Designation: E 646 – ~~98~~00

Standard Test Method for Tensile Strain-Hardening Exponents (*n* -Values) of Metallic Sheet Materials[1]

This standard is issued under the fixed designation E 646; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

INTRODUCTION

This test method for determining tensile strain-hardening exponents *n* utilizes stress-stain data obtained in a uniaxial tension test. Tensile data are obtained in a continuous and rate-controlled manner via displacement or strain control. The strain-hardening exponents are determined from an empirical representation over the range of interest of the true-stress versus true-strain curve. The mathematical representation used in this method is a power curve (Note 1) of the form **(1)**[2]:

$$\sigma = K\epsilon^n$$

where:

σ = true stress,
ϵ = true plastic strain,
K = strength coefficient, and
n = strain-hardening exponent

1. Scope

1.1 This test method covers the determination of a strain-hardening exponent by tension testing of metallic sheet materials for which plastic-flow behavior obeys the power curve given in the Introduction.

NOTE 1—A single power curve may not fit the entire stress-strain curve between yield and necking. If such is the case, more than one value of the strain-hardening exponent can be obtained **(2)**.

1.2 This test method is for metallic sheet materials with thicknesses of at least 0.005 in. (0.13 mm) but not greater than 0.25 in. (6.4 mm).

1.3 The values stated in inch-pound units are to be regarded as the standard. The SI equivalents shown may be approximate.

1.4 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

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[1] This test method is under the jurisdiction of ASTM Committee E-28 on Mechanical Testing, and is the direct responsibility of Subcommittee E28.02 on Ductility and Flexure.

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[2] The boldface numbers in parentheses refer to the list of references appended to this method.

E 4 Practices for Force Verification of Testing Machines[3]
E 6 Terminology Relating to Methods of Mechanical Testing[3]
E 8 Test Methods for Tension Testing of Metallic Materials[3]
E 29 Practice for Using Significant Digits in Test Data to Determine Conformance with Specifications[4]
E 83 Practice for Verification and Classification of Extensometers[3]
E 177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods[4]

3. Terminology

3.1 *Definitions:*

3.1.1 The definitions of terms given in Terminology E 6 shall apply, with the addition of the following special terms used in this method.

3.1.2 *engineering strain (e)*—a dimensionless value that is the change in length (ΔL) per unit length of original linear dimension (L_0) along the loading axis of the specimen; that is, $e = (\Delta L)/L_0$.

3.1.3 *engineering stress (S)* [FL^{-2}]—the normal stress, expressed in units of applied force, F, per unit of original cross-sectional area, A_0; that is, $S = F/A_0$.

3.1.4 *necking*—the onset of nonuniform or localized plastic deformation, resulting in a localized reduction of cross-sectional area.

3.1.5 *strain-hardening (n)*—an increase in hardness and strength caused by plastic deformation.

3.1.6 *strength coefficient (K)* [FL^{-2}]—an experimental constant, computed from the fit of the data to the assumed power curve, that is numerically equal to the extrapolated value of true stress at a true strain of 1.00.

3.1.7 *true strain* (ϵ)—the natural logarithm of the ratio of instantaneous gage length, L, to the original gage length, L_0; that is, $\epsilon = 1n\,(L / L_0)$ or $\epsilon = 1n\,(1+e)$.

3.1.8 *true stress* (σ) [FL^{-2}]—the instantaneous normal stress, calculated on the basis of the instantaneous cross-sectional area, A; that is, $\sigma = F/A$; if no necking has occurred, $\sigma = S(1+e)$.

4. Summary of Test Method

4.1 This test method applies to materials exhibiting a continuous stress-strain curve in the plastic region. The displacement or strain is applied in a continuous and rate-controlled manner while the normal tensile load and strain are monitored. The instantaneous cross-sectional area may be monitored or calculated by assuming constancy of volume in the plastic region. Equations are presented that permit the calculation of the true stress, σ, true strain, ϵ, strain-hardening exponent, n, and strength coefficient, K, for that continuous portion of the true-stress versus true-strain curve which follows the empirical relationships described.

NOTE 2—The test method is recommended for use only in the plastic range for metallic sheet material for which the true-stress true-strain data follow the stated relationship.

5. Significance and Use

5.1 This test method is useful for estimating the strain at the onset of necking in a uniaxial tension test **(1)**. Practically, it provides an empirical parameter for appraising the relative stretch formability of similar metallic systems. The strain-hardening exponent is also a measure of the increase in strength of a material due to plastic deformation.

5.2 The strain-hardening exponent may be determined over the entire plastic stress-strain curve or any portion(s) of the stress-strain curve specified in a product specification.

NOTE 3—The strain interval 10–20% is commonly utilized for determining the n-value of formable low carbon steel products.

5.3 This test method is not intended to apply to any portion of the true-stress versus true-strain curve that exhibits discontinuous behavior; however, the method may be applied by curve-smoothing techniques as agreed upon.

NOTE ~~3~~4—For example, those portions of the stress-strain curves for mild steel or aluminum alloys which exhibit yield-point elongation or Lüders bands may be characterized as behaving discontinuously.

NOTE ~~4~~5—Caution should be observed in the use of curve-smoothing techniques as they may affect the *n*-value.

5.4 This test method is suitable for determining the tensile stress-strain response of metallic sheet materials in the plastic region prior to the onset of necking.

5.5 The *n*-value may vary with the displacement rate or strain rate used, depending on the metal and test temperature.

6. Apparatus

6.1 *Testing Machines*—Machines used for tension testing shall conform to the requirements of Practices E 4. The loads used to determine stress shall be within the loading range of the testing machine as defined in Practices E 4.

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[3] *Annual Book of ASTM Standards*, Vol 03.01.

[4] *Annual Book of ASTM Standards*, Vol 14.02.

6.2 *Strain-Measurement Equipment*—Equipment for measurement of extension shall conform to the requirements of Class C or better as defined in Practice E 83.

7. Sampling

7.1 Samples shall be taken from the material as specified in the applicable product specification.

8. Test Specimens

8.1 *Selection and Preparation of Specimens*:

8.1.1 In the selection of specimen blanks, special care shall be taken to assure obtaining representative material that is flat and uniform in thickness.

8.1.2 In the preparation of specimens, special care shall be taken to prevent the introduction of residual stresses.

8.2 *Dimensions*—One of the specimen configurations shown in Fig. 1 shall be used.

9. Procedure

9.1 Measure and record the original thickness of the reduced section of the specimen to at least the nearest 0.0005 in. (0.013 mm) and the width of the reduced section to at least the nearest 0.001 in. (0.025 mm).

NOTE ~~5~~6—The rounding-off method given in Practice E 29 shall be used for all measurements.

9.2 Grip the specimen in the testing machine in a manner to ensure axial alignment of the specimen as noted in Test Methods E 8 and attach the extensometer.

NOTE ~~6~~7—The order of this step may be reversed if required by the design of the extensometer or the specimen grips, or both.

9.3 *Speed of Testing*:

9.3.1 The speed of testing shall be such that the loads and strains are accurately indicated.

9.3.2 The test speed, defined in terms of rate of separation of heads during tests, free running crosshead speed, or rate of straining shall be between 0.05 and 0.50 in./in. (m/m) of the length of the reduced section per minute (see Test Methods E 8). The speed setting shall not be changed during the strain interval over which *n* is to be determined.

NOTE ~~7~~8—The mode of control and the rate used may affect the values obtained.

9.3.3 If the yield point, yield-point elongation, yield strength, or any combination of these is to be determined also, the rate of stress application or crosshead separation during this portion of the test shall be within the range permitted by Methods E 8 or any other specific value. After exceeding the strain necessary for this information, adjust the crosshead speed to within the range specified prior to the next step.

9.4 Record the load and corresponding strain for at least five approximately equally spaced levels of strain (Note~~ 9)~~ 10) encompassing the range of interest specified in the product specification. Usually, the greatest of these strains is at or slightly prior to the strain at which the maximum load occurs, and usually the lower bound of these strains is the yield strain (for continuous-yielding material) or the end of yield-point extension (for discontinuous-yielding material). See Fig. 2.

NOTE ~~8~~9—There is a statistical basis for points equally spaced in a reference frame.

NOTE ~~9~~10—The requirement that at least five load-strain data pairs be recorded is met with an autographic recording and the selection of five or more pairs from that curve.

NOTE 1~~0~~1—The test is not valid if less than five data pairs are obtained.

9.4.1 If multiple *n*-values are to be determined (Note 1), use at least five stress and strain values for the calculation of *n* in each interval of strain.

9.4.2 Other parameters may be recorded in place of loads and strains provided that they can ultimately be transformed into true stress and true strain at least as accurately as those measured using the techniques already described in this test method.

10. Calculations

10.1 Determine the strain-hardening exponent from the logarithmic form of the power curve representation of the true-stress versus true-strain curve within the plastic range (Note 1~~1~~2):

$$\log \sigma = \log K + n \log \epsilon$$

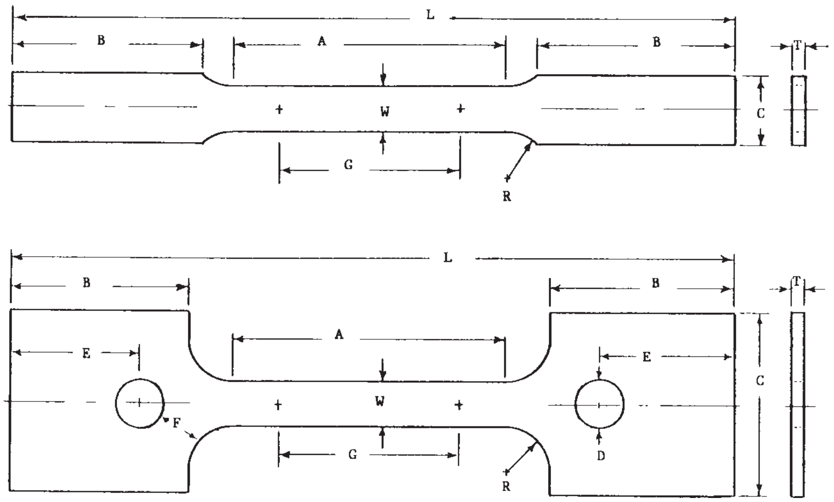
Calculate values of true stress and true strain from the following:

$$\text{True stress } \sigma = S(1+e)$$

$$\text{True strain } \epsilon = \ln(1+e)$$

where:

(σ, ϵ) = a true-stress versus true-strain pair in the selected interval,
S = engineering stress, and
e = engineering strain.



Dimensions

Required Dimensions for Reduced Section of Specimen		
	Dimensions	
	in.	mm
G Gage length	2.000 ± 0.005	50.0 ± 0.10
W Width (Note 1)	0.500 ± 0.010	12.5 ± 0.25
T Thickness (Note 2)	thickness of material	
R Radius of fillet, min	½	13
L Overall length, min	8	200
A Length of reduced section, min	2¼	60
B Length of grip section, min	2	50
Suggested Dimensions for Ends of Specimen		
"Plain-End" Specimens		
C Width of grip section (Note 3 and Note 4)	¾	20
"Pin-End" Specimens		
C Width of grip section, approximate (Note 5)	2	50
D Diameter of hole for pin (Note 6)	½	13
E Distance of center of pin from end, approximate	1½	38
F Distance of edge of hole from fillet, min	½	13

NOTE 1—The width of the reduced section shall be parallel to within ±0.001 in. (±0.025 mm).

NOTE 2—The thickness of the reduced section shall not vary by more than ±0.0005 in. (0.013 mm) or 1 %, whichever is larger, within the gage length, G.

NOTE 3—It is desirable, if possible, that the grip sections be long enough to extend into the grips a distance equal to two-thirds or more the length of the grips.

NOTE 4—Narrower grip sections may be used. If desired, the width may be 0.500± 0.010 in. (12.5 ± 0.25 mm) throughout the length of the specimen, but the requirement for dimensional tolerance in the central reduced section stated in Note 1 shall apply. The ends of the specimen shall be symmetrical with the center line of the reduced section within 0.01 in. (0.25 mm).

NOTE 5—The ends of the specimen shall be symmetrical with the center line of the reduced section within 0.01 in. (0.25 mm).

NOTE 6—Holes shall be on the centerline of the reduced section, within ±0.002 in. (±0.05 mm).

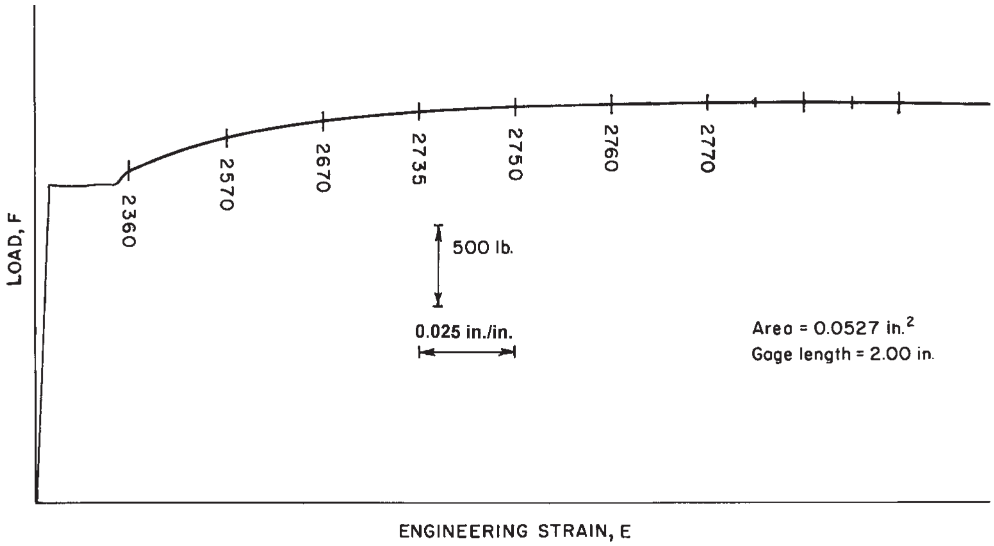
FIG. 1 Specimen for Determining *n* -Values

NOTE 1~~1~~2—Any logarithmic base may be used in these calculations unless otherwise noted. The use of the term "log" does not imply the use of base 10.

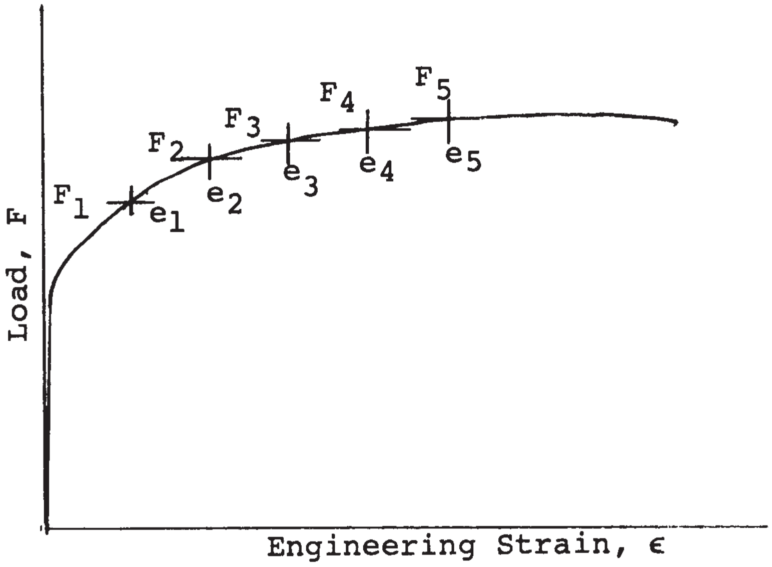
NOTE 1~~2~~3—For convenience when the elastic strain is less than 10 % of the total strain, it is not necessary to subtract the elastic strain. Elastic strain may be calculated by dividing the true stress by the nominal value of modulus of elasticity. All data pairs used to calculate an *n*-value must be treated in the same manner.

10.2 Obtain the logarithms of the true-stress versus true-strain pairs calculated in 10.1. From these paired sets of (log σ, log ϵ), calculate, via linear regression analysis of log σ versus log ϵ, the slope, *n*, and the standard error of the slope **(3, 4)**.

10.3 The equation for calculating the linear regression is as follows:



(*a*) *Material with initial discontinuous-yielding region*



(*b*) *Material with no discontinuous yielding*

FIG. 2 Examples Showing Load-Strain Data Pairs

$$n = \frac{N\sum_{i=1}^{N}(\log \epsilon_i \log \sigma_i) - (\sum_{i=1}^{N}\log \epsilon_i \sum_{i=1}^{N}\log \sigma_i)}{N(\log \epsilon_i)^2 - (\sum_{i=1}^{N}\log \epsilon_i)^2}$$

where N = the number of data pairs.

10.4 The equations for calculating the slope, the intercept, and the standard deviation of the slope are made convenient by symbolic representations as follows:

y = log σ N = number of data pairs
x = log ϵ SD = standard deviation of the *n*-value
b = log K n = *n*-value

10.5 The equation for the slope of the linear regression line provides the strain-hardening exponent as follows.

$$n = \frac{N\,\Sigma xy - \Sigma x \Sigma y}{N\,\Sigma x^2 - (\Sigma x)^2}$$

NOTE 1~~3~~4—Ref **(5)** is one source for the above equation and the basis for others that follow.

10.6 The equations for the strength coefficient (5.7) are as follows.

$$b = \frac{\Sigma y - n\,\Sigma x}{NM}$$

$$K = \exp\,[b]$$

10.7 The calculation of the standard deviation of the *n*-value is based upon the variance of the slope of the regression line. This measure of variability contains the computed *n*-value (10.5) and the computed strength coefficient (10.6).

$$\mathrm{SD} = \left\{\frac{\Sigma(y - b - nx)^2}{N\Sigma x^2 - (\Sigma x)^2} \times \frac{N}{N - 2}\right\}^{1/2}$$

10.8 An example of a worksheet for manually calculating these values is found in Appendix X1.

11. Report

11.1 The report shall include the following:

11.1.1 The material represented by commercial standard nomenclature. Materials that have no commercial standard shall be so indicated.

11.1.2 The strain interval(s) over which the *n*-value(s) were determined.

11.1.2.1 The *n*-value and its associated standard error, as reported, shall be considered applicable only over the strain interval for which it was determined.

11.1.2.2 The number of data pairs selected for computing the *n*-value.

11.1.3 The direction of testing relative to the principal rolling direction.

11.1.4 Precision and bias in accordance with Section 12.

11.1.5 Any special conditions that are believed to have affected the test result.

12. Precision and Bias [5]

12.1 The precision of a reported *n*-value depends on the precision and bias of the original stress and strain data.

12.2 The relative precision may be expressed as the coefficient of variation, COV; that is, the ratio of the standard deviation of *n* to its estimated value.

12.2.1 The standard deviation of *n* depends on the level of strain over which *n* is being determined and on the number of points, *N*, employed in the linear regression analysis (see 10.4 and Recommended Practice E 177).

12.3 No estimate of bias of an *n*-value is possible by presupposed experience. Neither are such bias estimates possible without examination or analysis of the data used in its resolution.

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[5] Supporting data available from ASTM Headquarters. Request RR:-E28–1002.

APPENDIX

(Nonmandatory Information)

X1. BASIC WORKSHEET FOR THE LINEAR RELATIONSHIP

X1.1 Table X1.1The sequence of steps in Table X1.2 develops in the logic of hand computations. Likewise, the same or similar steps exist in most computer programs.

X1.2 Common mini-computers exist to provide logarithmic transformations. Statistical models may readily provide slope and intercept values, but may not immediately provide intercept and slope variances.

X1.3 If data are manually recorded, sufficient decimal places must be carried to avoid losing significant figures in the subtraction of Steps (1) through (9) in Table X1.2.

TABLE X1.1 An Example of Calculating the Strain-Hardening Exponent, Strength Coefficient and Standard Deviation

Tabulation of Data from Test and Preliminary Evaluation

Data Pair	Load, P, kips[A]	Engineering Stress, ksi[B]	True Stress, σ, ksi[C]	Y $\log_{10}\sigma$	Y^2	Extension, in[A]	Engineering Strain, ϵ[D]	True Strain, ϵ[E]	X $\log_{10}\epsilon$	X^2	XY
1	2.360	44.81	45.93	1.66209	2.76254	0.050	0.025	0.02469	−1.60743	2.58384	−2.67170
2	2.570	48.80	51.24	1.70958	2.92265	0.100	0.050	0.04879	−1.31167	1.72047	−2.24240
3	2.670	50.69	54.50	1.73637	3.01499	0.150	0.075	0.07232	−1.14074	1.30128	−1.98075
4	2.735	51.93	57.12	1.75680	3.08636	0.200	0.100	0.09531	−1.02086	1.04216	−1.79345
5	2.750	52.21	58.74	1.76894	3.12914	0.250	0.125	0.11778	−0.92892	0.86289	−1.64320
6	2.760	52.40	60.26	1.78006	3.16861	0.300	0.150	0.13976	−0.85461	0.73036	−1.52126
7	2.770	52.59	61.80	1.79097	3.20758	0.350	0.175	0.16127	−0.79245	0.62798	−1.41926
$N = 7$				$\Sigma Y = 12.20481$; $\Sigma Y^2 = 21.29187$					$\Sigma X = -7.65668$; $\Sigma X^2 = 8.86898$; $\Sigma XY = -13.27200$		

[A]—Values are obtained from Fig. 2a.

[B]—Area = 0.504 × 0.1045 = 0.052668 in.^{2}

[C]—True stress = (engineering stress) × (1 + engineering strain).

[D]—Engineering strain = (extension) ÷ gage length; gage length = 2.00 in.

[E]—True strain = ln (1 + engineering strain). Total strain has been used. Elastic strain was not subtracted in this example.

All calculations shown were performed on a calculator that uses 10 significant figures although only five places to the right of the decimal were displayed.

TABLE X1.2 An Example of a Worksheet for Calculating the Strain Hardening Exponent, Strength Coefficient and Standard Deviation

where:

X denotes log ϵ

Y denotes log σ

n denotes strain hardening exponent

b denotes log of the strength coefficient

Data operated upon in this example are taken from Fig. 2a and evaluated in Table X1.1. The number of data-pairs, N, is 7. All logarithms used in the example are base 10.

From Table X1.1:

$$\Sigma X = \Sigma(\log \epsilon) = -7.65668; \bar{X} = \frac{\Sigma X}{N} = \frac{-7.65668}{7} = -1.09381$$

$$\Sigma X^2 = 8.86898$$

$$\Sigma Y = \Sigma(\log \sigma) = 12.20481; \bar{Y} = \frac{\Sigma Y}{N} = \frac{12.20481}{7} = 1.74354$$

$$\Sigma Y^2 = 21.29187 \text{ and}$$

$$\Sigma X \times Y = -13.27200$$

The calculations for n and b:

$$\frac{\Sigma X \times \Sigma Y}{N} = \frac{(-7.65668)(12.20481)}{7} = -13.34976 \qquad \text{Step 1}$$

$$Sxy = \Sigma XY - \text{Step 1} = -13.26999 - (-13.349767) = 0.07776 \qquad \text{Step 2}$$

$$\frac{(\Sigma X)^2}{N} = \frac{(-7.65668)^2}{7} = 8.37496 \qquad \text{Step 3}$$

$$Sxx = \Sigma X^2 - \text{Step 3} = 8.86898 - 8.37496 = 0.49402 \qquad \text{Step 4}$$

$$n = \frac{Sxy}{Sxx} = \frac{\text{Step 2}}{\text{Step 4}} = \frac{0.07776}{0.49402} = 0.15739 = 0.157 \qquad \text{Step 5}$$

$$n\bar{X} = (0.15739)(-1.09381) = -0.17216 \qquad \text{Step 6}$$

$$b = \bar{Y} - n\bar{X} = 1.74354 - \text{Step 6} = 1.74354 - (-0.17216) = 1.91570 \qquad \text{Step 7}$$

$$K = 10^b = 10^{1.91570} = 82.35690 = 82.36; \text{ see Note 1} \qquad \text{Step 7a}$$

X1.1.3 The calculations for the standard deviation

$$\frac{(Sxy)^2}{Sxx} = \text{Step 5} \times \text{Step 2} = (0.15739) \times (0.07776) = 0.0122386; \text{ see X1.3} \qquad \text{Step 8}$$

$$Syy = \Sigma Y^2 - \frac{(\Sigma Y)^2}{N} = 21.29187 - \frac{(12.20481)^2}{7} = 0.0122433 \qquad \text{Step 9}$$

$$S^2y = \frac{\text{Step 9} - \text{Step 8}}{N-2} = \frac{0.01224 - 0.01224}{7-2} = \frac{4.7 \times 10^{-6}}{5} = 9.4 \times 10^{-7} \qquad \text{Step 10}$$

$$S_n^2 = \frac{S^2y}{Sxx} = \frac{\text{Step 10}}{\text{Step 4}} = \frac{9.4 \times 10^{-7}}{4.9402 \times 10^{-1}} = 1.9 \times 10^{-6} \qquad \text{Step 11}$$

$$\text{Standard deviation} = S_n = \sqrt{\text{Step 11}} = 1.3 \times 10^{-3} \qquad \text{Step 12}$$

$$S_b^2 = S^2y \left\{\frac{1}{N} + \frac{\bar{X}^2}{Sxx}\right\} = 9.4 \times 10^{-7} \left\{\frac{1}{7} + \frac{(-1.09381)^2}{0.49402}\right\} = 2.41078 \times 10^{-6}$$

$$S_b = 1.55 \times 10^{-3}$$

REFERENCES

(1) Kleemola, H. J., and Nieminen, M. A., "On the Strain-Hardening Parameters of Metals," *Metallurgical Transactions,* Vol 5, August 1974, pp. 1863–1866.

(2) Morrison, W. B., "Effect of Grain Size on the Stress-Strain Relationship of Low Carbon Steel," *Transactions of the American Society for Metals,* TASEA, Vol 59, 1966, pp. 824–846.

(3) Crow, E. L., Davis, F. A., and Maxfield, M. W.," Statistics Manual," U.S. Naval Ordinance Test Station, *NAVORD Report 3369,* NOTS 948, 1955, p. 165.

(4) Bowker and Lieberman, *Engineering Statistics,* Prentice-Hall, Inc., Englewood Cliffs, NJ, 1959, Chapter 9, Fitting Straight Lines.

(5) Volk, William, *Applied Statistics for Engineers,* McGraw-Hill Book Company, Inc., New York, NY, 1958.

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