Designation: D 4460 – 97 (Reapproved 2004)

Standard Practice for Calculating Precision Limits Where Values are Calculated from Other Test Methods[1]

This standard is issued under the fixed designation D 4460; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice describes techniques for calculating precision limits when values are calculated from two other methods having precision limits.

1.2 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:* [2]

D 1188 Test Method for Bulk Specific Gravity and Density of Compacted Bituminous Mixtures Using Paraffin-Coated Specimens

D 2041 Test Method for Theoretical Maximum Specific Gravity and Density of Bituminous Paving Mixtures

D 3203 Test Method for Percent Air Voids in Compacted Dense and Open Bituminous Paving Mixtures

E 177 Practice for Use of the Terms Precision and Bias in ASTM Test Methods

3. Terminology Definitions

3.1 For definitions of terms used in this document, consult Practice E 177, or a standard dictionary, or a statistical text from Refs. **(1, 2, and 3)**.[3]

4. Significance and Use

4.1 Precision limits for a test result which is calculated by addition, subtraction, multiplication, or division of two other test results that have valid precision limits can be calculated directly. This saves the cost and delay of conducting an interlaboratory study.

4.2 At the heart of statistical theory is the concept of a frequency distribution of a random variable. The precision limit of the random variable is determined by the standard deviation of the variable. The standard deviation of a random variable that is the sum, difference, product, or quotient of two other random variables can be calculated simply so long as the individual variables are independent and the standard deviations are small relative to their mean values. These restrictions are usually met in ASTM methods. In those cases where these restrictions are not met, other methods can be used. Only cases complying with the restrictions are covered in this standard.

5. Procedure

5.1 The standard deviation on which precision limits for a test result are based can be calculated from the following equations:

$$\sigma_{x \pm y} = \sqrt{\sigma_x^{\ 2} + \sigma_y^{\ 2}} \tag{1}$$

where:

$\sigma_{x \pm y}$ = standard deviation for determining precision limits of a test result for a new standard based on either an addition or subtraction of test results from two other standards,

σ_x = standard deviation from precision statement of one of the standards on which new standard is based, and

σ_y = standard deviation from precision statement of other standard on which new standard is based.

The distributions of the test results from the two standards should be independent.

$$\sigma_{xy} = \sqrt{\bar{y}^2\sigma_x^{\ 2} + \bar{x}^2\sigma_y^{\ 2}} \tag{2}$$

where:

$\sigma_{x\,y}$ = standard deviation for determining precision limits of test results for a new standard based on the products of two other test results from two other standards,

σ_x = standard deviation from precision statement of one of the standards on which new standard is based,

$\bar{x}$ = mean or average value of *X* variable,

σ_y = standard deviation from precision statement of other standard on which new standard is based, and

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[1] This practice is under the jurisdiction of ASTM Committee D04 on Road and Paving Materials and is the direct responsibility of Subcommittee D04.94 on Statistical Procedures and Evaluation of Data.

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[2] For referenced ASTM standards, visit the ASTM website, www.astm.org, or contact ASTM Customer Service at service@astm.org. For *Annual Book of ASTM Standards* volume information, refer to the standard's Document Summary page on the ASTM website.

[3] The boldface numbers in parentheses refer to the list of references at the end of this standard.

$\bar{y}$ = mean or average value of *Y* variable.

$$\sigma_{\frac{x}{y}} = \sqrt{\frac{\bar{y}^2\sigma_x{}^2 + \bar{x}^2\sigma_y{}^2}{\bar{y}^4}} \quad (3)$$

where

$\sigma_{x/y}$ = standard deviation for determining precision limits of test results for a new standard based on the quotient of two test results from two other standards.

$$\sigma_x, \sigma_y, \bar{x}, \bar{y} = \text{definitions given above.} \quad (4)$$

An example in Appendix X1. illustrates how the equations are applied.

6. Keywords

6.1 precision limits; standard deviation

APPENDIX

(Nonmandatory Information)

X1. EXAMPLE OF CALCULATED PRECISION LIMITS

X1.1 Test Method D 3203 calculates the percent air voids in paving mixtures by using values obtained from Test Methods D 1188 and D 2041. Both of these test methods have precision statements so a precision statement can be calculated for Test Method D 3203.

X1.2 *Calculating Air Voids:*

X1.2.1 The equation for calculating air voids is:

$$\text{Percent air voids} = 100\,(1 - (\text{bulk sp gr/theoretical maximum sp gr})) \quad (X1.1)$$

The bulk specific gravity of Test Method D 1188 is divided by the theoretical maximum specific gravity of Test Method D 2041. Therefore, Eq 3 involving a quotient is the proper one to use in determining a precision statement for air voids.

X1.2.2 Test Method D 1188 has one of the older precision statements which does not give the standard deviation but only gives the D2S limit for multilaboratory precision. The testing operations in Test Methods D 1188 and D 2041 are very similar so it is not surprising that the D2S limits for multilaboratory precision are almost identical being 0.02 for Test Method D 1188 and 0.019 for Test Method D 2041. Therefore, the table for nonporous aggregate from Test Method D 2041 is used as the basis for both test methods as follows:

	Standard Deviation	Acceptable Range of Two Results
Test and type index	(1S)	(D2S)
Single-operator precision	0.0040	0.011
Multilaboratory precision	0.0067	0.019

During a testing program, the bulk specific gravity (Test Method D 1188) of an asphalt mixture was determined to be 2.423 and the theoretical maximum specific gravity (Test Method D 2041) was determined to be 2.523.

X1.3 Eq 3 was used to calculate the single-operator precision standard deviation for these specific gravity values. Standard deviation for Test Method D 3203 is as follows:

Single-operator precision:

$$\sigma_{\frac{x}{y}} = \sqrt{\frac{(2.523)^2(0.004)^2 + (2.423)^2(0.004)^2}{(2.523)^4}} \quad (X1.2)$$

$$= 0.0022$$

Multilaboratory precision:

$$\sigma_{\frac{x}{y}} = \sqrt{\frac{(2.523)^2(0.0064)^2 + (2.423)^2(0.0064)^2}{(2.523)^4}} \quad (X1.3)$$

$$= 0.0035$$

These standard deviations are in terms of specific gravity. Since Test Method D 3203 is reported in percent they must be multiplied by 100 to get the required percentages yielding the following values:

	Standard Deviation	Range of Two Results
Test and Type Index	(1S %)	(D2S %)
Single-operator precision	0.22	0.62
Multilaboratory precision	0.35	0.99

This is the complete precision statement for Test Method D 3203 where nonporous aggregates are used.

REFERENCES

(**1**) Geary, R. C., "The Frequency Distribution of a Quotient," *Journal of the Royal Statistical Society*, Vol 93, 1930, pp. 442–446.

(**2**) Fieller, E. C., "The Distribution of the Index in a Normal Bivariate Population," *Biometrika*, Vol 24, 1932, pp. 428–440

(**3**) Ku, H. H., "Notes on the Use of Propagation of Error Formulas," *Journal of Research of the National Bureau of Standards*, Vol 70C, No. 4, 1966, pp. 331–341.