Designation: C 1045 – 9701

Standard Practice for
Calculating Thermal Transmission Properties Under Steady-State Conditions[1]

This standard is issued under the fixed designation C 1045; the number immediately following the designation indicates the year of original adoption or, in the case of revision, the year of last revision. A number in parentheses indicates the year of last reapproval. A superscript epsilon (ε) indicates an editorial change since the last revision or reapproval.

1. Scope

1.1 This practice covers requirements and guidelines provides the user with a uniform procedure for calculating the determination of thermal transmission properties based upon steady-state one dimensional heat transfer tests on of a thermal insulation material or system for which values of from data generated by steady state, one dimensional test methods used to determine heat flux, and surface o temperatures. This practice is intended to eliminate the need for similaur calculation sections in Test Methods C 177, C 335, C 518, C 976, C 1033, C 1114 and specimen geometry are reported from C 1363 by permitting use of these standard test methods. calculation forms by reference.

1.2 The thermal transmission properties described include: thermal conductance, thermal resistance, apparent thermal conductivity, apparent thermal resistivity, surface conductance, surface resistance, and overall thermal resistance or transmittance.

1.3 This practice is restricted to calculation of provides the method for developing the apparent thermal transmission properties conductivity as a function of temperature relationship for a specimen from heat transfer data generated by standard test methods. These methods include: (1) planar geometries such as those at small or large temperature differences. This relationship can be used in Test Methods C 177, C 236, C 518, C 745, C 976, to characterize material for comparison to material specifications and C 1114, and (2) radial geometries for use in calculation programs such as those Practice C 680.

1.4 The SI unit values used in Test Methods C 335 and C 1033.

1.4 This this practice are considered standard.

1.5 This practice includes a discussion of the procedure definitions and underlying assumptions for development the calculation of thermal conductivity as a function of temperature equation transmission properties. Tests to detect deviations from data generated by standard test methods.

1.5 The values stated in SI units these assumptions are described. This practice also considers the complicating effects of uncertainties due to be regarded as the standard.

1.6 The attached appendixes provide discussions measurement processes and material variability. See Section 7.

1.6 This practice is not intended to cover all possible aspects of thermal properties data base development. For new materials, the user should investigate the variations in thermal properties seen in similar materials. The information contained in Section 7, the Appendix and the technical papers listed in the References section of this practice may be helpful in determining whether the material under study has thermal properties that can be described by equlations using m this practice. Some examples where this method hals, limited application include: (1) the development onset of convection in insulation as described in Reference **(21)**; (2) a phase change of one of the basic relationships used insulation system components such as a blowing gas in this practice, foam; and examples (3) the influence of t heat flow direction and temperature difference changes for reflective insulations.

2. Referenced Documents

2.1 *ASTM Standards:*

C 168 Terminology Relating to Thermal Insulating Materials[2]
C 177 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Guarded-Hot-Plate Apparatus[2]
C 236 Test Method for Steady-State Thermal Performance of Building Assemblies by Means of a Guarded Hot Box[2]
C 335 Test 335 Test Method for Steady-State Heat Transfer Properties of Horizontal Pipe Insulations[2]
C 518 Test Method for Steady-State Heat Flux Measurements and Thermal Transmission Properties by Means of the Heat Flow Meter Apparatus[2]
C 680 Practice for Determination of Heat Gain or Loss and the Surface Temperature of Insulated Pipe and Equipment Surfaces by the Use of a Computer Program[2]
C 745 Test Method for Heat Flux Through Evacuated Insulations Using a Guarded Flat Plate Boiloff Calorimeter,[2]
C 976 Test 976 Test Method for Steady-State Thermal Performance of Building Assemblies by Means of a Calibrated Hot Box[2]
C 1033 Test Method for Steady-State Heat Transfer Properties of Pipe Insulation Installed Vertically[2]
C 1058 Practice for Selecting Temperatures for Evaluating and Reporting Properties of Thermal Insulation[2]
C 1114 Test Method for Steady-State Thermal Transmission Properties by Means of the Thin-Heater Apparatus[2]
C 1199 Test Method for Measuring the Steady-State Thermal Transmittance of Fenestration Systems Using Hot Box Methods[2]
C 1363 Test Method for Thermal Performance of Building Assemblies by Means of a Hot Box Apparatus[2]
E 122 Practice for Choice of Sample Size to Estimate the Average Quality of a Lot or Process[3]

3. Terminology

3.1 *Definitions*— The definitions and terminology of this practice are intended to be consistent with Terminology C 168. However, because exact definitions are critical to the use of this practice, the following equations are defined here for use in the calculations section of this practice.

3.2 *Symbols*—The symbols, terms and units used in this practice are the following:
C hchhLLp qQ r rarinroutR T2 Th Tin TmT out ΔT

A	=	specimen area normal to heat flux direction, m^2,
λC	=	thermal conductivity or apparent thermal conductivity, conductance, $W/(m^2 \cdot K)$,
$\lambda(T)$	=	the functional relationship between thermal conductivity and temperature, $W/(m\ K)$,
h_c λ_{exp}	=	the experimental thermal conductivity, $W/(m\ K)$,
λ_m	=	mean thermal conductivity, averaged with respect to temperature from surface heat transfer coefficient, cold side, T_c to T_h, $W/(m \cdot K)$,
C	=	thermal conductance, $W/(m^2K)$, $W/(m^2 \cdot K)$,
h_h	=	surface heat transfer coefficient, hot side, $W/(m^2 \cdot K)$,
$h_c L$	=	surface coefficient, cold side, $W/(m^2K)$, thickness of a slab in heat transfer direction, m,
lL_p	=	metering area length in the axial direction, m,
q	=	one-dimensional heat flux (time rate of heat flow through metering area divided by the apparatus metering area A), W/m^2,
Q	=	time rate of one-dimensional heat flow through the metering area of the test apparatus, W,
$r_i\ r$	=	thermal resistivity, $K \cdot m/K$,
$r_n\ r$	=	radius of a hollow cylinder at the i th surface, m, apparent thermal resistivity, $K \cdot m/K$,
$r' r_{in}$	=	thermal resistivity, $K\ m/W$, inside radius of a hollow cylinder, m,
r_{out}	=	outside radius of a hollow cylinder, m,
R	=	thermal resistance, $m^2 \cdot K/W$,
$R_h\ R_c$	=	surface thermal resistance, hot cold side, $m^2 \cdot K/W$,
R_{ch}	=	surface thermal resistance, cold hot side, $m^2 \cdot K/W$,
R_u	=	overall thermal resistance, $m^2 \cdot K/W$,
T	=	temperature, K,
T_1	=	area-weighted air temperature 75 mm or more from the hot side surface, K,
T_{h_2}	=	area-weighted air temperature of specimen hot 75 mm or more from the cold side surface, K,
T_c	=	area-weighted temperature of the specimen cold surface, K,
$T2_h$	=	area-weighted air temperature 75 mm or more from the cold side of specimen hot surface, K,
T_{in}	=	temperature at the inner radius, K,

T_m = specimen mean temperature, average of two opposite surface temperatures, $(T_h + T_c)/2$, K,

$\Delta T T_{out}$ = temperature ~~difference~~, at the outer radius, K,

$\overline{\Delta T_{s\text{-}s}}$

ΔT = temperature difference, ~~surface to surface, $(T_h - T_c)$~~, K,

$\Delta T_{a\text{-}a}$ = temperature difference, air to air, $(T_1 - T_2)$, K,

$\Delta T_{s\text{-}s}$ = temperature difference, surface to surface, $(T_h - T_c)$, K,

U = thermal transmittance, $W/(m^2 \cdot K)$, and

x = linear dimension in the heat flow direction, m,

λ = thermal conductivity, $W/(m \cdot K)$,

λ_a = apparent thermal conductivity, $W/(m \cdot K)$,

$\lambda(T)$ = the functional relationship between thermal conductivity and temperature, $W/(m \cdot K)$,

λ_{exp} = the experimental thermal conductivity, $W/(m \cdot K)$,

λ_m = mean thermal conductivity, averaged with respect to temperature from T_c to T_h, $W/(m \cdot K)$, (see sections 6.4.1 and Appendix X3).

NOTE 1—Subscripts h and c are used to differentiate between hot side and cold side surfaces.

3.3 *Thermal Transmission Property Equations*:

3.3.1 *Thermal~~-r~~ Resistance*, *R*, is defined in Terminology C 168. It is not necessarily a unique function of temperature or material, but is rather a property determined by the specific thickness of the specimen and by the specific ~~range~~ set of hot-side and cold-side temperatures used to measure the thermal resistance.

$$R = \frac{A\,(T_h - T_c)}{Q} \tag{1}$$

3.3.2 *Thermal Conductance, C*:

$$C = \frac{Q}{A\,(T_h - T_c)} \tag{2}$$

$$C = \frac{Q}{A\,(T_h - T_c)} = \frac{1}{R} \tag{2}$$

NOTE ~~1~~2—Thermal resistance, *R*, and the corresponding thermal conductance, *C*, are reciprocals~~,~~; that is, their product is unity. These terms apply to specific bodies or constructions as used, either homogeneous or heterogeneous, between two specified isothermal surfaces.

NOTE ~~2—Subscripts h~~ 3—Eq 1, Eq 2, and ~~c~~ Eq 11-13 are for rectangular coordinate systems only. Similar equations for resistance, etc. can be developed for a cylindrical coordinate system providing the differen~~t~~ce in a~~t~~re~~ b~~as is considered. In prac~~t~~wice, for cylindrical systems such as piping runs, th~~e~~o~~e~~ thermal resi~~de~~stan~~d~~ce should be bas~~i~~ed upon the pipe external surface area since that area does not change with different insulation thickness.

3.3.3 *Apparent thermal conductivity*, λ_a, is defined in Terminology C 168.
Rectangular coordinates:

$$\lambda = \frac{Q\,L}{A\,(T_h - T_c)} \tag{3}$$

$$\lambda_a = \frac{Q\,L}{A\,(T_h - T_c)} \tag{3}$$

Cylindrical coordinates:

$$\lambda = \frac{Q\,\ln(r_2/r_1)}{2\,\pi\,l\,(T_h - T_c)} \tag{4}$$

$$\lambda_a = \frac{Q\,\ln(r_{out}/r_{in})}{2\,\pi\,L_p\,(T_{in} - T_{out})} \tag{4}$$

~~where:~~
~~r_1~~   = ~~inner radius,~~
~~T_h~~   = ~~temperature at the inner radius,~~
~~r_2~~   = ~~outer radius, and~~
~~T_c~~   = ~~temperature at the outer radius.~~

3.3.4 *Apparent~~t~~ Thermal~~-r~~ Resistivity*,~~-r~~ r_a, is defined in Terminology C 168.
Rectangular~~-c~~ Coordinates:

$$r' = \frac{A\,(T_h - T_c)}{Q\,L} \tag{5}$$

$$r_a = \frac{A\,(T_h - T_c)}{Q\,L} = \frac{1}{\lambda_a} \tag{5}$$

Cylindrical~~-c~~ Coordinates:

$$r' = \frac{2\pi l (T_h - T_c)}{Q \ln (r_2 / r_1)} \tag{6}$$

$$r_a = \frac{2\pi L_p (T_{in} - T_{out})}{Q \ln (r_{out} / r_{in})} = \frac{1}{\lambda_a} \tag{6}$$

NOTE ~~3~~4—~~The apparent~~ thermal resistivity, ~~r'~~r_a, and the corresponding thermal conductivity, λ_a, are reciprocals, that is, their product is unity. These terms apply to specific materials tested between two specified isothermal surfaces. For this practice, materials are considered homogeneous when the value of the thermal conductivity or thermal resistivity is not significantly affected by variations in the thickness or area of the sample within the normally used range of those variables.

3.4 ~~Thermal Transmission~~Transmission *Property Equations for Convective Boundary Conditions*:

3.4.1 *Surface Thermal Resistance, R_i,* the quantity determined by the temperature difference at steady-state between an isothermal surface and its surrounding air that induces a unit heat flow rate per unit area to or from the surface. Typically, this parameter includes the combined effects of conduction, convection, and radiation. Surface resistances are calculated as follows:

$$R_h = \frac{A (T_1 - T_h)}{Q} \tag{7}$$

$$R_c = \frac{A (T_c - T_2)}{Q} \tag{8}$$

~~NOTE 4—Subscripts 1 and 2 are used to differentiate between the hot and cold side air, respectively.~~

3.4.2 *Surface ~~c~~ Heat Transfer Coefficient, h_i,* is often called the film coefficient. These coefficients are calculated as follows:

$$h_h = \frac{Q}{A (T_1 - T_h)} \tag{9}$$

$$h_h = \frac{Q}{A (T_1 - T_h)} = \frac{1}{R_h} \tag{9}$$

$$h_c = \frac{Q}{A (T_c - T_2)} \tag{10}$$

$$h_c = \frac{Q}{A (T_c - T_2)} = \frac{1}{R_c} \tag{10}$$

NOTE 5—The surface heat transfer coefficient, h_i, and the corresponding surface thermal resistance, R_i, are reciprocals, that is, their product is unity. These properties are measured at a specific set of ambient conditions and are therefore only correct for the specified conditions of the test.

3.4.3 *Overall ~~t~~ Thermal ~~r~~ Resistance, R_u,* —~~t~~The quantity determined by the temperature difference, at steady-state, between the air temperatures on the two sides of a body or assembly that induces a unit time rate of heat flow per unit area through the body. It is the sum of the resistance~~s~~ of the body or assembly and of the two surface resistances and may be calculated as follows:

$$R_u = \frac{A (T_1 - T_2)}{Q} \tag{11}$$

$$= R_c + R + R_h$$

3.4.4 *Thermal ~~t~~ Transmittance, U* (sometimes called overall coefficient of ~~therma~~thermal ~~t~~ transfer), is calculated as follows:

$$U = \frac{Q}{A (T_1 - T_2)} \tag{12}$$

$$U = \frac{Q}{A (T_1 - T_2)} = \frac{1}{R_u} \tag{12}$$

The transmittance can be calculated from the thermal conductance and the surface coefficients as follows:

$$1/U = (1/h_h) + (1/C) + (1/h_c) \tag{13}$$

NOTE 6—Thermal transmittance, U, and the corresponding overall thermal resistance, R_u, are reciprocals~~,~~; that is, their product is unity. These properties are measured at a specific set of ambient conditions and are therefore only correct for the specified conditions of the test.

4. Significance and Use

4.1 ASTM thermal test method descriptions are complex because of added apparatus details necessary to ensure accurate results. As a result, many users find it difficult to locate the data reduction details necessary to reduce the data obtained from these tests. This practice is designed to be referenced in the thermal test methods, thus allowing th~~e~~ose test methods to concentrate on experimental details rather than data reduction.

4.2 This practice is intended to provide the user with a uniform procedure for calculating the thermal transmission properties of a material or system from standard test methods used to determine heat flux and surface temperatures. This practice is intended to ~~replace~~ eliminate the need for similar calculation sections ~~of~~ in the ASTM Test Methods ~~C 177, C 236,~~ (C 177, C 335, C 518, ~~C 745,~~ C 976, C 1033, C 1114, C 1199, and ~~C 1114.~~ C 1363) by permitting use of these standard calculation forms by reference.

4.3 This practice provides the method for developing the thermal conductivity as a function of temperature for a specimen from data taken at small or large temperature differences. This relationship can be used to characterize material for comparison to material specifications and for use in calculations programs such as Practice C 680.

4.4 Two general solutions to the problem of establishing thermal transmission properties for application to end-use conditions are outlined in Practice C 1058. (Practice C 1058 should be reviewed prior to use of this practice.) One is to measure each product for at each end-use condition. This solution is rather straightforward, but burdensome, and needs no other elaboration. The second is to measure each product over the entire temperature range of application conditions and to use these data to establish the thermal transmission property dependencies on at the various end-use conditions. One advantage of the second approach is that once these dependencies have been established, they serve as the basis for estimating the performance for a given product to at other conditions.

4.5 **Precaution**—The use of a thermal conductivity curves developed in Section 6 must be limited to a temperature range that does not extend to less than beyond the range of highest and lowest test surface temperature or higher than temperatures in the highest surface temperature for the test data set used to generate the curves.

5. Determination of Thermal Transmission Properties for a Specific Set of Temperature Conditions

5.1 UChoosinge the appropriate test method of interest, determine the steady-state heat flux and temperature data for the test.

NOTE 7—The calculation of thermal properties requires that: (1) the thermal insulation specimen is homogeneous, as defined in Terminology C 168 or, as a minimum, appears uniform across the test area; (2) the measurements are taken only after steady-state has been established; (3) the heat flows in a direction normal to the isothermal surfaces of the specimen; (4) the rate of flow of heat is known; (5) the specimen dimensions, that is, heat flow path length parallel to heat flow, and area perpendicular to heat flow, are known; and (6) both specimen surface temperatures (and equivalently, the temperature difference across the specimen) are known. In the case of a hot box systems test, both air curtain temperatures must be known.

5.2 Choose the thermal test parameter (λ or r', R or C, U or R_u) to be calculated from the test results.

5.3 Calculate results. List any additional information required by that calculation i.e. heat flux, temperatures, dimensions. Recall that the selected test parameter might limit the selection of the thermal test method used in 5.2.

5.2 Select the appropriate test method that provides the thermal test data that can be used to determine the thermal transmission property of interest using for the sample material being studied. (See referenced papers and Appendix X1 for help with this determination.

5.3 Using that test method, determine the required steady-state heat flux and temperature data from at the selected test condition.

NOTE 7—The calculation of specific thermal transmission properties requires that: (*1*) the thermal insulation specimen is homogeneous, as described defined in 5.1, Terminology C 168 or, as a minimum, appears uniform across the test area; (*2*) the measurements are taken only after steady-state has been established; (*3*) the heat flows in a direction normal to the isothermal surfaces of the specimen; (*4*) the rate of flow of heat is known; (*5*) the specimen dimensions, that is, heat flow path length parallel to heat flow, and area perpendicular to heat flow, are known; and (*6*) both specimen surface temperatures (and equivalently, the temperature difference across the specimen) are known; and in the case of a hot box systems test, both air curtain temperatures must be known.

5.4 Calculate the thermal property using the data gathered in 5.2 and 5.3, and the appropriate equation in 3.3 or 3.4.

5.4 Using 3.4 above. The user of this practice is responsible for insuring that the input data from the tests conducted are consistent with the defined properties of the test parameter prior to parameter calculation. A review of the information in Section 7 will help in this evaluation. For example, data should be examined for consistency in such areas as heat flow stability, heat flow orientation, metering area, geometry limits, surface temperature definition and others.

5.5 Using the data from the test as described in 5.13, determine the test mean temperature for the thermal property of 5.3 5.4 using the following equation: Eq 14:

$$T_m = (T_h + T_c)/2 \qquad (14)$$

NOTE 8—The thermal transmission properties determined in 5.3 5.4 are applicable only for the conditions of the test. Further analysis is required using data from multiple tests if the relationship for the thermal transmission property variation with mean test temperature is to be determined. If this relationship is required, the analysis to be followed is presented in Section 6.

5.56 *An Example of a Example: Computation of Thermal Conductivity Measured in a Two-Sided Guarded Hot Plate*:

5.56.1 For a guarded hot plate apparatus in the normal, double-sided mode of operation, the heat developed in the metered area heater passes through two specimens. To reflect this fact, Eq 3 for the operational definition of the mean thermal conductivity of the pair of specimens must be modified to read:

$$\lambda_{exp} = \frac{Q}{A\,[(\Delta T_{/L})_1 + (\Delta T_{/L})_2]} \qquad (15)$$

$$\lambda_{exp} = \frac{Q}{A\,[(\Delta T_{s-s}/L)_1 + (\Delta T_{s-s}/L)_2]} \qquad (15)$$

where:
$(\Delta T_{s-s}/L)_1$ = the ratio of surface-to-surface temperature difference to thickness for Specimen 1. A similar expression is used for Specimen 2.

5.56.2 In many experimental situations, the two temperature differences are very nearly equal (within well under 1 %), and the two thicknesses are also nearly equal (within 1 %), so that Eq 15 may be well approximated by a simpler form:

$$\lambda_{exp} = \frac{Q\,L_{average}}{2A\,\Delta T_{average}} \qquad (16)$$

where:

$\Delta T_{average}$ = the ~~arithmetic~~ mean temperature difference, $((\Delta T_{s-s})_1 + (\Delta T_{s-s})_2)/2$,

$L_{average}$ = $(L_1 + L_2)/2$ is the ~~arithmetic~~ mean of the two specimen thicknesses, and

$2A$ = occurs because the metered power flows out through two surfaces of the metered area for this apparatus. For clarity in later discussions, use of this simpler form, Eq 16, will be assumed.

NOTE 9—The mean thermal conductivity, λ_m, is usually not the same as the thermal conductivity, $\lambda(T_m)$, at the mean temperature T_m. The mean thermal conductivity, λ_m, and the thermal conductivity at the mean temperature, $\lambda(T_m)$, are equal only in the special case where $\lambda(T)$ is a constant or linear function of temperature (1),; that is, when there is no curvature (nonlinearity) in the conductivity-temperature relation. In all other cases, the conductivity, λ_{exp}, as determined by Eq 3 is not simply a function of mean temperature, but depends on the values of both T_h and T_c. This is the reason the experimental value, λ_{exp}, of thermal conductivity for a large temperature difference is not, in general, the same as that for a small difference at the same mean temperature. The discrepancy between the mean thermal conductivity and the thermal conductivity at the mean temperature increases as ΔT increases. Treatment of these differences is discussed in Section 6.

5.56.3 When ΔT is so large that the mean (experimental) thermal conductivity differs from the thermal conductivity at the mean specimen temperature by more than 1 %, the derived thermal conductivity (Eq 3) shall be identified as a mean value, λ_m, over the range from T_c to T_h. For example, for the insulation material presented in X3.4, the 1 % limit is exceeded for temperature differences greater than 125 K at a temperature of 475 K. Reference (1) describes a method for establishing the actual λ versus T dependency from mean thermal conductivity measurements. Proofs of the above statements, along with some illustrative examples, are given in Appendix X3.

6. Determination of the Thermal Conductivity Relationship for a Temperature Range

6.1 Consult Practice C 1058 for the selection of appropriate test temperatures. Using the appropriate test method of interest, determine the steady-state heat flux and temperature data for each test covering the temperature range of interest.

6.2 *Small tWhen Temperature ~~d~~ Differences are Small*—The use of Eq ~~15~~ 3 or Eq ~~16~~ 4 is valid for determining the thermal conductivity versus ~~mean~~ temperature only if the temperature difference between the hot and cold surfaces is small. For the purpose of this practice, experience with most insulation materials at temperatures above ambient shows that the maximum ΔT should be 25 K or 5 % of the ~~absolute~~ mean temperature (K), whichever is greater. At temperatures below ambient, the temperature difference should be less than 10 percent of the absolute mean temperature. (See Reference (1)). The procedure given in section 6.2.1 may be followed only when th~~is~~ese temperature difference ~~standard is~~ conditions are met. The procedure of section 6.3 may be used for all test data reduction.

NOTE 10—One exception to this temperature difference conditions is testing of insulation materials exhibiting inflection points due to the change of state of insulating gases. For these materials, testing shall be conducted with sufficiently small temperature differences and at closely spaced mean temperatures. The selection of test temperatures will depend on the vapor pressure versus temperature relationship of the gases involved and the ability of the test apparatus to provide accurate measurements at low temperature differences. Another exception occurs with the onset of convection within the specimen. At this point, the thermal conductivity of the specimen is no longer defined at these conditions and the thermal parameter of choice to be calculated is either thermal resistance or thermal conductance.

6.2.1 The quantities on the right-hand side of Eq ~~15~~ 3 are known for each data point; from these quantities $\lambda(T)$ may be calculated if ΔT is sufficiently small (see 6.2), for normal insulation applications. The value of $\lambda(T)$ so obtained is an approximation, its accuracy depends on the curvature (non-linearity) of the thermal conductivity-temperature relationship (1). It is conventional to associate the value of λ_{exp} obtained from Eq 16 with the mean temperature T_m at the given data point. For data obtained at a number of mean temperatures, a functional dependence of λ with T may be obtained, with functional coefficients to be determined from the data. ~~For~~ In order to apply a least squares fit to the ~~situation where~~ data, the number of data points ~~is~~ shall be greater than the number of coefficients in the ~~function, the quantities calculated from the right-hand side over determine the coefficients, and a least-squares fit~~ function to ~~the data may be used to~~ obtain the functional dependence of the thermal conductivity λ ~~with~~ T on temperature, T. The accuracy of the coefficients thus obtained depend not only on the experimental imprecision, but also on the extent to which the thermal conductivity-temperature relationship departs from ~~a linear one~~ the true relationship over the temperature range defined by the isothermal boundaries of the specimen during the tests.

6.3 *Computation of ~~t~~ Thermal ~~c~~ Conductivity ~~w~~ When ~~t~~ Temperature Differences are ~~l~~ Large*—The following sections apply to all testing results and are specifically required when the temperature difference ~~is greater than about 5 % of~~ exceeds the ~~absolute mean temperature.~~ limits stated in 6.2. This situation typically occurs during measurements of ~~thermal~~ transmission in pipe insulation, Test Method C 335, but may also occur with measurements using other apparatus. Eq 17 and ~~Eq~~ 18 are developed in Appendix X2, but are presented here for continuity of this practice.

6.3.1 The dependence of λ on T for flat-slab geometry is:

$$\lambda_{\mathrm{m}} = \frac{1}{\Delta T}\int_{T_c}^{T_h} \lambda(T)\, dT$$

$$\lambda_{\mathrm{m}} = \frac{1}{\Delta T}\int_{T_c}^{T_h} \lambda(T)\, \delta T$$

or:

$$\lambda_{\mathrm{m}} = QL \, / \, [2\,A\,(T_h - T_c)] \tag{17}$$

The quantities T_h, T_c, Q, and $(L/2A)$ on the right-hand side are known for each data point obtained by the user.

6.3.2 The dependence of λ on T for cylindrical geometry is:

$$\lambda_{\mathrm{m}} = \frac{1}{\Delta T}\int_{T_c}^{T_h} \lambda(T)\, dT$$

$$\lambda_{\mathrm{m}} = \frac{1}{\Delta T}\int_{T_{out}}^{T_{in}} \lambda(T)\, \delta T$$

or:

$$\lambda_{\mathrm{m}} = \frac{Q\ln(r_2/r_1)}{2\,\pi\,l\,(T_h - T_c)} \tag{18}$$

$$\lambda_{\mathrm{m}} = \frac{Q\ln(r_{out}/r_{in})}{2\,\pi\,L_p\,(T_{in} - T_{out})} \tag{18}$$

The quantities T_{in}, T_{out}, Q, $ln(r_{out}/r_{in})$ and $2\pi L_p$ on the right-hand side, are known for each data point obtained by the user.

6.4 *Thermal Conductivity Integral (TCI) Method*—To obtain the dependence of thermal conductivity on temperature from Eq 17 or Eq 18, a specific functional dependence to represent the conductivity-temperature relation must first be chosen. This Practice recommends that the functional form of the describing equation closely describe the physical phenomena governing the heat transfer through the sample. While not absolutely necessary, choosing the physically correct equation form can provide better understanding of the physical forces governing the heat flow behavior. After the form of the thermal conductivity equation is chosen, steps 6.4.1-6.4.3 are followed to determine the coefficients for that equation.

6.4.1 Integrate the selected thermal conductivity function with respect to temperature. For example, if the selected function $\lambda(T)$ were a polynomial function of the form

$$\lambda(T) = A_o + A_n T^n + A_m T^m, \tag{19}$$

$$\lambda(T) = a_o + a_n T^n + a_m T^m, \tag{19}$$

then, from Eq 18, the temperature-averaged thermal conductivity would be:

$$\lambda_{\mathrm{m}} = \frac{A_o(T_h - T_c)}{(T_h - T_c)} + \frac{A_n(T_h^{n+1} - T_c^{n+1})}{(n+1)(T_h - T_c)} + \frac{A_m(T_h^{m+1} - T_c^{m+1})}{(m+1)(T_h - T_c)} \tag{20}$$

$$\lambda_{\mathrm{m}} = a_o + \frac{a_n(T_h^{n+1} - T_c^{n+1})}{(n+1)(T_h - T_c)} + \frac{a_m(T_h^{m+1} - T_c^{m+1})}{(m+1)(T_h - T_c)} \tag{20}$$

6.4.2 By means of any standard least-squares fitting routine, the right-hand side of Eq 20 is fitted against the values of experimental thermal conductivity, λ_{exp}. This fit determines the coefficients (a_o, a_n, a_m) for the selected n and m in the thermal conductivity function, Eq 19 in this case.

6.4.3 Use the coefficients obtained in 6.4.2 to describe the assumed thermal conductivity function, Eq 19. Each data point is then conventionally plotted at the corresponding mean specimen temperature. When the function is plotted, it may not pass exactly through the data points. This is because each data point represents mean conductivity, λ_{m}, and this is not equal to the value of the thermal conductivity, $\lambda(T_{\mathrm{m}})$, at the mean temperature. The offset between a data point and the fitted curve depends on the size of test ΔT and on the nonlinearity of the thermal conductivity function.

NOTE 1—Many equation forms other than Eq 19 can be used to represent the thermal conductivity function. If possible, the equation chosen to represent the thermal conductivity versus temperature relationship should be easily integrated with respect to temperature. However, in some instances it may be desirable to choose a form for $\lambda(T)$ that is not easily integrated. Such equations may be found to fit the data over a much wider range of temperature. Also, the user is not restricted to the use of polynomial equations to represent $\lambda(T)$, but only to equation forms that can be integrated either analytically or numerically. In cases where direct integration is not possible, one can carry out the same procedure using numerical integration.

6.5 *TCI Method—A Summary*—The thermal conductivity integral method of analysis is summarized in the following steps:

6.5.1 Measure several sets of λ_{exp}, T_h, and T_c over a range of temperatures.

6.5.2 Select a functional form for $\lambda(T)$ as in Eq 19, and integrate it with respect to temperature to obtain the equivalent of Eq 20.

6.5.3 Perform a least-squares fit to the experimental data of the integral of the functional form obtained in 6.5.2 to obtain the best values of the coefficients.

6.5.4 Use these coefficients to complete the $\lambda(T)$ equation as defined in 6.5.2. Remember that the thermal conductivity equation derived herein is good only over the range of temperatures encompassed by the test data. Extrapolation of the test results to a

temperature range not covered by the data is not acceptable.

7. Consideration of Test Result Significance

7.1 A final step in the analysis and reporting of test results requires that the data be reviewed for significance and accuracy. It is not the intent of this practice to cover all aspects of the strategy of experimental design, but only identify areas of concern. Some additional information is provided in the Appendix but the interested reader is referred to the reference section for more detailed information. The following areas should be considered in the evaluation of the test results produced using a Practice C 1045 analysis.

7.2 *Assessment of ~~a~~ Apparatus ~~u~~ Uncertainty*—The determination of apparatus uncertainty should be performed as required by the appropriate apparatus test method.

7.3 *Material ~~i~~ Inhomogeneity*—The uncertainty caused by specimen inhomogeneity can seriously alter the measured dependencies. To establish the possible consequences of material inhomogeneity on the interpretation of the results, the user shall measure an adequate fraction of the product over the entire range of product manufacture variations. ~~The specimen area is generally much smaller than the surface area of the delivered product.~~ If possible, several specimens should be measured to sample a sufficient portion of the product. The resultant mean value of the measurements is representative of the product to within the uncertainty of the mean, while the range of the results is indicative of the product's inhomogeneity. Additional information regarding sampling procedures can be found in Practice E 122.

7.4 *Test ~~g~~ Grid*—The thermal transmission properties ~~o~~ determined for an insulation~~s~~ are dependent on several variables, including product classification, temperature, density, plate emittance, fill-gas pressure, temperature difference, and fill-gas species. The effect of the insulation material variability (inhomogeneity) is an important parameter in asessing the significance of results and their application to design or quality control. A ~~total~~ complete characterization of these dependencies would require~~s~~ the measurement of ~~the~~ thermal transmission for all possible combinations of these ~~variables at an adequate number~~ variables. Analysis of this magnitude demands the use of statistical experimental design in order to develop sufficient data while minimizing testing costs. However, since the producer and consumer of a product are seldom interested in the entire range of properties possible, most industr~~y~~ial specifications ~~generally~~ require specific test conditions on representative ~~samples. It must be emphasized that the effect of material variability (inhomogeneity) is an important parameter in assessing the significance of results from the test grid.~~ samples.

7.5 *Range of ~~t~~ Test ~~t~~ Temperatures*—The test temperature range for each variable shall include the entire range of application to avoid extrapolation of any measured dependency. Guidance for this selection is presented in Table 3 of Practice C 1058.

8. Report

8.1 The report of thermal transmission properties shall include all necessary items specified by the test method followed.

8.2 The total uncertainty of the thermal transmission properties shall be calculated ~~in accordance with~~ according to the test method and reported.

8.3 The report shall include any test conditions on which the thermal transmission properties are dependent.

8.4 ~~Any corrections shall be specified, and the basis of these corrections shall be given.~~

~~8.5 If~~

8.4 If mean values are reported for tests employing large temperature differences ~~(larger than 5 % of~~ (see 6.2), the ~~absolute temperature), the range of~~ temperature differences shall be reported.

8.~~6~~5 When the thermal conductivity versus temperature relationship has been determined, report the equation with its coefficients, the method of data analysis and regression, and the range of temperatures that were used to determine the coefficients.

8.~~7~~6 The temperature range of usefulness for the equation coefficients shall be ~~specified.~~

~~8.8 Results shall be reported in either SI or conventional units as specified by~~ specified. For example, using the ~~requester. If not~~ data of Table X3.1 yields a temperature range of usefulness of the coefficients of 286 K to 707 K.

8.7 Unless otherwise specified, the calculation and reporting of C 1045 results shall be ~~reported~~ in SI units.

9. Using C 1045 in Specifications

9.1 Material specifications can benefit from the use of C 1045 in specifying the apparent thermal conductivity relationship desired. It is important that the material be specified by intrinsic properties that are independent of test conditions to insure that the method of test, or the conditions used during the test do not influence the results. Practice C 1045 provides a method of identifying the material's relationship between temperature and thermal properties independent of temperature difference. To insure that C 1045 is used properly, the following paragraphs are recommended for inclusion in material specifications when specifying thermal properties that are a function of temperature.

9.1.1 The apparent thermal conductivity as a function of temperature for the representative specimens shall be determined with data obtained from a series of thermal tests utilizing test methods C 177, C 335, C 518, C 976, C 1033, C 1114, C 1199 or C 1363 as appropriate for the material under study.

9.1.2 The test method selected shall have proven correlation with C 177 over the temperature range of conditions used. In cases of dispute, C 177 shall be considered as the final authority for materials having flat geometry, while C 335 shall be used for materials having cylindrical geometry.

9.1.3 Practice C 1058 may be used to obtain recommended test temperature combinations for data generation.

9.1.4 As specified in C 1045, the range of test conditions shall include at least one test where the hot surface temperature is greater than, or equal to, the hot limit of the temperature usage range of desired data and at least one test where the cold surface temperature is less than, or equal to, the cold limit of the temperature usage range desired. At least two additional tests shall be distributed somewhat evenly over the rest of the temperature usage range.

NOTE 12—Many existing material specifications require that the two tests at the extremes of the temperature range have mean temperatures within 30K of the temperature limits. While not a requirement of C 1045, this practice is thought, by the material specification writers, to be helpful in obtaining more accurate results.

9.1.5 Final analysis of the thermal data shall be conducted in accordance with C 1045 to generate an apparent thermal conductivity versus temperature relationship for the specimen. The C 1045 analysis shall be conducted in SI or IP units as specified by the material specification.

9.1.6 Final output of the analysis shall be a table of data where the apparent thermal conductivity is calculated for the temperatures specified by the specification. Comparison to the specification can then be made directly.

9.1.7 The apparent thermal conductivity versus temperature equation may also be included for reporting purposes. Be aware, however, that no direct comparison of the equation coefficients can be made due to differences in the model used. Comparison shall be made on calculated values at selected temperature points within the range.

9.1.8 **Caution:** While it is recommended that the specification data be presented as apparent thermal conductivity versus temperature, several existing specifications contain mean temperature data from tests conducted at specific hot and cold surface temperatures. In these cases, the apparent thermal conductivity as a function of temperature from the C 1045 analysis may provide different results. In order to make a fair evaluation, a C 680 analysis will be required to determine the effective thermal conductivity for comparison to the specification requirements. The input data for the C 680 analysis would be apparent thermal conductivity versus temperature relationship from C 1045 and the specific hot and cold surface temperatures from the material specification.

10. Keywords

9.1 calculation; integral method;

10.1 calculation; thermal conductance; thermal conductivity; thermal properties; thermal resistance; thermal resistivity; thermal transmission

APPENDIXES

(Nonmandatory Information)

X1. GENERAL DISCUSSION OF THERMAL PROPERTIES MEASUREMENT

X1.1 Thermal transmission properties, that is, thermal conductivity and thermal resistivity, are considered to be intrinsic characteristics of a material. These intrinsic properties are dependent on temperature as well as the microscopic structure of the material. Furthermore, some external influences, such as pressure, may affect the structure of a material and, therefore, its thermal properties. For heterogeneous materials such as those composed of granules, fibers, or foams, additional conductive dependencies arise due to the presence of the fill-gas. As long as the heat flux mechanism is conductive, each of the dependencies is characteristic of the structure and constituents of the material. When only conductive heat flux is present, the measurement, calculation of thermal properties, and application of the results to end-use conditions is are well defined by the literature (**1-214**).

X1.2 The measurement of a thermal conductivity or thermal resistivity meeting the fundamental definition of an intrinsic property, requires the measurement of the true temperature gradient. Since it is impossible to measure the gradient at point within the insulation directly, an operational definition of these properties must be used. The operational definition replaces the gradient at a point with the overall temperature gradient defined as the overall temperature difference divided by the total thickness. So long as this substitution is adequate, the relationship is good. For purely conductive heat transfer, the adequacy of this treatment is accomplished by keeping the temperature difference small.

X1.3 In some materials, non-conductive heat fluxes are present that result in property dependencies on specimen dimensions, test temperature conditions, or apparatus parameters. This is not to be confused with the effect of measurement errors that are dependent on specimen or apparatus characteristics. The thermal conductivity of very pure metals at low temperature, for example, actually is dependent on the dimensions of the specimen when they are sufficiently small. This phenomenon is referred to as the size effect, and represents a deviation from conductive behavior. A similar phenomenon occurs in materials that are not totally opaque to radiation. The thermal transmission properties for such materials will be dependent on the specimen thickness and the test apparatus surface plate emittance. This is commonly referred to as the "thickness effect" (**3-9**). The heat flux in a heterogeneous material containing a fill-gas or fluid may, under certain conditions of porosity or temperature gradient, have a convective heat flux component (**10,11**). The resulting thermal transmission properties may exhibit dependencies on specimen size, geometry, orientation, and temperature difference.

X1.34 The existence of such non-intrinsic dependencies has caused considerable discussion regarding the utility of thermal transmission properties (12,13). From a practical standpoint, they are useful properties for two reasons. First, the transition from conductive to non-conductive behavior is a gradual and not an abrupt transition, and the dependencies on specimen size, geometry, and orientation are generally small. Second, pseudo thermal transmission properties can be calculated that apply to a restricted range of test conditions and are usually denoted by adding the modifier effective or apparent, for example, apparent thermal conductivity. For these pseudo properties to be useful, care must be exercised to specify the range of test conditions under which they are obtained.

X1.45 Some of the thermal-transmission property dependencies of interest may be quite small; however, others may be quite large. It is important that the uncertainties associated with the measurement procedure and material variability are known. Uncertainties caused by systematic errors can seriously alter the conclusions based on the measured dependencies. This point was clearly illustrated by an interlaboratory study on low-density fibrous glass insulation (14). In this round robin, each of five laboratories determined the thickness effect for insulation thickness from 2.54 to 10.2 cm thickness. cm. The lowest thickness effect observed was 2 %, while the highest was 6 %. The best estimate of the actual thickness effect clearly involvesd an in-depth analysis of the measurement errors of each laboratory. This analysis subsequently created the demand for the development of high R value transfer standards for use in calibrating these apparatus. Also, when a study of this type is undertaken, care must be taken to clearly identify the product involved so that the dependencies determined are assigned only to that product. For example, it is unlikely that the thickness dependencies of glass fiber and cellulosic insulations are identical.

X2. DEVELOPMENT OF EQUATIONS FOR PRACTICE C 1045 ANALYSIS

X2.1 This development of equations necessary to support Practice C 1045 applies to the flow of heat through a homogeneous insulation exhibiting a thermal conductivity that only depends on temperature. Existing methods of measurement of thermal conductivity account for various modes of heat transmission, that is, thermal conduction, convection and radiation, occurring within insulation under steady-state, one-dimensional heat flow conditions. Fourier's law of heat conduction has been derived in many heat transfer texts. Fourier's law is generally stated as the heat flux being proportional to the temperature gradient, or:

$$q = -\lambda(T)(dT/dp) \tag{X2.1}$$

$$q = -\lambda(T)(\delta T/\delta p) \tag{X2.1}$$

where the proportionality coefficient is the thermal conductivity as a function of temperature and p is the coordinate along which heat is flowing. Development of equations for test method heat flow in the slab (Test Methods C 177, C 518, C 1114, etc.) and radial heat flow in the hollow right circular cylinder (Test Method C 335) will be performed using the boundary conditions:

$$T = T_1 \text{ at } x = x_1, \text{ or } r = r_1 \tag{X2.2}$$

$$T = T_c \text{ at } x = x_c, \text{ or } r = r_c \tag{X2.2}$$

$$T = T_2 \text{ at } x = x_2, \text{ or } r = r_2$$

$$T = T_h \text{ at } x = x_h, \text{ or } r = r_h$$

X2.2 Case 1, Slab Insulation, substituting $p = x$ in Eq X2.1 and performing the indicated integration:

$$q \int_{x_1}^{x_2} dx = -\int_{T_1}^{T_2} \lambda (T) \, dt \tag{X2.3}$$

$$q \int_{x_c}^{x_h} \delta x = -\int_{T_c}^{T_h} \lambda (T) \, \delta t \tag{X2.3}$$

yields:

$$q = \lambda_{eff} \frac{(T_1 - T_2)}{(x_1 - x_2)} \tag{X2.4}$$

$$q = \lambda_{eff} \frac{(T_h - T_c)}{(x_h - x_c)} \tag{X2.4}$$

where:

$$\lambda_{\text{eff}} = \int_{T_1}^{T_2} \lambda (T) \, dt / (T_1 - T_2) \tag{X2.5}$$

$$\lambda_{\text{exp}} = \frac{1}{(T_h - T_c)} \int_{T_c}^{T_h} \lambda (T) \, \delta T \tag{X2.5}$$

X2.3 Radial heat flow in hollow cylinders, substituting $p = r$ in Eq X2.1, and letting:

$$q = \frac{Q}{2 \pi r l} \tag{X2.6}$$

$$q = \frac{Q}{2 \pi r L} \tag{X2.6}$$

and:

$$q = -\lambda (T)\frac{dt}{dr}$$

$$q = -\lambda (T)\frac{\delta t}{\delta r}$$

combining these two expressions and performing the indicated integration:

$$\frac{Q}{2 \pi l} \int_{r_1}^{r_2} \frac{dr}{r} = -\int_{T_1}^{T_2} \lambda (T)\, dT \tag{X2.7}$$

$$\frac{Q}{2 \pi L} \int_{r_{out}}^{r_{in}} \frac{\delta r}{r} = -\int_{T_{out}}^{T_{in}} \lambda (T)\, \delta T \tag{X2.7}$$

where:
r = radius.
therefore:

$$Q = \lambda_{eff} \frac{2 \pi l (T_1 - T_2)}{\ln (r_2/r_1)} \tag{X2.8}$$

$$Q = \lambda_{exp} \frac{2 \pi L (T_{in} - T_{out})}{\ln (r_{out}/r_{in})} \tag{X2.8}$$

X3. THERMAL CONDUCTIVITY VARIATIONS WITH MEAN TEMPERATURE

X3.1 The purpose of this appendix is to expand upon statements made in the body of this practice relative to the handling of data from the thermal conductivity tests. Some examples are given to clarify the difference between the analysis of thermal conductivity data taken at large temperature differences and the analysis of conductivity data taken at small temperature differences. The necessity for a difference in analysis method is based on the distinction between mean thermal conductivity, λ_m, and thermal conductivity at the mean temperature, $\lambda (T_m)$, when the conductivity varies nonlinearly with temperature. For this discussion, the arithmetic mean of a variable x is denoted by the subscript m.

X3.2 Eq X3.1 provides the mathematical definition of the mean value of the thermal conductivity with respect to temperature over the range of temperature from T_c to T_h:

$$\lambda_m = \frac{1}{(T_h - T_c)} \int_{T_c}^{T_h} \lambda (T)\, dT \tag{X3.1}$$

X3.3 *Example 1—Thermal Conductivity as a Polynomial Function*:

X3.3.1 The thermal conductivity versus temperature relationship for many typical insulation materials can be defined by a third order polynomial equation. Eq X3.2 describes this thermal conductivity correlation.

$$\lambda(T) = a_0 + a_1 T + a_2 T^2 + a_3 T^3 \tag{X3.2}$$

or in terms of T_h and T_c, where $T_m = (T_h + T_c)/2$:

$$\lambda(T_m) = a_0 + a_1 (T_h + T_c)/2 + a_2(T_h^2 + 2T_h T_c + T_c^2)/4 \tag{X3.3}$$

$$+ a_3 (T_h^3 + 3T_h^2 T_c + 3T_h T_c^2 + T_c^3)/8$$

NOTE X3.1—In this and succeeding examples, the coefficients a_i ($i = 0,1,2,...$) are constants.

X3.3.2 Substituting Eq X3.2 into Eq X3.1 and integrating the thermal conductivity correlation over temperature, yields:

$$\lambda_m =$$
$$[a_0(T_h - T_c) + a_1 (T_h^2 - T_c^2)/ 2 + a_2 (T_h^3 - T_c^3)/ 3 + a_3 (T_h^4 - T_c^4) /4](T_h - T_c) \tag{X3.4}$$

$$\lambda_m = \frac{[a_0(T_h - T_c) + a_1 (T_h^2 - T_c^2)/ 2 + a_2 (T_h^3 - T_c^3)/ 3 + a_3 (T_h^4 - T_c^4) /4]}{(T_h - T_c)} \tag{X3.4}$$

$$= a_0 + a_1 (T_h + T_c)/2 + a_2(T_h^2 + T_h T_c + T_c^2)/$$
$$3 + a_3(T_h^2 + T_c^2) (T_h + T_c)/ 4$$

X3.3.3 The difference between mean thermal conductivity, λ_m, and thermal conductivity at the mean temperature, $\lambda (T_m)$, as defined by Eq X3.2 and X3.3 and X3.4 yields:

$$\lambda_m - \lambda(T_m) = (T_h - T_c)^2 [a_2/12 + a_3/8 (T_h + T_c)] \tag{X3.4}$$

$$\lambda_m - \lambda(T_m) = (T_h - T_c)^2 [a_2/12 + (a_3/8)(T_h + T_c)] \tag{X3.5}$$

X3.3.4 Eq X3.45 shows that this difference between the mean thermal conductivity and the thermal conductivity at the mean temperature is zero, regardless independent of the values of the constants a_0 and a_1, and is therefore valid also zero for the special cases (1) constant thermal conductivity with temperature ($\lambda = a_0$ = constant, that is, the terms a_1, a_2, and a_3 are zero), and (2) linear thermal conductivity ($\lambda = a_0 + a_1 T$, that is, a_2 and a_3 are zero), as well.

X3.3.5 Eq X3.45 also shows that for materials where the coefficients a_2 and a_3 are not zero, the difference between the mean thermal conductivity and the thermal conductivity at the mean temperature is a strong function of the temperature difference, $(T_h - T_c)^2$.

X3.4 *Example 2—"Real" Data*

X3.4.1 The final example illustrates the magnitude of the difference, $\lambda_m - \lambda(T_m)$, based on data for temperatures ranging from 13 286 to 435°C 707 K, for fibrous board insulation. a 292 kg/m³ insulation board. This data, presented in Table X3.1, were acquired from measurements on the same specimen set at both limited ($\Delta T < 110K$) and variable (ΔT up to $\leq$ 360 K) temperature differences. The fibrous board insulation has been represented by an equation of the form:

$$\lambda(T) = a_0 + a_1 T + a_3 T^3 \tag{X3.6}$$

X3.4.2 Combining Eq X3.1 and Eq X3.56, the equation for λ_m becomes:

$$\lambda_m = a_0 + a_1(T_h + T_c)/2 + a_3(T_h^2 + T_c^2)(T_h + T_c)/4 \tag{X3.6}$$

$$\lambda_m = a_0 + a_1(T_h + T_c)/2 + a_3(T_h^2 + T_c^2)(T_h + T_c)/4 \tag{X3.7}$$

X3.4.3 Using the data in Table X3.1 and Eq X3.67 and solving for coefficients of Eq X3.6 using a standard statistical analysis program yields the following values for the coefficients for the fibrous board insulation described by Eq X3.56:

$$a_0 = 0.03044$$

$$a_0 = 31.7408$$

$$a_1 = 1.287E-04$$

$$a_1 = -3.1308E-2$$

(X3.9) $a3 = 1.078E-09$

$$a_3 = 4.5377E-7$$

where the temperatures are in degrees celsius. Kelvin and the thermal conductivity is in (mW/m · K). Note that the standard estimate of error provided by the spreadsheet analysis for the correlation of this data below was 0.0003 W/mK 0.66 mW/m · K

X3.4.4 Fig. X3.1 shows how the test data compares with the final data regression equation as a function of mean temperature. N Table X3.2 compares the thermal conductivity λ (T_m) calculated at the mean temperature with the experimental thermal conductivity λ_{exp} values and the difference, $\lambda_{exp} - \lambda(T_m)$, between the two values. Fig. X3.2 shows that as the temperature difference of increases, the data increase, the difference between in the regression line and data points thermal conductivity increases.

TABLE X3.1 MExpeasurimed Apparent Thermal Conductivity (λ_{exp}) versus Hot (T$_h$) and Cold F (T$_c$) Surface Temperatures

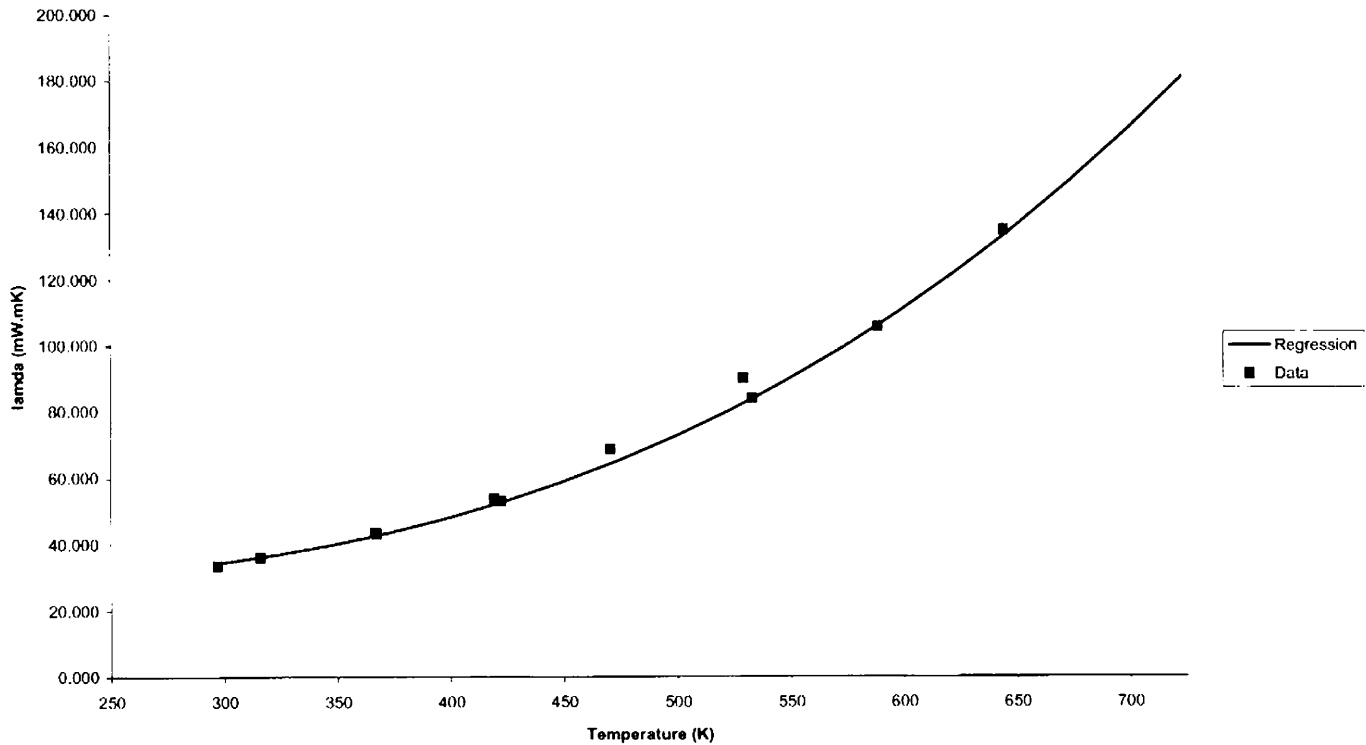| Hot F Surface Temperature (°CT$_h$) (K)	Cold F Surface Temperature (°CT$_c$) (K)	Thermal Conductivity (mW/m·K)
35.0	12.8	0.0336
308.2	285.9	33.6
60.1	25.2	0.0362
333.3	298.4	36.2
121.1	65.6	0.0436
394.3	338.7	43.6
154.6	33.3	0.0434
427.8	306.4	43.4
176.1	121.1	0.0531
449.8	394.3	53.1
247.3	44.4	0.0539
520.5	317.5	53.9
315.6	204.4	0.0841
588.7	477.6	84.1
336.4	58.2	0.0687
609.6	331.3	68.7
371.1	260.0	0.1057
644.3	533.2	105.7
426.7	315.6	0.1346
699.8	588.7	134.6
434.5	77.4	0.0900
707.7	350.6	90.0

FIG. X3.1 Test Data versus Regression Plot

X3.5 *Summary*:

X3.5.1 The above examples reveals the method by which one can obtain an apparent thermal conductivity versus temperature relationship, λ (*T*) from measurements of at large temperature differences. The method described here is referred to as the integral method and is described in detail in Ref (**12**). First, note that any experimental value of thermal conductivity, λ_{exp}, obtained using Eq 2 and measured values of *q*, *T*, and *L*, is really a value of the thermal conductivity averaged over the temperature range, λ_m and not the thermal conductivity at the mean temperature, $\lambda(T_m)$. In addition, note that values of T_h and T_c are available from the experiment. Therefore all of the variables in Eq X3.67 except the coefficients have been experimentally determined. If the experiment is repeated over a range of values of T_h and T_c, the entire data set can be used to evaluate the best values of the coefficients by normal least-squares fitting procedures. Once these coefficients have been determined, they are equally applicable to Eq X3.56, and λ(*T*) is therefore known.

NOTE X3.2—The process described above works for thermal transmission properties that show a gradual change with temperature. The practice may not work for such possibilities as (*1*) the onset of convection as observed in Reference (**21**); (*2*) abrupt phase change in one of the insulation components caused by a blowing gas condensation; and (*3*) heat flow direction abnormalities found in reflective insulations.

NOTE X3.3—This procedure is based on the assumption that a unique dependence of thermal conductivity on temperature exists for the material. Such a unique dependence may only be approximate, depending on the coupling effects of the underlying heat transfer mechanisms or irreversible changes in the material during the measurement process. The most convenient check to determine the existence of such effects is to intermix data of both small and large temperature differences in the fit of Eq X3.11. X3.6. If the deviations of these data from values calculated from Eq X3.11 X3.6 are systematically dependent on the temperature difference, two possibilities must shall be considered: (1) (*a*) a unique temperature dependence does not exist and the systematic dependence on temperature difference is a measure of this inconsistency; or (2) (*b*) the apparatus or measurement procedure produces a systematic bias that depends on temperature difference. To determine which of the two possibilities is the cause of the indicated inconsistency, a detailed examination of the apparatus and procedure, along with further experimentation, is necessary.
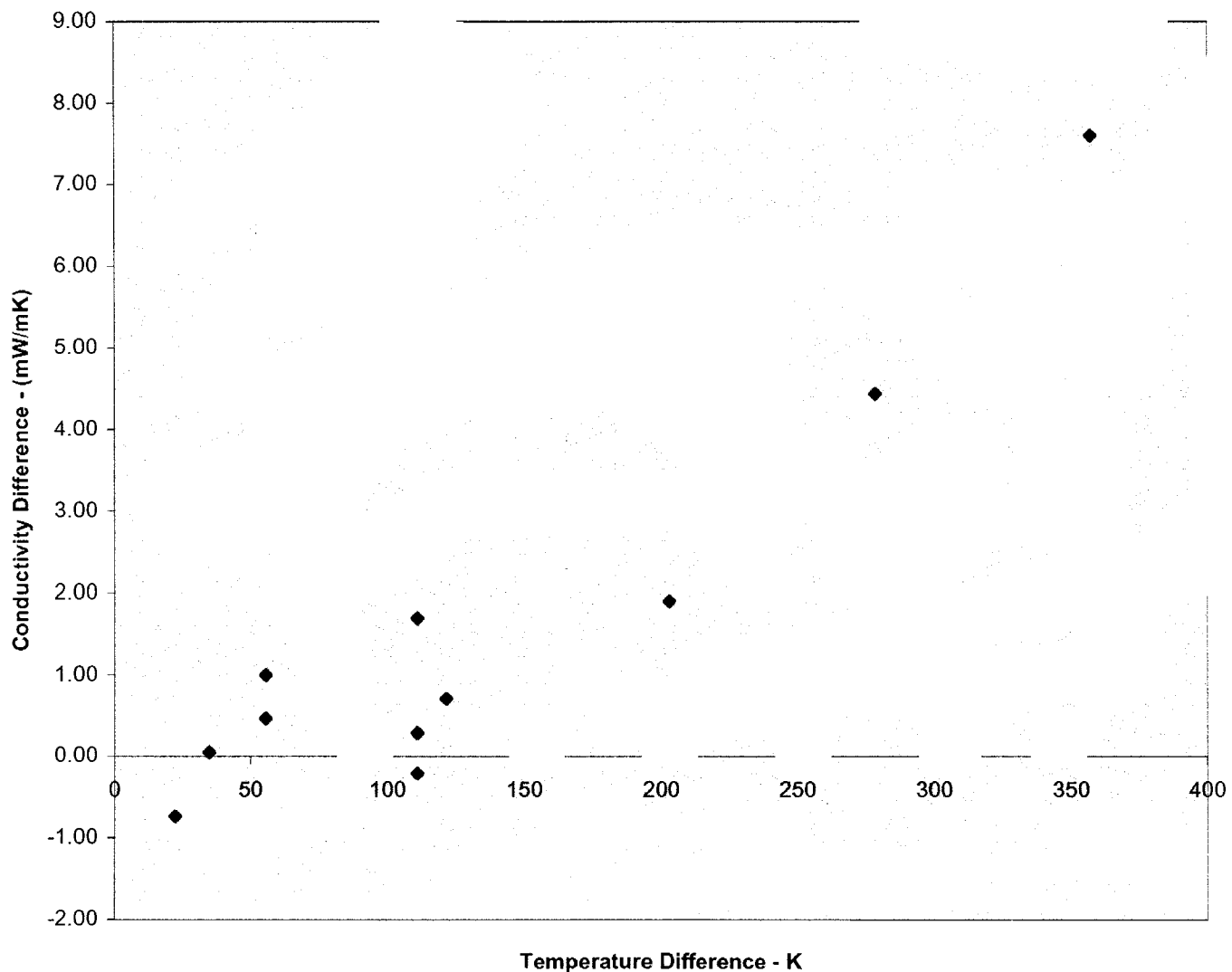
FIG. X3.2 Conductivity Difference versus Test Temperature Difference

TABLE X3.2 Thermal Conductivity Calculated Using Eq X3.6 as a Function of Temperature and Its Difference from the Experimental Thermal Conductivity versus Mean Temperature

Test Point	Mean Temperature	Calculated Thermal Conductivity Using Eq X3.6	Test Thermal Conductivity	Thermal Conductivity Difference	Test Temperature Difference
	(T_m) (K)	$\lambda(T_m)$ (mW/m·K)	λ_{exp}, (mW/m·K)	$\lambda_{exp} - \lambda(T_m)$ (mW/m·K)	ΔT (K)
1	297.05	34.3	33.6	−0.73	22.3
2	315.85	36.2	36.2	0.05	34.9
5	422.05	52.6	53.1	0.46	55.5
3	366.50	42.6	43.6	0.99	55.6
7	533.15	83.8	84.1	0.28	111.1
9	588.75	105.9	105.7	−0.21	111.1
10	644.25	132.9	134.6	1.69	111.1
4	367.10	42.7	43.4	0.70	121.4
6	419.00	52.0	53.9	1.90	203.0
8	470.45	64.3	68.7	4.44	278.3
11	529.15	82.4	90.0	7.59	357.1

X4. A HISTORY OF THE DEVELOPMENT OF THE ASTM C 1045 STANDARD

The history of the development of this standard has been prepared for inclusion in the document. The following discussion, while not detailed or complete, provides a brief overview of the changes that have taken place over the years since the standard was first written. lt is the intent of the task group to expand on the information contained below in the next revision of this practice.

X4.1 *First Edition*—The first published version of the ASTM C 1045 standard practice was written in the early 1980's and published in Volume 04.06 of the ASTM Book of Standards in August of 1985. As stated in the original scope: "This practice provides requirements and guidelines for the determination of thermal transmission properties based upon heat flux measurements under a variety of conditions. The practice is directed particularly toward a description of the heat flux and associated measurements necessary to obtain useful properties that are applicable to end-use conditions." The standard was initially developed as a way to consolidate the common background discussion that had been included in several of the Thermal Measurements Subcommittee C16.30 test methods into a single document that could be reference in those methods and others being developed. The original concept was to have the theoretical basis for the calculation of thermal properties, including the limitations associated with those properties, in this Standard Practice and retain the equations in the Test Methods.

X4.2 *Revision #1*—The first revision of the document, published as ASTM C 1045-90, did not substantially change the Practice but added text to help with it's understanding. The primary addition was an Appendix that gave some mathematical definitions of the equation variables and an example of how the practice could be used.

X4.3 *Revision #2*—The second revision of the standard practice was approved in July 1997. This revision was motivated by the complaints from many users of C 1045 that the previous versions of the standard was difficult, if not impossible, to understand and of no practical use. In this revision, much of the educational information was moved to the Appendix portion of the document so that only the "cookbook" materials necessary to make the fundamental calculations in support of the thermal test methods remained in the body of the standard. Unfortunately, some of the educational information, thought to be too theoretical and not practical, was dropped in this editing.

X4.4 *Revision #3*—This latest revision, started in 1998 and finally approved in 2001, was aimed at reaching a compromise between the "theoretical" and the "practical" factions on the task group. While each side had strong arguments for their version of the practice, compromise was necessary and finally available. Unfortunately, during the last stages of this revision, Bradley Peavy who had been a strong voice in the development of the technical basis for this practice, passed away. We have attempted to carry on his concerns in the development of this version of the standard and will miss his guidance.

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