Designation: E 1170 – 97 (Reapproved 2001)

Standard Practices for Simulating Vehicular Response to Longitudinal Profiles of Traveled Surfaces[1]

This standard is issued under the fixed designation E 1170; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ϵ) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 These practices cover the calculation of vehicular response to longitudinal profiles of traveled surface roughness.

1.2 These practices utilize computer simulations to obtain two vehicle responses: (*1*) axle-body (sprung mass) motion or (*2*) body (sprung mass) acceleration, as a function of time or distance.

1.3 These practices present standard vehicle simulations (quarter, half, and full car) for use in the calculations.

1.4 The values stated in SI units are to be regarded as the standard.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

E 867 Terminology Relating to Traveled Surface Characteristics[2]

E 950 Test Method for Measuring the Longitudinal Profile of Traveled Surfaces with an Accelerometer Established Inertial Profiling Reference[2]

E 1364 Test Method for Measuring Road Roughness by Static Level Method[2]

E 1489 Practice for Computing Ride Number of Roads from Longitudinal Profile Measurements Made by an Inertial Profile Measuring Device[2]

E 1926 Practice for Computing International Roughness Index from Longitudinal Profile Measurements[2]

2.2 *ISO Standard:*

2631 Guide for the Evaluation of Human Exposure to Whole-Body Vibration[3]

3. Summary of Practices

3.1 These practices use a measured profile (see Test Method E 950) or a synthesized profile as part of a vehicle simulation to obtain vehicle response.

3.2 The first practice for obtaining vehicle response uses simulation of a quarter-car or half-car model. The output is the accumulated relative motion between the sprung and unsprung vehicle masses, of the simulated vehicle, for a predetermined distance. The units are accumulated relative motion per unit of distance traveled (m/km or in./mile). For example, the quarter-car simulation is used when a Bureau of Public Roads BPR/roadmeter is to be simulated, and the half-car model (or the quarter car with the average of the left and right elevation profile input) is used when a road meter is to be simulated.

3.3 The second practice uses either a quarter-car, half-car, or full-car simulation to obtain vehicle body acceleration. The acceleration history can be computed as a function of time or distance, or both. One application of this practice is to use the acceleration history in a ride quality evaluation, such as the ISO Guide 2631.

3.4 For all calculations, a vehicle test speed is selected and maintained throughout the calculation. Pertinent information affecting the results must be noted.

4. Significance and Use

4.1 These practices provide a means for evaluating traveled surface-roughness characteristics directly from a measured profile. The calculated values represent vehicular response to traveled surface roughness.

4.2 These practices provide a means of calibrating response-type road-roughness measuring equipment.[4]

5. Apparatus

5.1 *Computer*—The computer is used to calculate acceleration and displacement of vehicle response to a traveled surface profile, using a synthesized profile or a profile obtained in accordance with Test Method E 950 as the input. Filtering shall be provided to permit calculation, without attenuation, at frequencies as small as 0.1 Hz at speeds of 15 to 90 Km/h (10 to 55 mph). Computation may be analog or digital. Noise

[1] These practices are under the jurisdiction of ASTM Committee E17 on Vehicle-Pavement Systemsand are the direct responsibility of Subcommittee E17.33 on Methodology for Measuring Pavement Roughness.

Current edition approved Dec. 10, 1997. Published April 1998. Originally published as E 1170 – 87. Last previous edition E 1170 – 92.

[2] *Annual Book of ASTM Standards*, Vol. 04.03.

[3] Available from American National Standards Institute, 11 W. 42nd St., 13th Floor, New York, NY 10036.

[4] Gillespie, T. D., Sayers, M. W., and Segel, L., “Calibration and Correlation of Response-Type Road Roughness Measuring Systems,” *NCHRP* Report 228, 1980.

within the computer shall be no more than one quarter of the intended resolution. It is recommended that a 16-bit or better digital computer be used.

5.2 *Data-Storage Device*—A data-storage device shall be provided for the reading of profiles and the recording and long-term storage of computed data. Profile data shall be scaled to maintain resolution of 0.025 mm (0.001 in.) and to accommodate the full range of amplitudes encountered during normal profile-measuring operations. The device shall not contribute to the recorded data any noise amplitude larger than 0.025 mm (0.001 in.).

5.3 *Digital Profile Recordings*—Road-roughness profiles shall be obtained in accordance with Test Method E 950 or synthesized. The profile must be recorded at intervals no greater than one third of the wavelength required for accurate representation of the traveled surface for the intended use of the data. For most applications a sample interval of 600 mm (2 ft) will give a valid representation for all types of road surfaces except where the roughness is extremely localized and therefore could be missed, in which case a sample interval of 150 mm (6 in.) should be used. When more than one path of a traveled surface is measured, the recorded profile data for the paths shall be at the same longitudinal location along the measured profiles. The recorded profile shall include all of the noted field data described in the Procedure (Data Acquisition) and Report sections of Test Method E 950. The length of the road-roughness profile must be reported with the results; however, caution must be exercised to ensure that transients in the simulation do not influence the results. It is recommended that at least 160 m (0.1 miles) of profile, preceding the test section, plus the desired test section be used as input in simulation to eliminate the effects of transients.

6. Vehicle Simulation Programs

6.1 These practices use one of four vehicle simulations:[5] a quarter car, a half car, a full car with four-wheel independent suspension, and a full car with a solid rear axle. Although several methods for solving the differential equations are available, the Runge-Kutta is described in NCHRP Report 228.[4]. The parametric models in Figs. 1-4 (such as the lumped parameter model) and the coordinate system defined constitute the standard practice. The analytic representation of the model and the methods of implementation need not be the same as outlined in the appendix.

6.1.1 *Quarter-Car Simulation Model*:

6.1.1.1 The quarter car is modeled as shown in Fig. 1, with z_1, as the vehicle-body (sprung mass) displacement, z_2 as the tire (unsprung mass) displacement, and the z_p as the longitudinal profile.

6.1.1.2 The relative motion between the body and the axle, Z', is defined as:

$$Z' = z_1 - z_2 \tag{1}$$

The equation of motion for the quarter-car model is given in X1.1. The parameters used for the quarter-car model are normalized by the body mass, M_1. The other vehicle parameters are: the vehicle spring constant, K_1; the damper value, C_1; the axle-wheel mass, M_2; the tire stiffness, K_2; and the tire damping constant, C_2. Values for these parameters are given in Table 1.

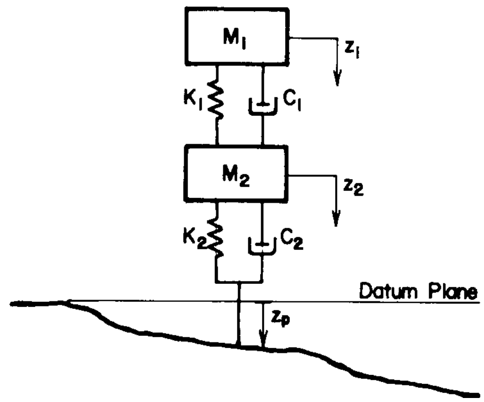


FIG. 1 Quarter-Car Simulation Model

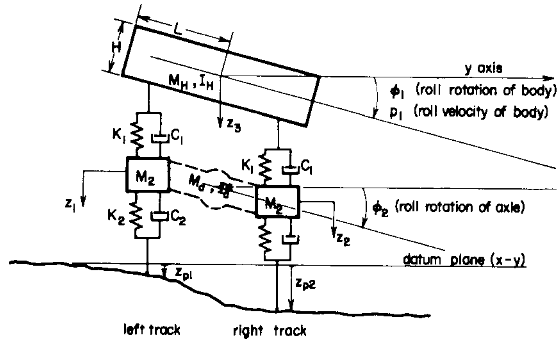


FIG. 2 Half-Car Model

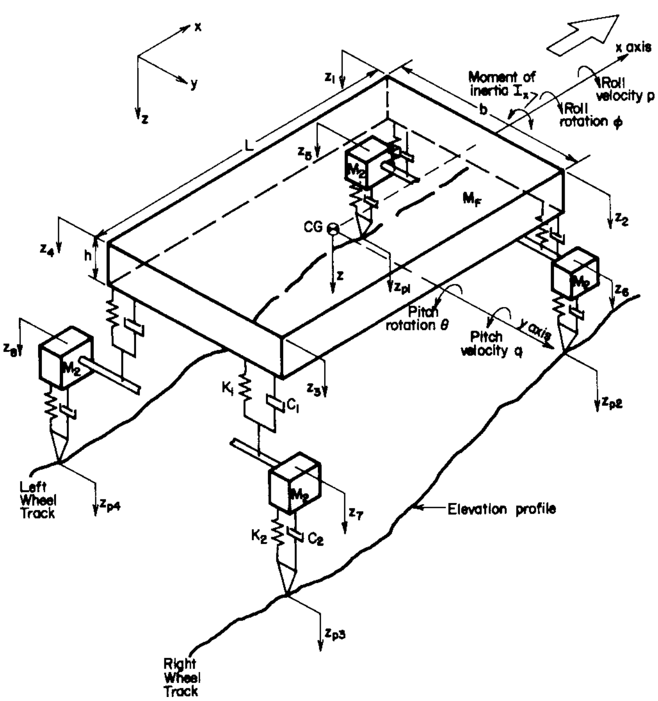


FIG. 3 Full-Car With Independent Suspension

[5] Wambold, J. C., Henry, J. J., and Yeh, E. C., "Methodology for Analyzing Pavement Condition Data" (Volume I and II, Final Report), Report No. FHWA/RD-83/094 and FHWA/RD-83/095, Federal Highway Administration, January 1984.

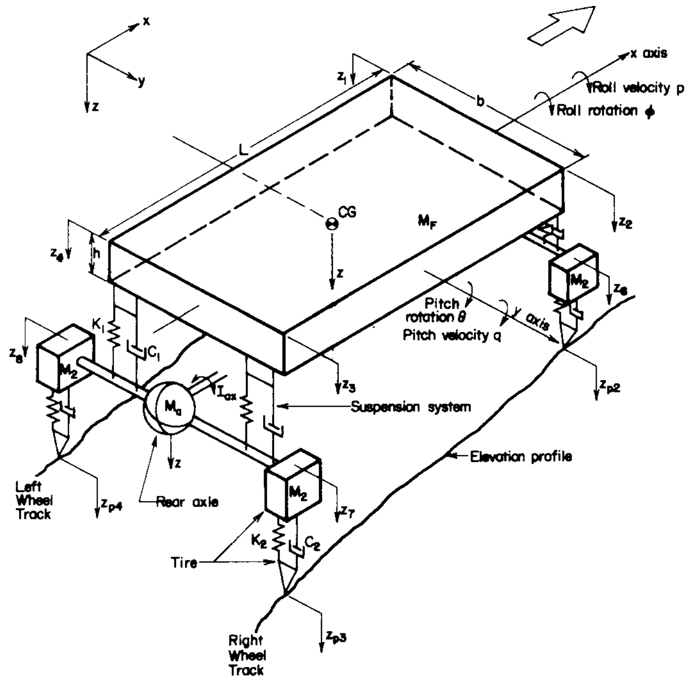


FIG. 4 Full-Car Model With Solid Rear Axle

TABLE 1 Quarter-Car Vehicle Physical Constants

	Simulated Vehicle			
Parameter	BPR Roughometer	Ride Meter-Vehicle Mounted	Ride Meter-Trailer	IRI
K_1/M_1	129 s^{-2}	63 s^{-2}	125 s^{-2}	63.3
K_2/M_1	643 s^{-2}	653 s^{-2}	622 s^{-2}	653
M_2/M_1	0.16	0.15	0.26	0.15
C_1/M_1	3.9 s^{-1}	6.0 s^{-1}	8.0 s^{-1}	6.0
C_2/M_1	0	0	0	0

6.2 *Half-Car Simulation Model*—The half-car model is constructed by using one half of a rigid vehicle and is made up of two quarter cars at the right and left tracks. The model for the half car is shown in Fig. 2, and the associated parameters are given in Table 2. The equation of motion is given in X1.2. The relative motion between the body and the axle, Z', is defined as $Z' = z_3 - ½ (z_1 + z_2)$. The mass of the axle, M_a and the moment of inertia of the axle, I_a must be set to zero when the half car being modeled has an independent wheel suspension. I_b represents the moment of inertia of the car body and b represents the wheel track.

TABLE 2 Half-Car Vehicle Physical Constants[A]

Parameter	Ride-Meter Vehicle Mounted	Ride-Meter Trailer
K_1/M_H	32 s^{-2}	57.5 s^{-2}
K_2/M_H	326 s^{-2}	311 s^{-2}
M_2/M_H	0.075	0.125
C_1/M_H	3 s^{-1}	4 s^{-1}
C_2/M_H	0	0
M_a/M_H	0.30	0.50
	(for model with rear axle)	
	0	
	(for model with independent rear suspension)	
$I_H/(M_H b^2)$	0.42	0.42
I_a/I_H	0.36	0.6
	(for model with rear axle)	
	0	
	(for model with independent rear suspension)	
b	1.8 m	1.8 m

[A] The values apply to the rear half of a vehicle.

6.3 *Full-Car Simulation Model with Four-Wheel Independent Suspension*:

6.3.1 This model is an expansion of the half-car simulation model. Two more wheel and pitch motions are added to make it a seven-degree-of-freedom model. This model is shown in Fig. 3 and the vehicle parameters are given in Table 3.

6.3.2 The equation of motion is developed similarly to that in the half-car model and the tire damping is again taken as zero to simplify the equations. The equations are given in X1.3.

6.4 *Full-Car Simulation Model with a Rear Axle*:

6.4.1 This model is a modification of the full-car model to change the rear suspension to a solid axle. The model is shown in Fig. 4. Again, the tire damping is taken as zero to simplify the equations. The equations are given in X1.4.

6.4.2 The values of the parameters I_x, I_w, and M_F are the same as in the model for the full car with independent suspension, except that the additional parameter, axle moment of inertia, I_{ax} is used.

7. Example Applications

7.1 *Displacement per Length of Travel*:

TABLE 3 Full-Car Vehicle Physical Constants

Parameter	Value	Parameter	Value
K_1/M_F	16 s^{-2}	$I_x/M_F b^2$	0.14
K_2/M_F	163 s^{-2}	$I_y/M_F L^2$ [A]	0.19
M_2/M_F	0.038	b	1.8 m
C_1/M_F	1.5 s^{-1}	L/b[A]	1.44
C_2/M_F	0	h/b[A]	0.5
		$I_{ax}/M_F b^2$	
		with axle	0.022
		without axle	0

[A] The wheel base is L, and the body height (center of gravity (cg) above suspension) is h.

7.1.1 *Inches per Mile*—An improved method of computing inches per mile (IPM) has been proposed by Gillespie, Sayers, and Segel.[4] Quantization, as used in current road meters, does not truly reflect the axle-body movement. Therefore, IPM is defined as:

$$IPM = \sum_{i=1}^{N} | Z'_i - Z'_{i+1}| / distance \qquad (2)$$

where:

Z'_i = relative maximum or minimum value of the axle-body movement.

7.1.2 *International Roughness Index (IRI)*—The IRI comes from the 1982 World Bank International Road Roughness Experiment in Brazil. The IRI is the measurement of the displacement of the sprung mass to unsprung mass of a quarter car model and is reported in units of displacement per length of travel. The method uses a standard quarter car model's response to longitudinal profile measurements.

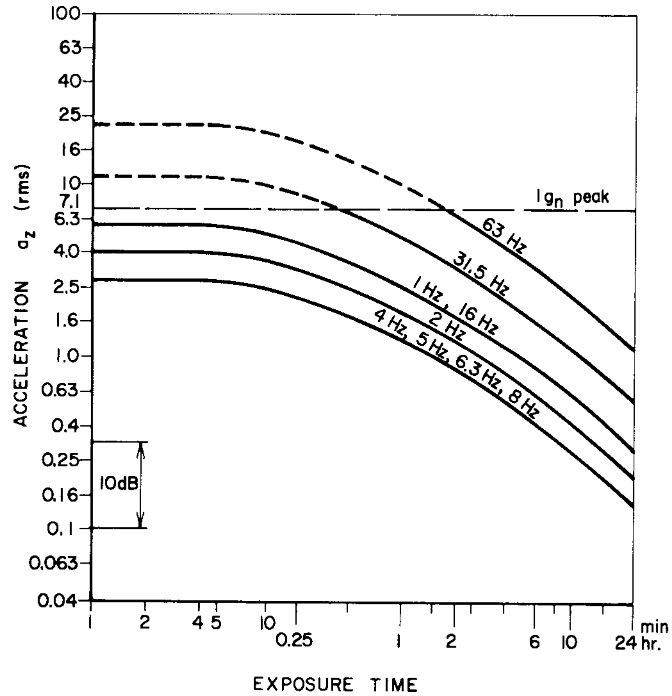
7.1.3 These IPM values are calculated on a continuous basis rather than in increments, and are considerably different from those obtained by current road meters.

7.2 *Ride Quality Analysis*:

7.2.1 The most commonly used standard is ISO 2631, that has a tabular format and uses human-body acceleration to predict the exposure time for human discomfort or fatigue. ISO 2631 can be converted to an index system by calculating the time-to-discomfort for every frequency interval from 1 Hz to 80 Hz. For ISO 2631, the usual input to the program is vehicle-body (sprung mass) acceleration. The analysis uses a Fast Fourier Transform (FFT) to obtain the space frequency spectrum of the acceleration history. The selected vehicle specifications and speed produce the vehicle-body acceleration spectrum. The seat is considered as having negligible effect on the human-body acceleration in the range of 1 Hz to 80 Hz.[4]

7.2.2 *Ride Number (RN)*—During the 1980's the ride number concept for estimating pavement ride quality from surface profile measurements was developed in a National Cooperative Highway research project. Various papers have compared the performance of ride number transforms and found it to be superior to other ride quality transforms, producing estimates of pavement ride quality with the highest correlation to the measured subjective ride quality and with he lowest Standard error.

7.2.3 After the acceleration frequency spectrum is calculated, the model in ISO 2631 is applied. This model determines the exposure time of reduced-comfort boundary or the fatigue of a human body from the frequency spectrum of the seat vertical acceleration (Fig. 5). The details for calculating the exposure times for reduced comfort or fatigue are given in NCHRP Report 228.[4] An alternative for calculating a ride index, developed at the University of Virginia,[6] is also presented in NCHRP Report 228.[4]



NOTE 1—Vertical (a_z) acceleration limits as a function of exposure time and frequency (center frequency of a third-octave band): "fatigue-decreased proficiency boundary." This graph was taken from ISO 2631.

FIG. 5 Model for Ride Quality Analysis

8. Calibration

8.1 If a digital analysis is used, calibration is required when the system is installed. If an analog computer is used, the system shall be calibrated on a periodic basis. At present, no standard road profile is available for such a calibration. It is suggested that each agency adopt a range of profile records for use in calibrating its complete system.

9. Report

9.1 Report the following information for each practice:

9.1.1 Data from profiles obtained in accordance with Test Method E 950 including date, the time of day of the measurement, or the date of the synthesized profile,

9.1.2 Vehicle simulation program used,

9.1.3 Speed of simulations,

9.1.4 Vehicle-parameter values used if other than those specified in these practices, and

9.1.5 Results of the analysis.

[6] Richards, L. G., Jacobson, I. D., and Pepler, R. D., "Ride Quality Models for Diverse Transportation Systems," *Transportation Research Record* 774 (1980), pp. 39–45.

APPENDIX

(Nonmandatory Information)

X1. EQUATIONS OF MOTION FOR VEHICLE RESPONSES TO LONGITUDINAL PROFILES

X1.1 *Quarter-Car Model*—The equation of motion for this model can be represented as follows.

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -K1/M1 & K1/M1 & -C1/M1 & C1/M1 \\ K1/M2 & -(K1+K2)/M2 & C1/M2 & -(C1+C2)/M2 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ w_1 \\ w_2 \end{bmatrix} + \begin{bmatrix} 0 \\ C2/M2 \\ C1C2/M1M2 \\ -C1C2 - C2\ 2 + K2M2/M22 \end{bmatrix} [z_p]$$

where two new variables are introduced.

$w_1 = \dot{z}_1$, and

$w_2 = \dot{z}_2 - C2/M2 z_p$, so that

$w_1 = \ddot{z}_1$, and

$w_2 = \ddot{z}_2 - C2/M2 z_p$.

X1.1.1 The relative motion between body and axle (Z') is defined as:

$$Z' = z_1 - z_2 \quad \text{(X1.1)}$$

X1.2 *Half-Car Model*—The equation of motion for this model is represented as follows.

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ w_1 \\ w_2 \\ w_3 \\ \phi_1 \\ p_1 \end{bmatrix} = [A] \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ w_1 \\ w_2 \\ w_3 \\ \phi_1 \\ p_1 \end{bmatrix} + [B] \begin{bmatrix} z_{p1} \\ z_{p2} \end{bmatrix}$$

where:

$w_1 = \dot{z}_1 - C2/(M2 + 0.5ma)\ z_{p1}$

$w_2 = \dot{z}_2 - C2/(M2 + 0.5ma) z_{p2}$, and

$w_3 = \dot{z}_3$

The other symbols are as shown in Fig. 2.

X1.2.1 The relative motion between body and axle (Z') is defined as:

$$Z' = z_3 - 1/2(z_1 + z_2) \quad \text{(X1.2)}$$

X1.2.2 The matrix A is:

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ -(K1+K2)/(M2+0.5ma) & 0 & K1/(M2+0.5ma) & -(C1+C2)/(M2+0.5ma) & 0 & C1/(M2+0.5ma) & -K1b/2/(M2+0.5ma) & -C1b/2/(M2+0.5ma) \\ 0 & -(K1+K2)/(M2+0.5ma) & K1/(M2+0.5ma) & 0 & -(C1+C2)/(M2+0.5ma) & C1/(M2+0.5ma) & +K1b/2/(M2+0.5ma) & +C1b/2/(M2+0.5ma) \\ K1/MH & K1/MH & -2K1/MH & C1/MH & C1/MH & -2C1/MH & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ -K1b/2IH & +-K1b/2IH & 0 & -C1b/2IH & -C1b/2IH & 0 & -K1b\ 2/2IH & -C1b\ 2/2IH \end{bmatrix}$$

and the matrix B is:

$$
B = \begin{bmatrix}
C2/(M2 + 0.5ma) & 0 \\
0 & C2/(M2 + 0.5ma) \\
0 & 0 \\
-(C1C2 + C2\ 2 - K2M2)/(M2 + 0.5ma)2 & 0 \\
0 & -(C1C2 + C2\ 2 - K2M2)/(M2 + 0.5ma)2 \\
C1C2/MH(M2 + 0.5ma) & C1C2/MH(M2 + 0.5ma) \\
0 & 0 \\
C1C2b/2/2IH(M2 + 0.5ma) & C1C2b/2/2IH(M2 + 0.5ma)
\end{bmatrix}
$$

X1.3 *Full-Car Model with Independent Suspension*—The equation of motion for this model can be represented as follows.

$$g = Ag + Bf \quad \text{(X1.3)}$$

so that,

$$g = \begin{bmatrix} z \\ z_5 \\ z_6 \\ z_7 \\ z_8 \\ w_9 \\ w_{10} \\ w_{11} \\ w_{12} \\ w \\ \phi \\ p \\ \theta \\ q \end{bmatrix}$$

where:

z = body displacement,
z_5 = left front-wheel displacement,
z_6 = right front-wheel displacement,
z_7 = right rear-wheel displacement,
z_8 = left rear-wheel displacement,
w_9 = left front-wheel velocity,
w_{10} = right front-wheel velocity,
w_{11} = right rear-wheel velocity,
w_{12} = left rear-wheel velocity,
w = body velocity,
ϕ = roll angle,
p = roll velocity,
θ = pitch angle, and
q = pitch velocity.

and the matrix *B* is:

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ K_2/M_2 & 0 & 0 & 0 \\ 0 & K_2/M_2 & 0 & 0 \\ 0 & 0 & K_2/M_2 & 0 \\ 0 & 0 & 0 & K_2/M_2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

X1.3.1 The input vector, *f* is defined as:

$$f = \begin{bmatrix} z_{p1} \\ z_{p2} \\ z_{p3} \\ z_{p4} \end{bmatrix}$$

where:

z_{p1} *and* z_{p2} = doubles track profiles, and
z_{p3} *and* z_{p4} = delays of z_{p1} and z_{p2}.

X1.3.2 The matrix *A* is:

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
K1/M2 & -(K1+K2)/M2 & 0 & 0 & 0 & -C1/M2 & 0 & 0 & 0 & C1/M2 & -K1b/2M2 & -C1b/2M2 & -K1L/2M2 & -C1L/2M2 \\
K1/M2 & 0 & -(K1+K2)/M2 & 0 & 0 & 0 & -C1/M2 & 0 & 0 & C1/M2 & K1b/2M2 & C1b/2M2 & -K1L/2M2 & -C1L/2M2 \\
K1/M2 & 0 & 0 & -(K1+K2)/M2 & 0 & 0 & 0 & -C1/M2 & 0 & C1/M2 & K1b/2M2 & C1b/2M2 & K1L/2M2 & C1L/2M2 \\
K1/M2 & 0 & 0 & 0 & -(K1+K2/M2 & 0 & 0 & 0 & -C/M2 & C1/M2 & -K1b/2M2 & -C1b/2M2 & K1L/2M2 & C1L/2M2 \\
-4K1/MF & K1/MF & K1/MF & K1/MF & K1/MF & C1/MF & C1/MF & C1/MF & C1/MF & -4C1/MF & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & -K1b/2Ix & K1b/2Ix & K1b/2Ix & -K1b/2Ix & -C1b/2Ix & C1b/2Ix & C1b/2Ix & -C1b/2Ix & 0 & -K1b\,2/Ix & -C1b\,2/Ix & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & -K1L/2Iy & -K1L/2Iy & K1L/2Iy & K1L/2Iy & -C1L/2Iy & -C1L/2Iy & C1L/2Iy & C1L/2Iy & 0 & 0 & 0 & -K1L2/Iy & -C1L\,2/Iy
\end{bmatrix}
$$

X1.4 *Full-Car Model with Solid Rear Axle*—The equation of motion for this model is represented as follows:

$$\dot{h} = ah + Bf \qquad \text{(X1.4)}$$

so that,

$$
h = \begin{bmatrix} z \\ z_5 \\ z_6 \\ w_5 \\ w_6 \\ w \\ \phi_1 \\ P_1 \\ \theta \\ q \\ z_9 \\ w_9 \\ \phi_2 \\ p_2 \end{bmatrix}
$$

where:

z = body displacement,
z_5 = left front-wheel displacement,
z_6 = right front-wheel displacement,
w_5 = left front-wheel velocity,
w_6 = right front-wheel velocity,
w = body velocity,
ϕ_1 = body roll angle,
p_1 = body roll rate,
θ = pitch angle,
q = pitch rate,
z_9 = axle displacement,
w_9 = axle velocity,
ϕ_2 = axle roll angle, and
p_2 = axle roll rate.

X1.4.1 The matrix *A* is:

$$
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
K1/M2 & -(K1+K2)/M2 & 0 & -C1/M2 & 0 & C1/M2 & -K1b/2M2 & -C1b/2M2 & -K1L/2M2 & -C1L/2M2 & 0 & 0 & 0 & 0 \\
K1/M2 & 0 & -(K1+K2/M2 & 0 & -C1/M2 & C1/M2 & K1b/2M2 & C1b/2M2 & -K1L/2M2 & -C1L/2M2 & 0 & 0 & 0 & 0 \\
-4K1/MF & K1/MF & K1/MF & C1/MF & C1/MF & -4C1/MF & 0 & 0 & 0 & 0 & 2K1/MF & 2C1/MF & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -K1b/2Ix & K1b/2Ix & -C1b/2Ix & C1b/2Ix & 0 & -K1b2/Ix & -C1b2/Ix & 0 & 0 & 0 & 0 & K1b2/2Ix & C1b2/2Ix \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & -K1L/2Iy & -K1L/2Iy & -C1L/2Iy & -C1L/2Iy & 0 & 0 & 0 & -K1L2/Iy & -C1L2/Iy & K1L2/Iy & C1L2/Iy & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
2K1/2Ma & 0 & 0 & 0 & 0 & 2C1/2Ma & 0 & 0 & K1L/2Ma & C1L/Ma & -2(K1+K2)/Ma & -2C1/Ma & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & K1b2/2Iax & C1b2/2Iax & 0 & 0 & 0 & 0 & -b2(K1+K2)/2Iax & -b2C1/2Iax
\end{bmatrix}
$$

and matrix *B* is:

$$
B = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
K_2/M_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & K_2/2M_2 & K_2/2M_2 \\
0 & 0 & 0 & 0 \\
0 & 0 & -bK_2/2I_{ax} & -bK_2/2I_{ax}
\end{bmatrix}
$$

X1.4.2 The input vector *f* is the same as X1.3.2.