.85	0.560	2.36	2.48
0.370	4.09	4.62	0.435	3.06	3.39	0.500	2.62	2.81	0.565	2.34	2.46
0.375	3.97	4.47	0.440	3.02	3.33	0.505	2.59	2.78	0.570	2.32	2.44
0.380	3.85	4.33	0.445	2.97	3.27	0.510	2.57	2.74	0.575	2.31	2.42
0.385	3.74	4.22	0.450	2.93	3.22	0.515	2.54	2.71	0.580	2.29	2.40
0.390	3.64	4.11	0.455	2.89	3.17	0.520	2.52	2.68	0.585	2.27	2.38
0.395	3.55	4.01	0.460	2.85	3.13	0.525	2.50	2.66	0.590	2.25	2.36
0.400	3.47	3.91	0.465	2.82	3.08	0.530	2.48	2.63	0.595	2.24	2.35
0.405	3.39	3.82	0.470	2.79	3.04	0.535	2.46	2.60	0.600	2.23	2.33
0.410	3.33	3.75	0.475	2.76	3.00	0.540	2.44	2.58			

$^A v1/v2$ is moderately affected by clip gage span with less than ½ % error introduced by using 0.8-in. (20.3-mm) span instead of measurements on the crack line.

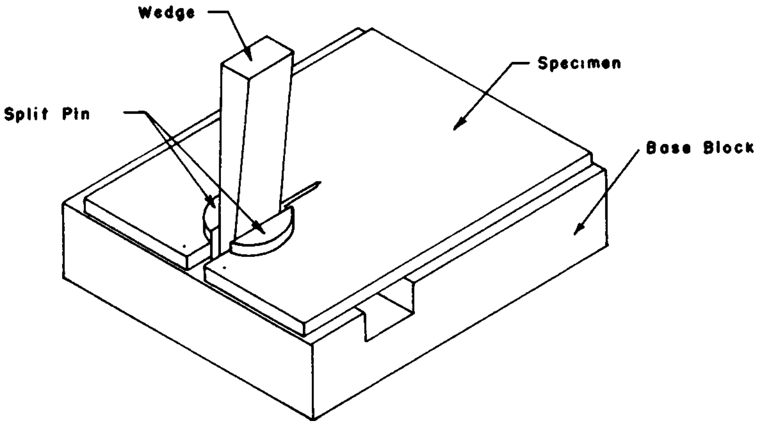


FIG. 3 Crack-Line-Loaded Specimen with Displacement-Controlled Wedge Loading

6.5.1 A recommended gage for use in M(T) panels, that is inserted into a drilled hole with a machined-in circular knife edge, is shown in Fig. 6 **(6)**. The diameter, d_i, is the gage length 2Y and it should be within 3 % of the dimension 2Y used in the calibration. Detail drawings on the gage are given in Fig. 7. Radius of the attachment tip should be less than the radius of the circular knife edge in the specimen. Proper construction techniques and required electronic procedures are specified in Test Method E 399.

6.5.2 The gage recommended in 6.5.1 is preferred from the standpoint of excellent linearity characteristics and ease of attachment. However, other types of gages used over different span lengths are equally acceptable provided the precision and accuracy requirements are retained. For example, the conventional clip gage of Test Method E 399 may be used with screw attached knife edges spanning the crack at a chosen span 2Y. In M(T) tests it is necessary to be cautious in choosing the proper compliance calibration curve to go with such arrangement, because compliance is a function of *Y/W.*

6.6 *Optical Equipment*—If the material being tested is sufficiently thin so that the crack-tip contour does not vary significantly from surface to midthickness, crack growth can be followed by surface observations using optical equipment. If load is sustained at given increments so that the crack stabilizes, crack length can be determined within 0.01 in. (0.2 mm) using a 30 to 50-power traveling-stage microscope. A movie camera recording system may be useful. A common technique is to record simultaneously load and crack growth using two synchronized cameras.

6.7 *Other Equipment*—Other methods of measuring crack length are available, such as eddy-current probes, which are most useful with nonferrous material, or electrical-resistance measurements, where the extension of the crack is determined from electrical potential differences.

7. Specimen Configuration, Dimensions, and Preparation

7.1 *Specimen Size*—In order for the *K* analysis to be valid, the specimen ligaments in the plane of the crack must be predominantly elastic at all values of applied load.

7.2 For the M(T) panel, the net section stress based on the physical crack size must be less than the yield strength of the material. The M(T) panel width, *W*, is optional provided the requirement of 7.1 is observed. The needed width to be below material yield may be estimated from the maximum expected plastic-zone size, r_Y (see 9.1.4), which is directly proportional to the square of the material toughness-to-yield strength ratio. As a guide, a specimen $27r_Y$ wide and ⅓ notched is expected to fail at a net section stress equal to the yield strength **(7)**. It therefore is desirable to have an estimate of the maximum *K* expected in the test before designing the specimen. As an aid, the following table lists minimum recommended M(T) sizes for assumed K_{max}-to-yield strength ratios.

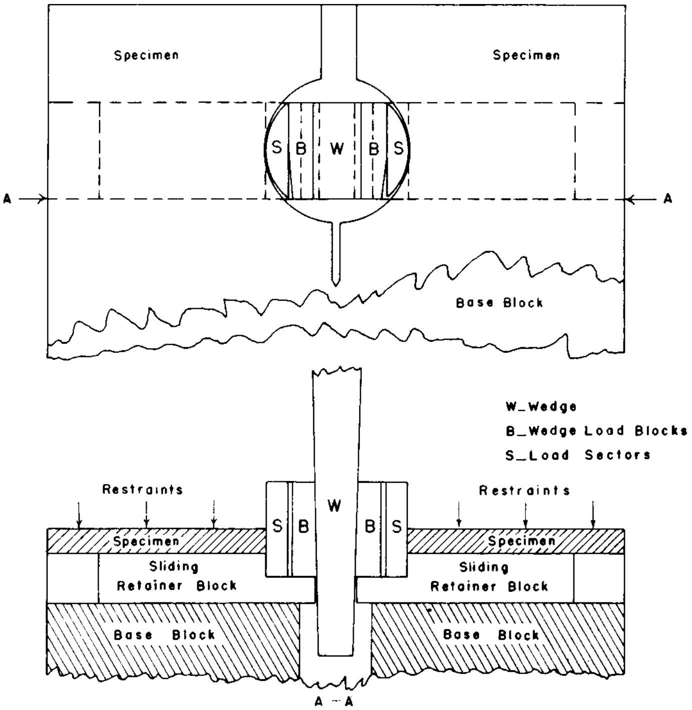


FIG. 4 Detail of Special Wedge and Split-Pin Setup Designed to Prevent Load-Line Shift

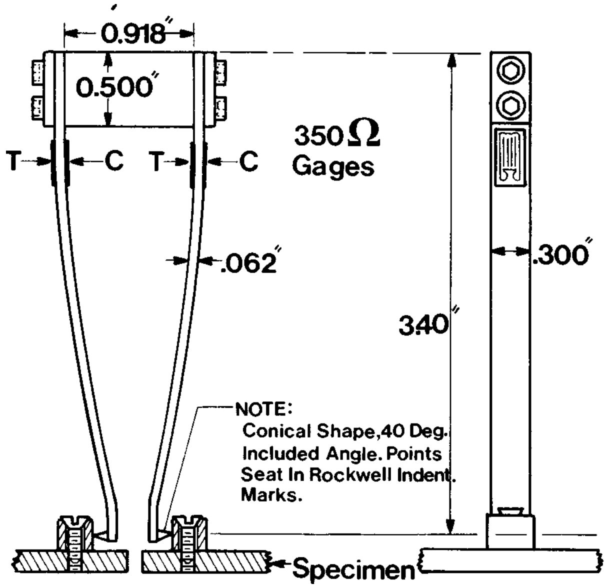


FIG. 5 Enlarged Clip Gage for Double Compliance Work

K_{max}/σ_Y, in.$^{1/2}$ (mm$^{1/2}$)	Width, in. (mm)	$2a_0$, in. (mm)	Length, in. (mm)[A]
0.5(0.80)	3.0(76)	1.0(25)	9(229)
1.0(1.6)	6.0(152)	2.0(51)	18(457)
1.5(2.4)	12.0(305)	4.0(102)	36(914)
2.0(3.2)	20.0(508)	6.7(170)	30(762)
3.0(4.8)	48.0(1219)	16.0(406)	72(1829)

[A] Length between pin centers of single pin loaded M(T) specimens is nominally 3*W*. Panels wider than 12 in. (305 mm) will require multiple pin grips and the length requirement is relaxed to 1.5*W*.

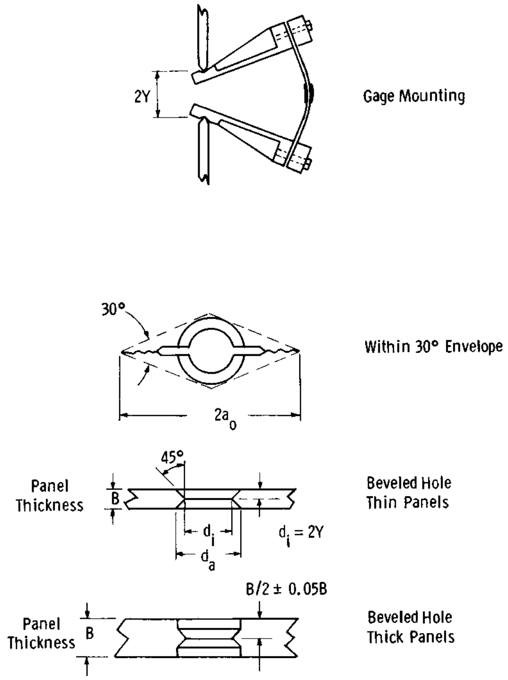


FIG. 6 Recommended Gage for Use in Drilled Hole M(T) Panels

7.3 The recommended C(T) specimen is shown in Fig. 8*a*. Crack-opening displacement is measured at a point $0.25W \pm 0.0006W$ or at $0.1576W$ in advance of the center line of the loading pins. Span of the gage is not critical so long as it is less than $W/4$. Alternative location of the gage is permitted but displacement values must be linearly extrapolated to $0.1576W$ in order to use the values given in Table 1 for compliance measurement.

7.4 The recommended C(W) specimen is shown in Fig. 8*b*. Hole size is proportioned according to specimen size. Some small amount of specimen brinelling at the hole can be tolerated. Clip gage placement is restricted to $0.1576W \pm 0.0006W$ in front and $0.303W \pm 0.0006W$ behind the load line.

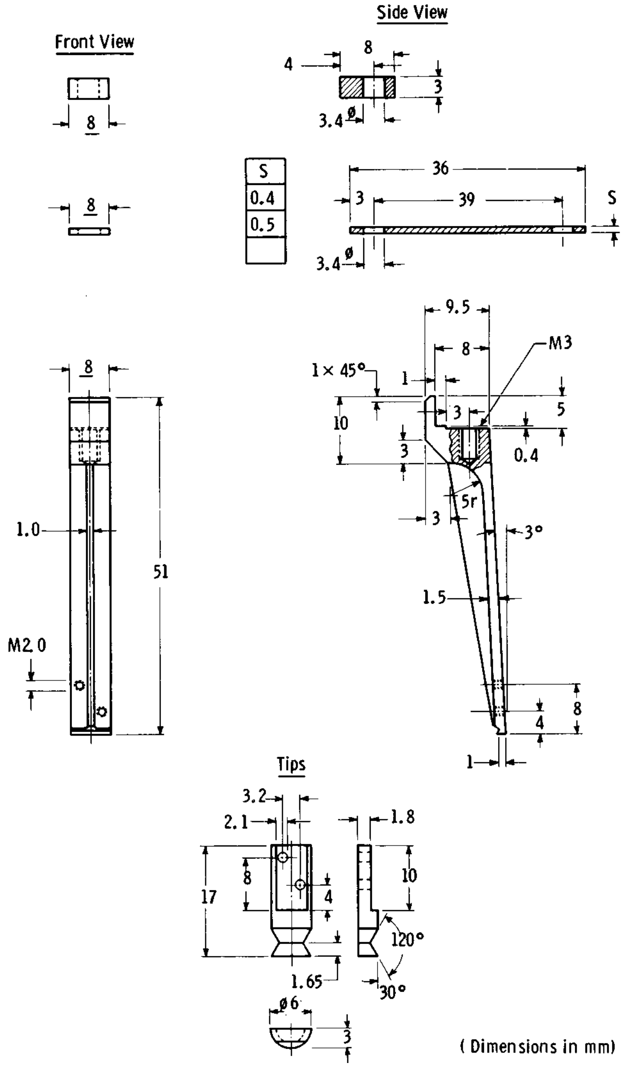


FIG. 7 Detail Drawings of M(T) Gage

Recommended gage span varies with specimen size as shown in the figure.

7.5 In order for a result to be considered valid for C(T) and C(W) specimens in accordance with this practice, it is required that the remaining uncracked ligament at the end of the test be at least equal to $(4/\pi)\,(K_{max}/\sigma_Y)^2$ where K_{max} is the maximum K level in the test and σ_Y is the 0.2 % offset yield strength of the material. As an aid, the following table lists maximum final crack size to width ratios:

$\frac{K_{max}}{\sigma_Y\sqrt{W}}$	Final a_p/W
0.686	0.40
0.657	0.45
0.626	0.50
0.594	0.55
0.560	0.60

Based on:

$$(W - a) \geq \frac{4}{\pi}\left(\frac{K_{max}}{\sigma_Y}\right)^2$$

The initial relative crack size, a_o/W, in C(T) and C(W) specimens shall be between 0.35 and 0.55. To maximize the valid data generation capacity of a specimen, choose the shortest possible initial crack size.

7.6 *Starting Notch*—The machined starter slot for any of the recommended specimens may be made by electrical-discharge machining, end milling, or saw cutting.

7.6.1 For the M(T) specimen, the machined notch shall be 25 to 40 % of W and shall be centered with respect to the specimen width within $0.002W$. The machined notch plus fatigue precrack shall be within the range of $0.25W$ to $0.4W$. It is advisable to have root radii at the ends of the slots of 0.003 in. (0.08 mm) or less to facilitate fatigue cracking. The starter slot must be extended by fatigue cracks not less than 0.05 in. (1.3 mm) in length (see Note 3). The slot must lie within an envelope described by Fig. 9.

NOTE 3—Fatigue cracks may be omitted only if it can be shown that the machined notch root radius effectively simulates the sharpness of a fatigue starter crack.

7.6.2 For the C(T) specimen, Fig. 10 shows the allowable notch types and envelope sizes. The machined slots must be extended by fatigue cracks not less than 0.05 in. (1.3 mm) in length.

7.7 All specimens shall be precracked in the final heat-treated condition. The length of the fatigue crack extension shall not be less than 0.05 in. (1.3 mm). Precracking may include two or more stages: crack initiation, intermediate propagation, and finishing. To avoid temporary growth retardation from a single step of load shielding, one or more intermediate levels may be added, if desired. The load reduction from the final intermediate stage to the finishing stage shall not be more than 30 %. The finishing stage shall be started at least 0.025 in. (0.65 mm) before the completion of precracking, and shall be performed at fixed cyclic load. The finishing stage shall be completed in no less than 5×10^3 cycles. Assuming 0.025 in. of finishing stage growth, more than 5×10^5 cycles to complete is unnecessarily conservative. As a guide, crack initiation can be started in most commercial materials at $K_{max}/E = 0.00083$ in.$^{1/2}$(0.00013 m$^{1/2}$). As a guide most commercial materials can be finished at $K_{max}/E = 0.0006$ in.$^{1/2}$ (0.0001 m$^{1/2}$). Stress ratio selection is optional, but R = 0.1 is most efficient and is recommended.

NOTE 4—Elastic (Young's) modulus, E, in units of ksi will yield K_{max} in units of ksi$\sqrt{\text{in.}}$ Elastic (Young's) modulus, E, in units of MPa will yield K_{max} in units of MPa$\sqrt{\text{m}}$.

8. Procedure

8.1 *Measurements*—Measure material thickness, B, to ±1 % of B at four locations near the crack plane. Measure specimen width, W, accurate to ±0.5 % of W.

8.2 *Number of Tests*—Replicate R-curves can be expected to vary as do other properties in mechanical tests such as Charpy-V energies or tensile properties. A curve plotted from a single determination may be a smoothly increasing function of crack extension, giving the impression that the single determination is an accurate representation. This is not necessarily so; make at least one additional confirming test.

8.3 *Loading Procedure*—Load the M(T), C(T), and C(W) specimens incrementally, allowing time between steps for the crack to stabilize before measuring load and crack length (see Note 5). Cracks stabilize in most materials within seconds of stopping the loading. However, when stopping near an instability condition, the crack may take several minutes to stabilize, depending upon the stiffness of the loading frame and other factors.

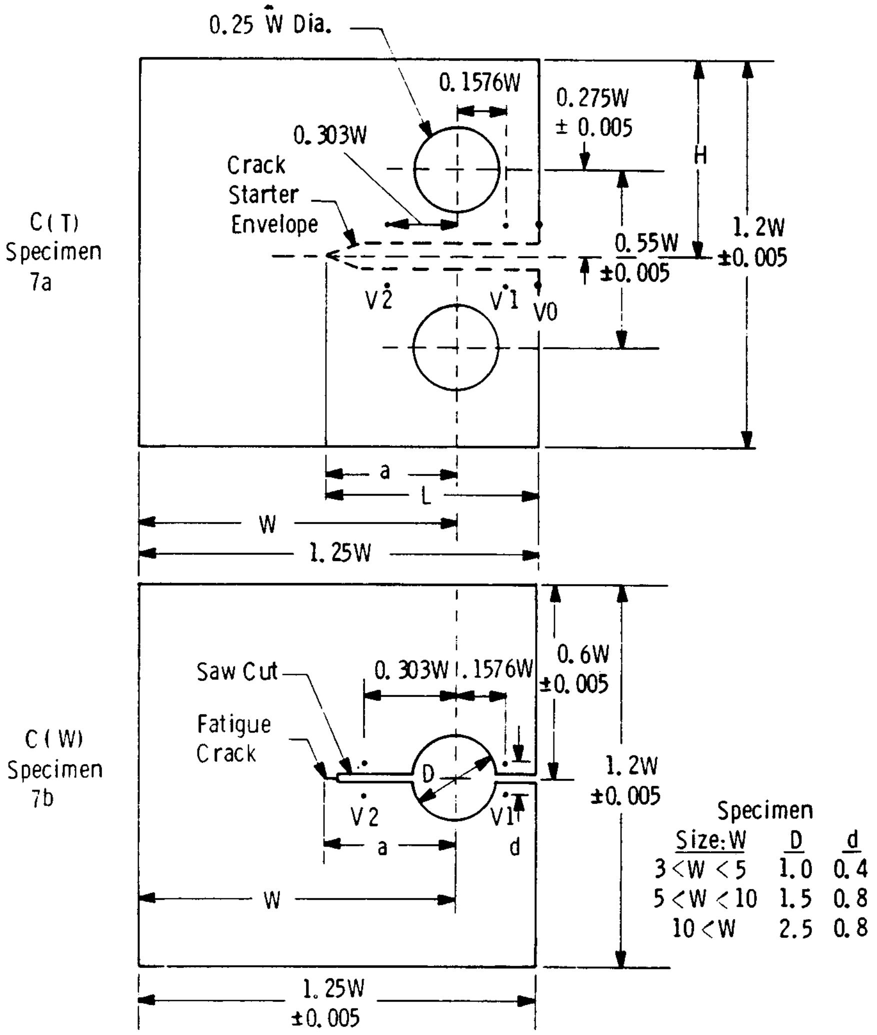


FIG. 8 Two Pin Hole Compact Tension and Crack-Line-Wedge-Loaded Compact Specimens

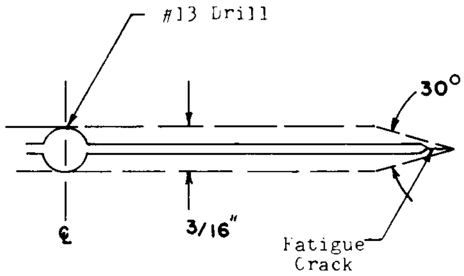


FIG. 9 Enlarged View of the Right Half of the Permitted Notch Envelope in M(T) Panels

NOTE 5—If autographic instrumentation is used, it is permitted to monitor load versus crack extension continuously under monotonic loading. Load rate must be slow enough so as not to introduce strain rate effects into the *R*-curve. Static K_R cannot be determined when the crack is steadily creeping or accelerating at or near instability.

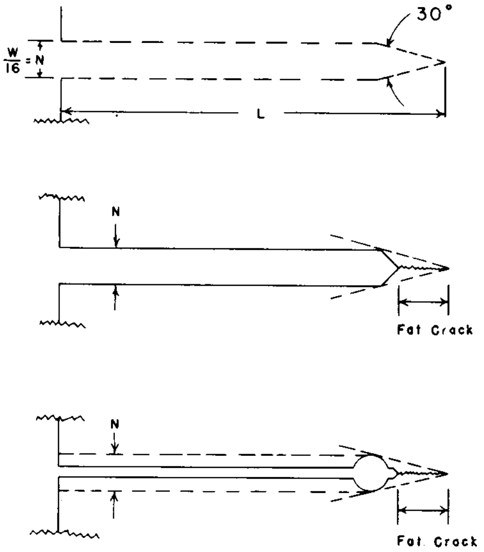
8.3.1 *Number of Data Points*—While *R*-curves can be developed with as few as four or five data points, ten to fifteen give improved confidence, and tougher materials usually require more data points.

8.4 *Physical Crack-Length Measurement,* a_p—Measure the physical crack length accurately to 0.01 in. (0.2 mm) at each step using suitable measuring devices described in 6.6 and 6.7. Physical crack length can also be measured with compliance techniques by partial unloading of the specimen after each increment, a technique described in 10.4. Adjust the physical crack length for plastic-zone, r_Y, to obtain effective crack length for calculating *K*.

8.4.1 In C(W) tests where the physical crack length is measured, determine the applied load or *K* from the relationship of Table 1 using an r_Y adjustment to crack length to enter the table. Since r_Y is a function of *K*, an iteration procedure is necessary.

8.5 *Effective Crack-Length Measurement,* a_e—Compliance measurements, *v/P*, made during the loading of specimens, can be used to determine effective crack length, a_e, directly. The crack is automatically plastic-zone corrected and these values can be used directly in the expressions for *K*.

8.5.1 Effective crack length can be determined directly in C(W) specimens using a double compliance technique (Note 6). By determining the displacements at two different locations, *V*1 and *V*2, along the crack line, as shown in Fig. 8*b*, an effective crack length-to-width ratio, a_e/W, can be found from the displacement ratio v_1/v_2 using Table 2. It is convenient to



NOTE 1—*N* need not be less than 1⁄16 in. (1.6 mm) but must not exceed *W*/10.

NOTE 2—The intersection of the crack-starter tips with the two specimen faces shall be equidistant from the top and bottom edges of the specimen within 0.005*W*.

FIG. 10 Envelope for Crack-Starter Notches and Examples of Notches Extended with Fatigue Cracks

plot autographically v_1 versus v_2 on an *X-Y* recorder at 100× and 200×, respectively. The load, *P*, can be calculated using a_e and displacement at *V*1 in conventional compliance relationships appearing in Table 1. In continuous *X-Y* plots, the wedge direction or load can be reversed at appropriate intervals to determine return slope $\Delta v/\Delta v_2$, which corresponds to physical crack length, using Table 2. In wedge systems, use a restraining jig to prevent withdrawal of the split pins along with the wedge.

NOTE 6—It is optional to use double compliance on C(T) specimens. The procedure is identical to that prescribed for C(W) testing, and effective crack lengths predicted should be identical to those predicted by single compliance. However, use the compliance relationships for C(T) loading as is noted in Table 1 and Table 2.

8.6 *Test Record Evaluation*—If compliance instrumentation is used, it is possible to determine when the specimen has developed undesirable buckling or when friction effects exist. The detection technique involves periodic partial unloading of the specimen as is shown schematically in Fig. 11 and Fig. 12. The initial part of the test record should have a linear portion which can be substantially retraced upon partial unloading. Should buckling or friction problems develop at some later stage in the test, the unloading and reloading slopes will tend to diverge. If the slopes differ by more than 2 % or if one or both have no linear range, then buckling or friction is present which is sufficient to cause significant error in compliance indicated crack lengths. Added confidence can be obtained by comparing the crack lengths predicted from return slopes, to physical crack length indicated with other more direct measurement methods.

8.7 Difficulties in the interpretation of test records will be encountered if the specimens are not flat prior to testing and if the plates contain regions of residual stress that are not negligible on a thickness average basis.

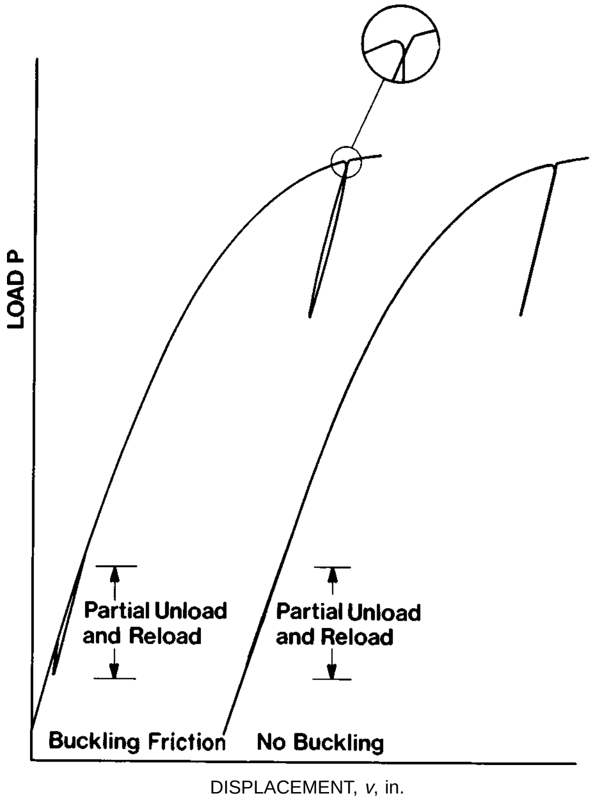


FIG. 11 Detection of Buckling from Compliance Test Records of M(T) and C(T) Specimens Compliance Test Records of M(T) and C(T) Specimens

8.8 *M(T) Specimen Testing*—Carefully align the specimens in the testing machine to eliminate eccentricity of loading. Misalignment can result in uncontrolled or spurious stress distribution in the specimen, which could be troublesome, particularly if compliance measurements are used to determine crack growth. Fixtures for measuring crack growth may be affixed to the specimen after applying a light preload. Starting crack length in a M(T) specimen is nominally 25 to 40 % of *W*, as established in 7.6.1. Measure this to the nearest 0.01 in. (0.2 mm).

8.9 *C(T) and C(W) Testing*—Starting crack length in a C(T) and C(W) specimen is nominally 35 to 55 % of *W*, as set forth in 7.5. The stress distribution in these crack-line-loaded types of specimens is such that the crack could deviate away from the original notch direction as the crack is driven **(8).** This is usually observed in materials that have appreciable anisotropy of toughness and where the crack is driven in the tougher direction. Accuracy of the elastic placement relationships decrease with deviation from the crack line; discard the data at deviation angles greater than 10°.

9. Calculation and Interpretation

9.1 To develop an *R*-curve, generate and use crack length and load data to calculate K_R.

9.1.1 For the middle-cracked tension specimen use either of the two following and equally appropriate expressions:

$$K_R = (P/WB)\sqrt{a}\cdot[1.77 - 0.177\,(2a/W) + 1.77\,(2a/W)^2]$$

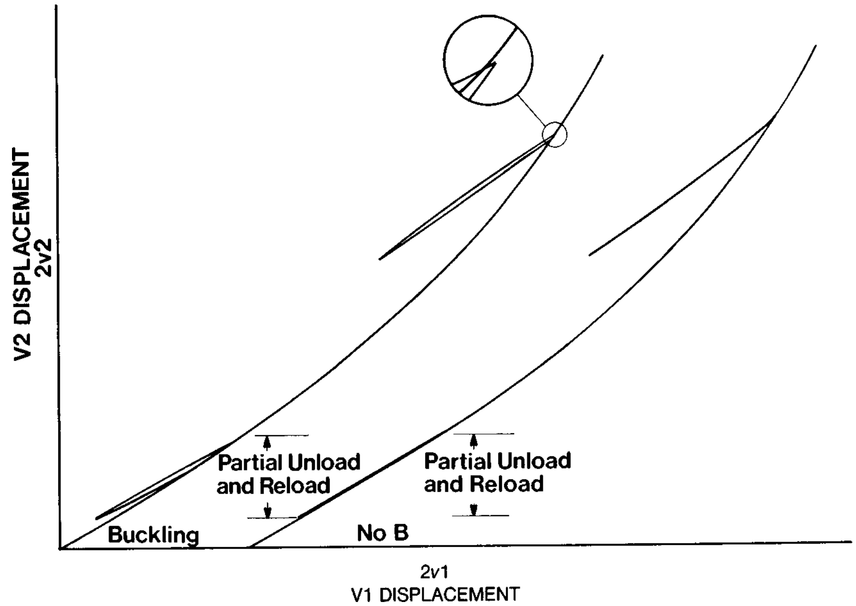


FIG. 12 Detection of Buckling from Double Compliance Test Records of C(W) Specimens

or

$$K_R = (P/WB)\,(\pi a \sec(\pi a/W))^{1/2}$$

where:

P = applied load,
B = specimen thickness,
W = total specimen width, and
a = a_e, the effective half crack size; the physical dimension plus plastic-zone adjustment.

9.1.2 For the C(T) and C(W) specimens, determine K_R as follows:

$$K_R = (P/B\sqrt{W}) \times f(a/W)$$

where:

$f(a/W)$ = $[(2 + a/W)/(1 - a/W)^{3/2}]\,[0.886 + 4.64\,(a/W) - 13.32\,(a/W)^2 + 14.72\,(a/W)^3 - 5.6\,(a/W)^4]$
Valid for any $a/W \geq 0.35$

a = plastic-zone corrected crack length measured from load line, a_e, and
W = specimen width measured from the load line.

9.1.3 Alternatively, values appearing in Table 1 may be used to calculate K_R.

9.1.4 The crack length used in the expressions of 9.1.1 and 9.1.2 is the effective crack length, which is the total physical crack length plus a correction for plastic zone, r_Y. Correct physically measured crack lengths as follows:

$$a_e = (a_0 + \Delta a_p + r_Y)$$

where:

a_0 = original half-crack length in an M(T) test or crack length in C(T) and C(W) tests,
Δa_p = physical crack growth at one crack tip, and
r_Y = plastic-zone adjustment

$$r_Y = (1/2\pi)(K_R^2/\sigma_Y^2)$$

9.1.5 The expression of 9.1.4 for r_Y is most accurate for high-strength materials of yield strength-to-density ratios above 700 000 psi/lb·in.$^{-3}$ (174 kPa/kg·m^{-3}). Lower-strength, high-toughness materials require increasing reliance on compliance methods to correct for plastic-zone effects.

10. Compliance Methods

10.1 *Determination of Effective Crack Length*—The compliance technique uses elastic-spring characteristics of the specimen calibrated over varied crack lengths **(9)**. A calibration curve may be developed experimentally by elastically loading specimens of varied crack sizes and determining the elastic reciprocal spring constant or reciprocal slopes of the various load versus displacement records, *v/P*. Normalize these reciprocal slopes for material thickness and elastic modulus and plot against crack length-to-specimen width ratio. An analytically developed expression for the compliance of the M(T) specimen, which can be used instead of an experimentally developed curve **(10)** is as follows:

$$\frac{E[v]}{\sigma W} = 2\{(\pi a/W)/\sin(\pi a/W)\}^{1/2} \left\{ \frac{2W}{\pi Y} \cosh^{-1}\left(\frac{\cosh \pi\, Y/W}{\cos \pi\, a/W}\right) - \frac{1+\mu}{\left[1 + \left(\frac{\sin \pi a/W}{\sinh \pi Y/W}\right)^2\right]^{1/2}} + \mu \right\} Y/W$$

$$\left(\text{valid for } 0.2 < \frac{2a}{W} < 0.8;\ \frac{Y}{W} \leq 0.5\right)$$

where:

E = Young's modulus,
v = center-opening displacement at center hole,
σ = gross stress, P/BW,
P = load,
B = specimen thickness,
W = total specimen width,
Y = half span of gage,
a = half-crack length, and

μ = Poisson's ratio.

10.2 The compliance calibration curve for a 16-in. (405-mm) wide M(T) panel using near-zero gage span is presented in Fig. 13. Note that the accompanying analytical curve for compliance was developed for a specific gage half-span-to-specimen width ratio, *Y/W.*

10.3 In testing to develop an *R*-curve, the test record of load versus clip-gage displacement for the M(T) and C(T) test, or the v_1 versus v_2 record for the C(W) test, will have an initial linear portion, the slope of which should correspond to the starting crack length in the specimen.

10.3.1 In M(T) and C(T) tests, determine the normalized compliance, *EBv/P* or *Ev/σW,* using the initial linear reciprocal slope of the test record, specimen thickness, and the best readily available value of the tensile elastic modulus, E, for the tested material. Look up the corresponding (predicted) crack size and if this value differs from the real crack size by more than 0.003*W,* a modulus adjustment is necessary. Determine a new value of E such that the normalized test record compliance predicts a crack size within 0.001*W* of the real crack size. This fitted modulus should not differ from an expected or theoretical value by more than 10 %, otherwise the test record is suspect and the data should be discarded. Make all subsequent crack size predictions (effective or physical) using the fitted modulus.

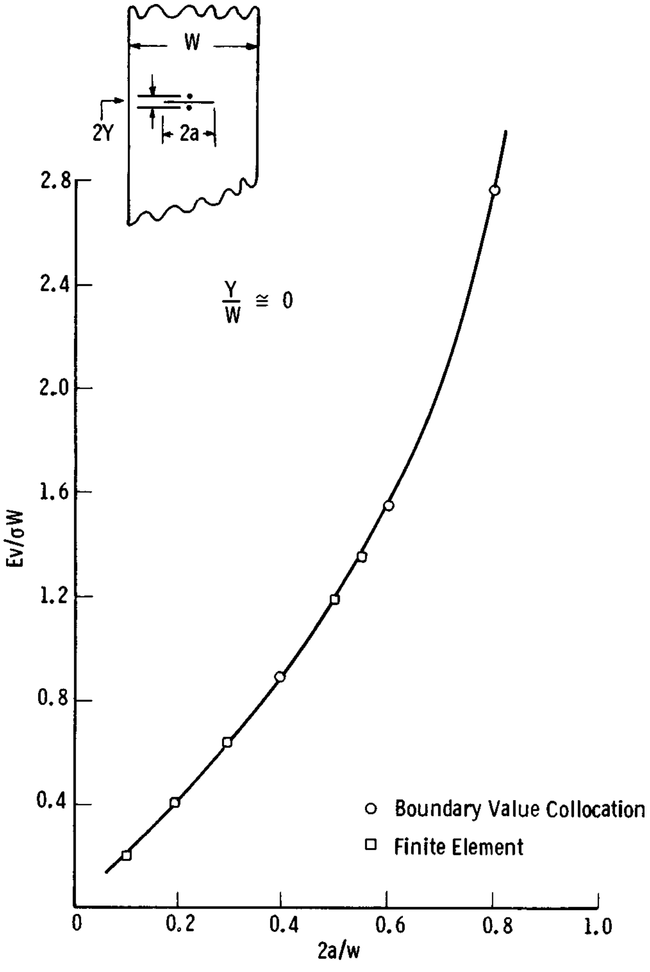


FIG. 13 Compliance Calibration Curve for a 16-in. Wide (405-mm) Middle-Cracked (M(T)) Panel with Near Zero Gage Span

NOTE 7—An alternative procedure that will give similar results is to use the above described values of *EBv/P* or *Ev/σW* and normalized initial crack size in the specimen, a_0/W, to establish one reference datum point for specimen compliance behavior (EBv/P, a_0/W). Plot the point and then adjust an overlay that contains the theoretical compliance curve to pass through this point. Normalized crack size is fixed at a_0/W and the adjustment is made on compliance only. Evaluation of crack growth increments in test are based upon progressive values of *EBv/P* applied to the transposed curve. This alternative procedure is not mathematically equivalent to the primary recommended procedure, but is acceptably comparable over an extensive change in *a/W* due to crack growth.

10.3.2 To develop an *R*-curve for either a M(T) or a C(T) test, draw secants to the test curve from the origin to arbitrarily selected points on the test record (load versus displacement) as shown in Fig. 14. The reciprocal slopes of these secants correspond to effective crack lengths at their points of intersection with the test record. Normalize the reciprocal slopes with fitted elastic modulus and material thickness and enter the calibration record to determine a_e/W.

10.4 An alternative procedure for M(T) specimens is to use normalized compliance in inverted form as is recommended in Test Method E 647. The following equations can be used, but it is important to recognize that they are an approximation to the more exact solution of paragraph 10.1. For greatest accuracy, determine an effective modulus, E_M, given an initial crack size $2a_o$, and initial elastic reciprocal slope, (v/P) of a test record:

$$X = -0.00929 + 0.96868(2a_o/W) - 0.402(2a_o/W)^2 + 0.44571(2a_o/W)^3 \quad (1)$$

$$E_M B(v/P) = \sqrt{4.584[\ln(1 - X)]^2 + (2Y/W)^2} \quad (2)$$

To predict crack size from compliance:

$$X = 1 - EXP\left[-\frac{\sqrt{(E_M Bv/P)^2 - (2Y/W)^2}}{2.141}\right] \quad (3)$$

$$2a/W = 1.2235X - 0.699032X^2 + 3.25584X^3 - 6.65042X^4 + 5.54X^5 - 1.66989X^6 \quad (4)$$

Accurate within 2 % for $0.3 < 2a/W < 0.8$ and $2Y/W < 0.5$.

10.5 In M(T) and C(T) tests, partial unloading at any given point in the test will result in a return slope different from the

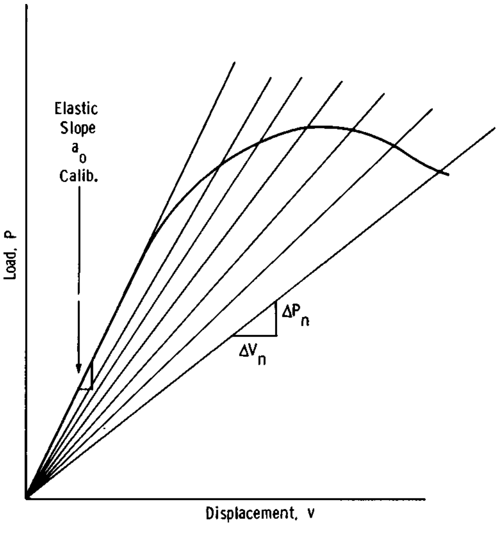


FIG. 14 Schematic Test Record for M(T) or C(T) Specimens

secant discussed in 10.3.2. The unloading slopes correspond to the physical crack length. This load reversal shall be only enough to establish the return slope accurately from which the physical crack length can be determined. Should the test record not return linearly immediately upon unloading, factors other than material behavior are influencing the test record and return slope measurements should be suspect.

10.6 In a C(W) test record **(11),** the initial linear relationship between displacements at locations *V*1 and *V*2 corresponds to the starting physical crack length in the specimen, and should be accurate within 0.005*W.* The *V*1/*V*2 double compliance calibration curve cannot be modulus adjusted as with the M(T) and C(T) specimen single compliance relationships. Despite possible error in prediction of initial crack length, a_0, the ability to determine increments of crack growth should remain unimpaired. However, if the starting crack length is in error by more than 3 % of a_0, the data shall be discarded and the test equipment checked for conformance to the requirements of this practice. Increments of crack growth are indicated by subtracting the compliance-indicated initial crack length from the crack lengths determined in succeeding increments.

10.7 Calculate K_R in accordance with expressions in 9.1.1 or 9.1.2 using compliance-determined effective crack lengths.

11. Report

11.1 Report the following information:

11.1.1 Type and size of specimen used,

11.1.2 Crack propagation direction (see Test Method E 399 for coding system),

11.1.3 Material thickness,

11.1.4 Yield strength, and

11.1.5 Percent oblique fracture (of value as supplementary information only).

11.2 The *R*-curve may be plotted in terms of either physical or effective crack extension. The legend shall contain the following information: (*a*) the method of plastic-zone adjustment to the physical crack length, and (*b*) whether the abscissa is given in terms of physical or effective crack extension. Instability predictions, the procedure for which is described in Section 4, can be made only from effective crack-extension plots.

12. Precision and Bias

12.1 The precision of *R*-Curve data is a complex synergistic function of the precision and accuracy of the instrumentation used, set up of the test fixtures, and the performance of the test. The latter is a matter of care and skill which cannot be prescribed in a standard method. An example of measurement precision that resulted from interlaboratory testing involving seven laboratories, each testing two materials, is given in Table 3. The two materials represent two levels of uniformity of behavior during stable crack growth; one presenting a slight tendency for crack pop-in. All laboratories participated with the compact, C(T), specimen, but plan-view size and initial crack size were varied as is allowed within the scope of this standard.

12.2 An *R*-Curve is not a single valued quantity, but a series of quantities dependent on crack growth. Hence, *R*-curves are not easily analyzed to statistical methods. Bias cannot be evaluated because there exists no reference value by which it is possible to identify a value of K_R at all of the possible levels of the effective crack growth, Δa_e.

TABLE 3 Variability in K_R at Four Selected Levels of Effective Crack Growth, Δ a_e Seven Labs.—Triplicate Tests

NOTE 1—The standard deviation has been pooled for all laboratories testing a given alloy. Data on the round robin results are on file at ASTM Headquarters, 1916 Race Street, Philadelphia, PA 19103. Request RR: E-24-1011.

	2024-T351 (σ_y = 48 ksi) K_R, ksi $\sqrt{in.}$			
Crack Growth, Δa_e	@ 0.1 in.	@ 0.2 in.	@ 0.3 in.	@ 0.4 in.
Grand Mean (21 specimens)	43.5	56.3	66.8	74.0
Standard Deviation	1.8	1.8	1.4	1.5
	7475-T7351 (σ_y = 59 ksi) K_R, ksi $\sqrt{in.}$			
Crack Growth, Δa_e	@ 0.1 in.	@ 0.2 in.	@ 0.3 in.	@ 0.4 in.
Grand Mean (20 specimens)	48.1	60.0	71.2	77.5
Standard Deviation	3.1	3.8	3.7	4.1

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