Designation: D 6028 – 96

AMERICAN SOCIETY FOR TESTING AND MATERIALS
100 Barr Harbor Dr., West Conshohocken, PA 19428
Reprinted from the Annual Book of ASTM Standards. Copyright ASTM

Standard Test Method (Analytical Procedure) for Determining Hydraulic Properties of a Confined Aquifer Taking into Consideration Storage of Water in Leaky Confining Beds by Modified Hantush Method[1]

1. Scope

1.1 This test method covers an analytical procedure for determining the transmissivity and storage coefficient of a confined aquifer taking into consideration the change in storage of water in overlying or underlying confining beds, or both. This test method is used to analyze water-level or head data collected from one or more observation wells or piezometers during the pumping of water from a control well at a constant rate. With appropriate changes in sign, this test method also can be used to analyze the effects of injecting water into a control well at a constant rate.

1.2 This analytical procedure is used in conjunction with Test Method D 4050.

1.3 *Limitations*—The valid use of the modified Hantush method **(1)**[2] is limited to the determination of hydraulic properties for aquifers in hydrogeologic settings with reasonable correspondence to the assumptions of the Hantush-Jacob method (Test Method D 6029) with the exception that in this case the gain or loss of water in storage in the confining beds is taken into consideration (see 5.1). All possible combinations of impermeable beds and source beds (for example, beds in which the head remains uniform) are considered on the distal side of the leaky beds that confine the aquifer of interest (see Fig. 1).

1.4 The values stated in SI units are to be regarded as standard.

1.5 *This standard does not purport to address all of the safety concerns, if any, associated with its use. It is the responsibility of the user of this standard to establish appropriate safety and health practices and determine the applicability of regulatory limitations prior to use.*

2. Referenced Documents

2.1 *ASTM Standards:*

D 653 Terminology Relating to Soil, Rocks, and Contained Fluids[3]

D 4050 Test Method (Field Procedure) for Withdrawal and Injection Well Tests for Determining Hydraulic Properties of Aquifer Systems[3]

D 4106 Test Method (Analytical Procedure) for Determining Transmissivity and Storage Coefficient of Nonleaky Confined Aquifers by the Theis Nonequilibrium Method[3]

D 6029 Test Method (Analytical Procedure) for Determining Hydraulic Properties of a Confined Aquifer and a Leaky Confining Bed with Negligible Storage by the Hantush Jacob Method[4]

3. Terminology

3.1 *Definitions:*

3.1.1 *aquifer, confined, n*—an aquifer bounded above and below by confining beds and in which the static head is above the top of the aquifer.

3.1.2 *aquifer, unconfined, n*—an aquifer is unconfined where it has a water table.

3.1.3 *coefficient of leakage, n*—see *leakance*.

3.1.4 *confining bed, n*—a hydrogeologic unit of less permeable material bounding one or more aquifers.

3.1.5 *control well, n*—well by which the head and flow in the aquifer is changed, for example, by pumping, injection, or change of head.

3.1.6 *drawdown, n*—vertical distance the static head is lowered due to the removal of water.

3.1.7 *head, n*—see *head, static*.

3.1.8 *head, static, n*—the height above a standard datum of the surface of a column of water (or other liquid) that can be supported by the static pressure at a given point.

3.1.9 *hydraulic conductivity, n*—(field aquifer test) the volume of water at the existing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit area measured at right angles to the direction of flow.

3.1.10 *leakance, n*—the ratio of the vertical hydraulic conductivity of a confining bed to its thickness.

3.1.11 *observation well, n*—a well open to all or part of an aquifer.

3.1.12 *piezometer, n*—a device used to measure static head at a point in the subsurface.

3.1.13 *specific storage, n*—the volume of water released

---

[1] This test method is under the jurisdiction of Committee D-18 on Soil and Rock and is the direct responsibility of Subcommittee D18.21 on Ground Water and Vadose Zone Investigations.

Current edition approved Oct. 10, 1996. Published April 1997.

[2] The boldface numbers in parentheses refer to a list of references at the end of this test method.

[3] *Annual Book of ASTM Standards*, Vol 04.08.

[4] *Annual Book of ASTM Standards*, Vol 04.09.

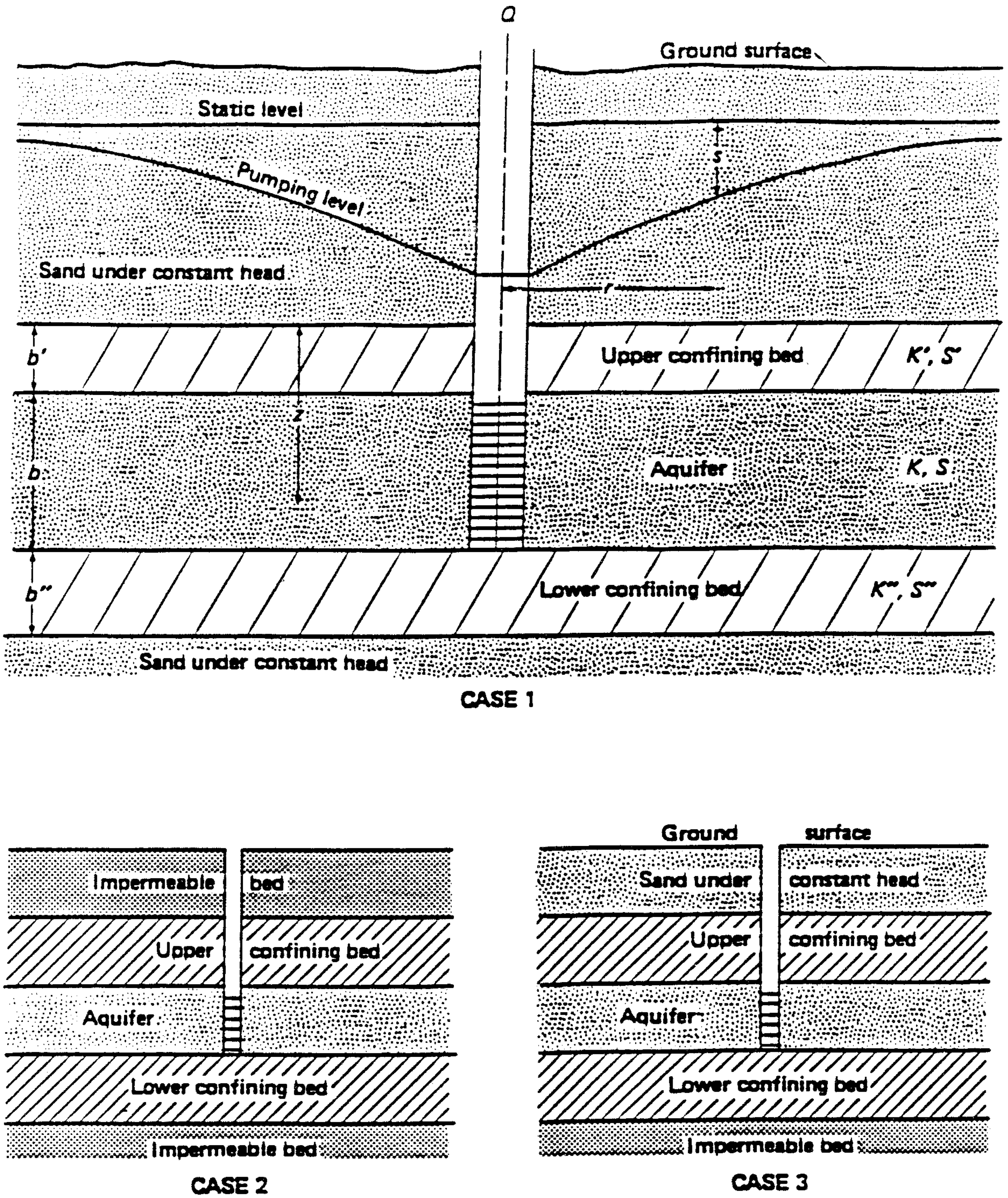

FIG. 1 Cross Sections Through Discharging Wells in Leaky Aquifers with Storage of Water in the Confining Beds, Illustrating Three Different Cases of Boundary Conditions (from Reed (2))

from or taken into storage per unit volume of the porous medium per unit change in head.

3.1.14 *storage coefficient*, *n*—the volume of water an aquifer releases from or takes into storage per unit surface area of the aquifer per unit change in head.

3.1.14.1 *Discussion*—For a confined aquifer, the storage coefficient is equal to the product of the specific storage and aquifer thickness. For an unconfined aquifer, the storage coefficient is approximately equal to the specific yield.

3.1.15 *transmissivity*, *n*—the volume of water at the prevailing kinematic viscosity that will move in a unit time under a unit hydraulic gradient through a unit width of the aquifer.

3.1.16 For definitions of other terms used in this test method, see Terminology D 653.

3.2 *Symbols:Symbols and Dimensions*:

3.2.1 $H(u,\beta)$—well function for leaky systems where water storage in confining beds is important [*nd*].

3.2.2 *K*—hydraulic conductivity of the aquifer [LT^{-1}].

3.2.2.1 *Discussion*—The use of the symbol K for the term hydraulic conductivity is the predominant usage in groundwater literature by hydrogeologists, whereas the symbol k is commonly used for this term in soil and rock mechanics and soil science.

3.2.3 K', K''—vertical hydraulic conductivities of the confining beds through which leakage can occur [LT^{-1}].

3.2.4 Q—discharge [L^3T^{-1}].

3.2.5 $S = bS_s$—storage coefficient of the aquifer [*nd*].

$\underline{3.2.6\ \ S' = b'S'_s}$ —storage coefficients of the confining beds [*nd*].

$\underline{S'' = b''S''_s}$

3.2.7 S_s—specific storage of the aquifer [L^{-1}].

3.2.8 $\underline{S'_s\ S''_s}$ —specific storages of the confining beds. $\underline{[L^{-1}]}$

3.2.9 T—transmissivity [L^2T^{-1}].

3.2.10 $u = \dfrac{r^2s}{4Tt}$ [*nd*].

3.2.11 *W(u,r/B)*—well function for leaky aquifer systems with negligible storage changes in confining beds [*nd*].

3.2.12 *W(u)*—well function for nonleaky aquifer systems [*nd*].

3.2.13 b—thickness of aquifer [L].

3.2.14 b', b''—thicknesses of the confining beds through which leakage can occur [L].

3.2.15 r—radial distance from control well [L].

3.2.16 s—drawdown [L].

3.2.17 $B = \sqrt{\dfrac{Tb'}{K'}}\,[L]$.

3.2.18 t—time since pumping or injection began [T].

3.2.19 $\beta = \dfrac{r}{4b}\left(\sqrt{\dfrac{K'S'}{b'KS_s}} + \right)\sqrt{\dfrac{K''S''}{b''KS_s}}\,[nd]$.

4. Summary of Test Method

4.1 This test method involves pumping a control well that is fully screened through the confined aquifer and measuring the water-level response in one or more observation wells or piezometers. The well is pumped at a constant rate. The water-level response in the aquifer is a function of the transmissivity and storage coefficient of the aquifer and the leakance coefficients and storage coefficients of the confining beds. Alternatively, the test method can be performed by injecting water at a constant rate into the control well. Analysis of buildup of water level in response to injection is similar to analysis of drawdown of water level in response to withdrawal in a confined aquifer. The water-level response data are analyzed using a set of type curves.

4.2 *Solution*—Hantush **(1)** gave solutions applicable to each of Cases 1, 2, and 3 shown in Fig. 1 for "relatively small" values of time and for "relatively large" values of time. The solution applicable for each case for relatively small values of time can be written as follows

$$s = \frac{Q}{4\pi T} H(u,\beta) \tag{1}$$

where:

$$u = \frac{r^2S}{4Tt} \tag{2}$$

and

$$\beta = \frac{r}{4b}\left(\frac{K'S'}{b'KS_s} + \frac{K''S''}{b''KS_s}\right) \tag{3}$$

$$H(u,\beta) = \int_u^\infty \frac{e^{-y}}{y}\, erfc \frac{\beta\sqrt{u}}{\sqrt{y(y-u)}}\, dy \tag{4}$$

$$erfc(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty e^{-y^2}\, dy \tag{5}$$

where y is the variable of integration.

4.2.1 The "relatively small" times when Eq 1 is applicable are when:

$$t < \frac{b'S'}{10K'} \text{ and } t < \frac{b''S''}{10K''} \tag{6}$$

Equation 1 is applicable at early times for each of the cases shown in Fig. 1 even though the conditions on the distal sides of the confining beds are quite different because for early times the solution in the aquifer is essentially independent of conditions on the distal side of the confining beds. The effects of those distant boundary conditions are not felt in the aquifer for a while. Eq 1-5 are the basis for the type curve solution that is described by this test method.

4.2.2 For relatively large values of time the solutions given by Hantush **(1)** can be written as:

4.2.2.1 *Case 1*—Heads in zones on the distal side of the confining beds remain constant and are unaffected by discharge of the pumped well. For times when

$$t > 5\frac{b'S'}{K'} \text{ and } t > 5\frac{b''S''}{K''} \tag{7}$$

are both satisfied, then

$$s = \frac{Q}{4\pi T} W(u\delta_1, \alpha) \tag{8}$$

where:

$$\delta_1 = 1 + \frac{(S' + S'')}{3S} \text{ and } \alpha = r\sqrt{\frac{K'}{Tb'} + \frac{K''}{Tb''}} \tag{9}$$

Hantush **(1)** notes that if K'', S', and S'' are taken as zero in the flow systems shown in Fig. 1 as Case 1 or Case 3, the resulting flow system is that of a confined aquifer overlying an impermeable bed and the aquifer being overlain by a confining bed in which the storage is negligible. Hantush gives the solution for that special case as follows:

$$s = \frac{Q}{4\pi T} W(u,r/B) \tag{10}$$

where:

$$\frac{r}{B} = r\sqrt{\frac{K'}{Tb'}}$$

Note that $W(u,r/B)$ is the well function for leaky systems with negligible storage in the confining beds given by Hantush and Jacob **(3)** and described in Test Method (D 6029). That function is defined as follows:

$$W(u,r/B) = \int_u^\infty \exp\left(-y - r^2/(4B^2y)\right)\frac{dy}{y} \tag{11}$$

4.2.2.2 *Case 2*—The materials in the zones on the distal sides of the confining beds are impermeable. For times when

$$t > 10\frac{b'S'}{K'} \text{ and } t > 10\frac{b''S''}{K''} \tag{12}$$

are both satisfied, then

$$s = \frac{Q}{4\pi T} W(u,\delta_2) \tag{13}$$

where:

$$\delta_2 = 1 + \frac{(S' + S'')}{S}$$

and where the function $W(u)$ is the well function for non-leaky aquifers that appears in the solution given by Theis **(4)** described in Test Method D 4106 for drawdowns in response to a well pumped at a constant rate from a non-leaky aquifer.

4.2.2.3 *Case 3*—The materials on the distal side of one confining bed are impermeable and the heads on the distal sides of the other confining bed remain constant and are unaffected by discharge of the pumped well. For times when

$$t > \frac{5b'S'}{K'} \text{ and } t > \frac{10b''S''}{K''} \tag{14}$$

are both satisfied, then

$$s = \frac{Q}{4\pi T} W\left(u\delta_3, r\sqrt{\frac{K'}{Tb'}}\right) = \frac{Q}{4\pi t} W(u\delta_3, r/B) \tag{15}$$

where:

$$\delta_3 = 1 + (S'' + {}_{S'/3})S \tag{16}$$

and $W(u,r/B)$ is defined in Case 1 (see Eq 11).

Hantush **(1)** did not develop expressions for the solutions to these cases for intermediate times (between“ small” and “large” times). Reed (**(2)** p. 26) notes that Neuman and Witherspoon (**(5)**, p. 250) developed a complete (that is, applicable for all times) solution for Case 1 (source beds on the distal sides of both confining beds) but did not tabulate it.

5. Significance and Use

5.1 *Assumptions*:

5.1.1 The control well discharges at a constant rate, Q.

5.1.2 The control well is of infinitesimal diameter and fully penetrates the aquifer.

5.1.3 The aquifer is homogeneous, isotropic, and areally extensive.

5.1.4 The aquifer remains saturated (that is, water level does not decline below the top of the aquifer).

5.1.5 The aquifer is overlain or underlain, or both, everywhere by confining beds individually having uniform hydraulic conductivities, specific storages, and thicknesses. The confining beds are bounded on the distal sides by one of the cases shown in Fig. 1.

5.1.6 Flow in the aquifer is two-dimensional and radial in the horizontal plane.

5.2 The geometry of the well and aquifer system is shown in Fig. 1.

5.3 *Implications of Assumptions*:

5.3.1 Paragraph 5.1.1 indicates that the discharge from the control well is at a constant rate. Paragraph 8.1 of Test Method D 4050 discusses the variation from a strictly constant rate that is acceptable. A continuous trend in the change of the discharge rate could result in misinterpretation of the water-level change data unless taken into consideration.

5.3.2 The leaky confining bed problem considered by the modified Hantush method requires that the control well has an infinitesimal diameter and has no storage. Moench **(6)** generalized the field situation addressed by the modified Hantush **(1)** method to include the well bore storage in the pumped well. The mathematical approach that he used to obtain a solution for that more general problem results in a Laplace transform solution whose analytical inversion has not been developed and probably would be very complicated, if possible, to evaluate. Moench **(6)** used a numerical Laplace inversion algorithm to develop type curves for selected situations. The situations considered by Moench indicate that large well bore storage may mask effects of leakage derived from storage changes in the confining beds. The particular combinations of aquifer and confining bed properties and well radius that result in such masking is not explicitly given. However, Moench (**(6)**, p. 1125) states “Thus observable effects of well bore storage are maximized, for a given well diameter, when aquifer transmissivity Kb and the storage coefficient S_sb are small.” Moench (p. 1129) notes that “...one way to reduce or effectively eliminate the masking effect of well bore storage is to isolate the aquifer of interest with hydraulic packers and repeat the pump test under pressurized conditions. Because well bore storage C will then be due to fluid compressibility rather than changing water levels in the well”...“ the dimensionless well bore storage parameter may be reduced by 4 to 5 orders of magnitude.”

5.3.3 The modified Hantush method assumes, for Cases 1 and 3 (see Fig. 1), that the heads in source layers on the distal side of confining beds remain constant. Neuman and Witherspoon **(7)** developed a solution for a case that could correspond to Hantush’s Case 1 with $K'' = O = S''$ except that they do not require the head in the unpumped aquifer to remain constant. For that case, they concluded that the drawdowns in the pumped aquifer would not be affected by the properties of the other, unpumped, aquifer when (Neuman and Witherspoon **(7)** p. 810) time satisfies:

$$t \leq 0.1 \frac{S'b'}{K'} \tag{17}$$

5.3.4 Implicit in the assumptions are the conditions that the flow in the confining beds is essentially vertical and in the aquifer is essentially horizontal. Hantush’s **(8)** analysis of an aquifer bounded only by one leaky confining bed suggested that these assumptions are acceptably accurate wherever

$$\frac{K}{K'} > 100 \frac{b}{b'} \tag{18}$$

That form of relation between aquifer and confining bed properties may also be a useful guide for the case of two leaky confining beds.

6. Apparatus

6.1 Analysis of data from the field procedure (see Test Method D 4050) by this test method requires that the control well and observation wells meet the requirements specified in the following paragraphs.

6.2 *Construction of Control Well*—Install the control well in the aquifer and equip with a pump capable of discharging water from the well at a constant rate for the duration of the test. Preferably, the control well should be open throughout the full thickness of the aquifer. If the control well partially penetrates the aquifer, take special precaution in the placement or design of observation wells.

6.3 *Construction and Location of Observation Wells and Piezometers*—Construct one or more observation wells or piezometers screened only in the pumped aquifer at a distance from the control well. Observation wells may be open through all or part of the thickness of the aquifer. Hantush (**(9)** p. 350) indicates that the effects of a partially penetrating control well can be neglected for

$$r > 1.5b\sqrt{\frac{K_r}{K_z}} \qquad (19)$$

where K_r and K_z are the aquifer hydraulic conductivities in the horizontal and vertical directions, respectively. Although that relationship was developed for an aquifer confined by a leaky confining bed in which storage is neglected, it may be a useful guideline for the cases where storage in the confining beds is important. If an observation well fully penetrates the aquifer, it's drawdown is not affected by a partially penetrating control well and it reacts as if the control well completely penetrated the aquifer (Hantush **(9)** p. 351).

7. Procedure

7.1 Pretest preparations are described in detail in Test Method D 4050. The overall test procedure consists of (*1*) conducting the field procedure for withdrawal or injection well tests (described in Test Method D 4050) and (*2*) analysis of the field data, which is addressed in Section 8.

8. Calculation and Interpretation of Test Data

8.1 *Aquifer*—Test data for "relatively small" values of time are analyzed using Eq 1-3. The graphical procedure used to calculate test results is based on the functional relations between $H(u,\beta)$ and s and between u and t/r^2.

NOTE 1—Because the $H(u,\beta)$ type curve method is based on the assumption that the duration of the test is such that the boundary conditions on the distal sides of the confining beds have not yet affected drawdowns in the pumped aquifer, only the relatively early-time drawdown data should be used in fitting the $H(u,\beta)$ curves. "Relatively late-time" drawdown data can be analyzed using Eq 8, Eq 13, or Eq 15 for field conditions described by Cases 1, 2, or 3, respectively. Equations 8 and Equations 15 correspond to the condition that there are no further changes in storage in the leaky confining beds bounded by constant head layers and leakage into the pumped aquifer though those confining beds by those times correspond entirely to water transmitted from the source (constant head) layers. That situation is discussed in Test Method D 6029. Reed (**(4)** p. 28–29) notes that the late-time data for Cases 1 and 3 will fall on the flat part of the $W(u,r/B)$ type curves and a time-drawdown plot match would be indeterminate. Equation 13 corresponds to non-leaky confined aquifers, and that situation is discussed in Test Method D 4106. Spane and Wurstner **(10)** discuss the advantage of supplementing the type curve plots of drawdown versus time by plots of the derivative of drawdown (with respect to an appropriate time function) versus time as an aid in selecting an aquifer interpretation model and in estimating the aquifer parameters. They discuss also an approach that transforms water-level recovery (that is, the response of water levels when the pump is shut off) data plots to a form that can be analyzed with drawdown data in constructing derivative plots. To apply the derivative methods requires that measurements be spaced closely enough that numerically developed time derivatives can be reasonably approximated.

8.1.1 Plot values of $H(u,\beta)$ versus $1/u$ for selected values of β on logarithmic-scale paper. This plot is referred to as the type curve plot. Table 1 gives a tabulation of values of $H(u,\beta)$ for selected values of u and β. Fig. 2 is a logarithmic plot of $H(u,\beta)$ versus $1/u$ for selected values of β (from Kruseman and deRidder **(11)**). If a set of type curves are inaccessible, these data can be used to develop type curves. A more extensive tabulation of $H(u,\beta)$ is given in Hantush **(12)**. Some readily available sources of these type curves are Lohman **(13)** and Reed **(2)**. Dawson and Istok **(14)** provide a diskette that contains a computer program that computes values of this function. A listing for a computer program to compute values of $H(u,\beta)$ is in Reed **(2)** and a diskette including that program is available from the National Water Information System (NWIS) office of the U.S. Geological Survey.

8.1.2 On logarithmic tracing paper of the same scale and size as the $H(u,\beta)$ versus $1/u$ type curves, plot values of drawdown, s, for each observation well on the vertical coordinate versus time divided by distance between the control well and the observation well squared, t/r^2, on the horizontal coordinate. This plot is referred to as the data plot.

8.1.3 Overlay the data plot on the type curve plot and, keeping the coordinate axes of the two plots parallel, shift the plot to the position where the data for each observation well falls either between one pair of the β curves, or along one of them. It is preferable for two or more observation wells to be at different distances from the control well. Recall the definition of β (see Eq 3). The advantages of having two or more observation wells is that the distance values, r, for the observation wells should fall on curves having proportional β values. For example, if data are available from three observation wells at 100, 200, and 800 ft from the control well, the data plots for the three wells should match curves having corresponding β values having the ratios 1:2:8. Weeks **(15)** notes that for values of β ranging from zero (this is the Theis curve which corresponds to a non-leaky case) to about 0.7, there is virtually no difference in the shape of the curves on the $H(u,\beta)$ versus $1/u$ plot. Weeks states that if β falls within this range for a given observation well it is impossible to determine unique values of transmissivity and storativity for the aquifer and β using only that well. The use of a composite plot involving more than one observation well at different distances, r, may permit a unique fit to be obtained.

NOTE 2—Moench **(6)** notes that it is desirable to also obtain data on water-level changes in the pumped well because it can "...be helpful in determining the presence or absence of leakage when compared with observation well data." However, data from the pumped well are affected by variations in the pumping rate, effects of well-bore storage, and the "skin" (a zone around the well hydraulically different from the native materials because of disturbance and alteration caused by well drilling and construction).

8.1.4 Select and record the values of $H(u,\beta)$, $1/u$, s, and t/r^2 at an arbitrary point, referred to as the match point, anywhere on the overlapping part of the type curve plot and the data plot. For convenience, the match point may be selected where $H(u,\beta)$ and $1/u$ are integer values. Record the value of β for each observation well's data.

8.1.5 Using the selected values, determine the transmissivity and storage coefficient from Eq 1 and Eq 2:

$$T = \frac{Q}{4\pi s} H(u,\beta) \qquad (20)$$

TABLE 1 Values of $H(u,\beta)$ for Selected Values of u and β (from Reed).

NOTE 1—From Hantush . Numbers in parentheses are powers of 10 by which the other numbers are multiplied (for example 963(−4) = 0.0963)

u	β							
	0.03	0.1	0.2	1	3	10	30	100
1×10^{-9}	12.3088	11.1051	10.0066	8.8030	7.7051	6.5033	5.4101	4.2221
2	11.9622	10.7585	9.6602	8.4566	7.3590	6.1579	5.0666	3.8839
3	11.7593	10.5558	9.4575	8.2540	7.1565	5.9561	4.8661	3.6874
5	11.5038	10.3003	9.2021	7.9987	6.9016	5.7020	4.6142	3.4413
7	11.3354	10.1321	9.0339	7.8306	6.7337	5.5348	4.4487	3.2804
1×10^{-8}	11.1569	9.9538	8.8556	7.6525	6.5558	5.3578	4.2737	3.1110
2	10.8100	9.6071	8.5091	7.3063	6.2104	5.0145	3.9352	2.7858
3	10.6070	9.4044	8.3065	7.1039	6.0085	4.8141	3.7383	2.5985
5	10.3511	9.1489	8.0512	6.8490	5.7544	4.5623	3.4919	2.3662
7	10.1825	8.9806	7.8830	6.6811	5.5872	4.3969	3.3307	2.2159
1×10^{-7}	10.0037	8.8021	7.7048	6.5032	5.4101	4.2221	3.1609	2.0591
2	9.6560	8.4554	7.3585	6.1578	5.0666	3.8839	2.8348	1.7633
3	9.4524	8.2525	7.1560	5.9559	4.8661	3.6874	2.6469	1.5966
5	9.1955	7.9968	6.9009	5.7018	4.6141	3.4413	2.4137	1.3944
7	9.0261	7.8283	6.7329	5.5346	4.4486	3.2804	2.2627	1.2666
1×10^{-6}	8.8463	7.6497	6.5549	5.3575	4.2736	3.1110	2.1051	1.1361
2	8.4960	7.3024	6.2091	5.0141	3.9350	2.7857	1.8074	0.8995
3	8.2904	7.0991	6.0069	4.8136	3.7382	2.5984	1.6395	0.7725
5	8.0304	6.8427	5.7523	4.5617	3.4917	2.3661	1.4354	0.6256
7	7.8584	6.6737	5.5847	4.3962	3.3304	2.2158	1.3061	0.5375
1×10^{-5}	7.6754	6.4944	5.4071	4.2212	3.1606	2.0590	1.1741	0.4519
2	7.3170	6.1453	5.0624	3.8827	2.8344	1.7632	0.9339	0.3091
3	7.1051	5.9406	4.8610	3.6858	2.6464	1.5965	0.8046	0.2402
5	6.8353	5.6821	4.6075	3.4394	2.4131	1.3943	0.6546	0.1635
7	6.6553	5.5113	4.4408	3.2781	2.2619	1.2664	0.5643	0.1300
1×10^{-4}	6.4623	5.3297	4.2643	3.1082	2.1042	1.1359	0.4763	963(−4)
2	6.0787	4.9747	3.9220	2.7819	1.8062	0.8992	0.3287	494(−4)
3	5.8479	4.7655	3.7222	2.5937	1.6380	0.7721	0.2570	315(−4)
5	5.5488	4.4996	3.4711	2.3601	1.4335	0.6252	0.1818	166(−4)
7	5.3458	4.3228	3.3062	2.2087	1.3039	0.5370	0.1412	103(−4)
1×10^{-3}	5.1247	4.1337	3.1317	2.0506	1.1715	0.4513	0.1055	390(−5)
2	4.6753	3.7598	2.7938	1.7516	0.9305	0.3084	551(−4)	169(−5)
3	4.3993	3.5363	2.5969	1.5825	0.8006	0.2394	355(−4)	713(−6)
5	4.0369	3.2483	2.3499	1.3767	0.6498	0.1677	190(−4)	205(−6)
7	3.7893	3.0542	2.1877	1.2460	0.5589	0.1292	120)−4)	821(−7)
1×10^{-2}	3.5195	2.8443	2.0164	1.1122	0.4702	955(−4)	695(−5)	274(−7)
2	2.9759	2.4227	1.6853	0.8677	0.3214	487(−4)	205(−5)	226(−8)
3	2.6487	2.1680	1.4932	0.7353	0.2491	308)−4)	888(−6)	
5	2.2312	1.8401	1.2535	0.5812	0.1733	160)−4)	261(−6)	
7	1.9558	1.6213	1.0979	0.4880	0.1325	982(−5)	106(−6)	
1×10^{-1}	1.6667	1.3893	0.9358	0.3970	966(−4)	552(−5)	365(−7)	
2	1.1278	0.9497	0.6352	0.2452	468(−4)	149(−5)	307(−8)	
3	0.8389	0.7103	0.4740	0.1729	281(−4)	592(−6)		
5	0.5207	0.4436	0.29556	0.1006	130(−4)	151(−6)		
7	0.3485	0.2980	0.1985	646(−4)	714(−5)	534(−7)		
1×1	0.2050	0.1758	0.1172	365(−4)	337(−5)	151(−7)		
2	458(−4)	395(−4)	264(−4)	760(−5)	487(−6)			
3	122(−4)	106(−4)	707(−5)	196(−5)	102(−6)			
5	108(−5)	934(−6)	624(−6)	167(−6)	672(−8)			
7	109(−6)	941(−7)	629(−7)	165(−7)				
1×10	391(−8)	339(−8)	227(−8)					
2								
3								
5								
7								

$$S = 4Tu \frac{t}{r^2} \tag{21}$$

Equation 3 indicates that if the aquifer of interest is overlain and underlain by leaky confining beds, the value of β characterizes a composite of the properties of the individual confining beds.

8.1.6 Reed (**(2)** p. 26–27) notes that for certain special situations, the β values may be used to characterize individual confining bed properties. For example, suppose that the hydrogeologic information for an area suggests that the value of K'' S'' for the underlying confining bed is negligible. This would occur if the bed is effectively impermeable and incompressible. For that situation Eq 3 reduces to:

$$\beta = \frac{r}{4b}\sqrt{\frac{K'S'}{b'KS_S}} \tag{22}$$

which can be manipulated to give that

$$K'S' = \frac{16\beta^2 b^2 KS_S}{r_2} b' \tag{23}$$

Recalling that $T = bK$ and $S = bS_s$ this can be rewritten as

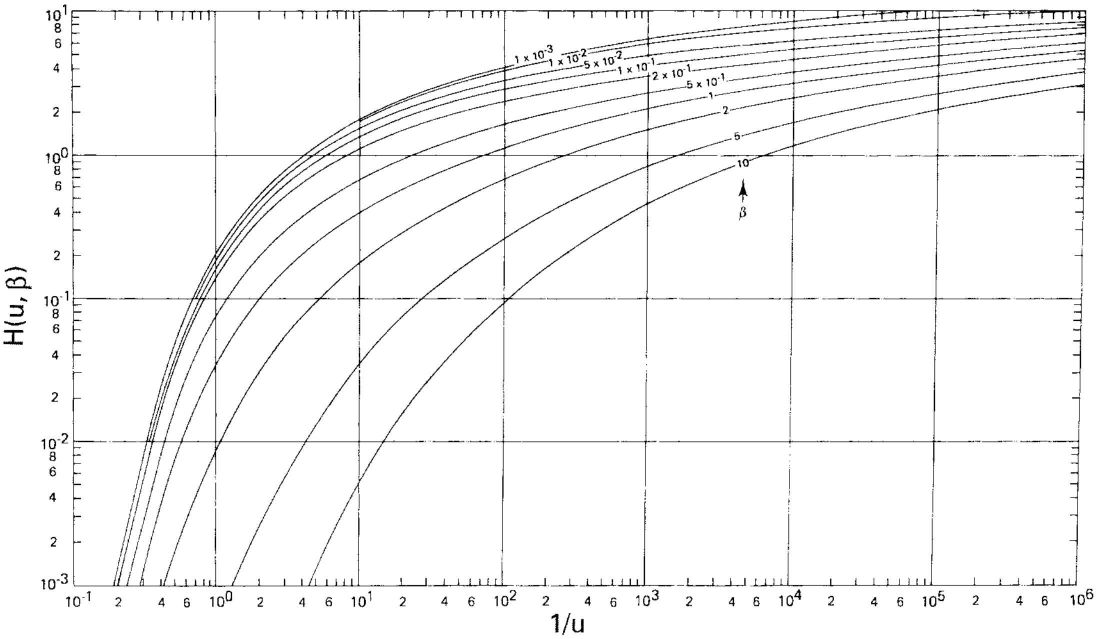

FIG. 2 Family of Curves of H(u,β) versus 1/u for Selected Values of β (from Kruseman and deRidder (11))

$$K'S' = \frac{16\beta^2\, TS}{r^2}\, b' \qquad (24)$$

Note that b' and r are measured and T, S, and β are estimated from the test analysis so that a value for $K'S'$ can be calculated. Reed **(2)** notes that if one expects $K''S'' = K'S'$ then Eq 3 can be manipulated to give that:

$$K'S' = \frac{16\beta^2}{r^2}\, TS\, \frac{b'b''}{b' + b'' + 2\sqrt{b'b''}} \qquad (25)$$

NOTE 3—Fig. 3 shows an application of the type-curve method using the modified Hantush method taken from Stallman and Weeks **(16)**. An example of a match of multiple observation-well data to the Hantush **(1)** type curves is shown in Fig. 3. This test, performed on a well near Houston, Texas, is an area where significant subsidence has been induced by ground-water withdrawal, and thus almost undoubtedly should fit the Hantush **(1)** curves. Note that differences in positions of the data curves are in the right direction to indicate the effects of confining-bed storage, but their departures from the Theis curve are too small to be totally convincing. Nonetheless, selection of the best-matching β curve values, the appropriate r values for the observation wells, and an estimate of S'_s results in a hydraulic conductivity of the confining layer comparable to that used in modeling the Houston aquifer (Jorgensen **(17)**). The lack of clear definition of the effects of confining-bed leakage in the data response curves suggests that an observation piezometer in the confining layer would have been desirable in order to apply the Neuman-Witherspoon **(18)** ratio method.

9. Report

9.1 *Introduction*—The introductory section presents the scope and purpose of the modified Hantush method. Summarize the field hydrogeologic conditions and the field equipment and instrumentation including the construction of the control well and observation wells and piezometers, or both, the method of measurement of discharge and water levels, and the duration of the test and pumping rate. Discuss the rationale for selecting the modified Hantush formulation which assumes that the gain or loss of water from storage in the confining bed(s) is significant.

9.2 *Hydrogeologic Setting*—Review the information available on the hydrogeology of the site. Include the driller's logs and geologists's description of drill cuttings. Interpret and describe the hydrogeology of the site as it pertains to the selection of the methods for conducting and analyzing an aquifer test. Compare the hydrogeologic characteristics of the site as it conforms and differs from the assumptions in the solution to the test method.

9.3 *Equipment*—Report the field installation and equipment for the aquifer test, including the construction, diameter, depth of screened interval, and location of control well and pumping equipment, and the construction, diameter, depth, and screened interval of observation wells or piezometers and their distances from the control well.

9.4 *Instrumentation*—Report the field instrumentation for observing water levels, pumping rate, barometric changes, and other environmental conditions pertinent to the test. Include a list of measuring devices used during the test; the manufacturer's name, model number, and basic specifications for each major item; and pertinent information on the method, including date, of the last calibration, if applicable.

9.5 *Testing Procedures*—State the steps taken in conducting pretest, drawdown, and recovery phases of the test. Include the frequency of measurements of discharge rate, water level in observation wells, and other environmental data recorded during the test procedure.

9.6 *Presentation and Interpretation of Test Results*:

9.6.1 *Data*—Present tables (and charts for graphically recorded data) of data collected during the test (pretest and recovery included). Show methods of adjusting water levels for barometric changes, tidal changes, or other background water level changes (interference with other operations and boundary conditions) and calculation of drawdown.

9.6.2 *Data Plots*—Present data plots used in analysis of the data. Show overlays of data plots and type curves with match points and corresponding values of parameters at match points.

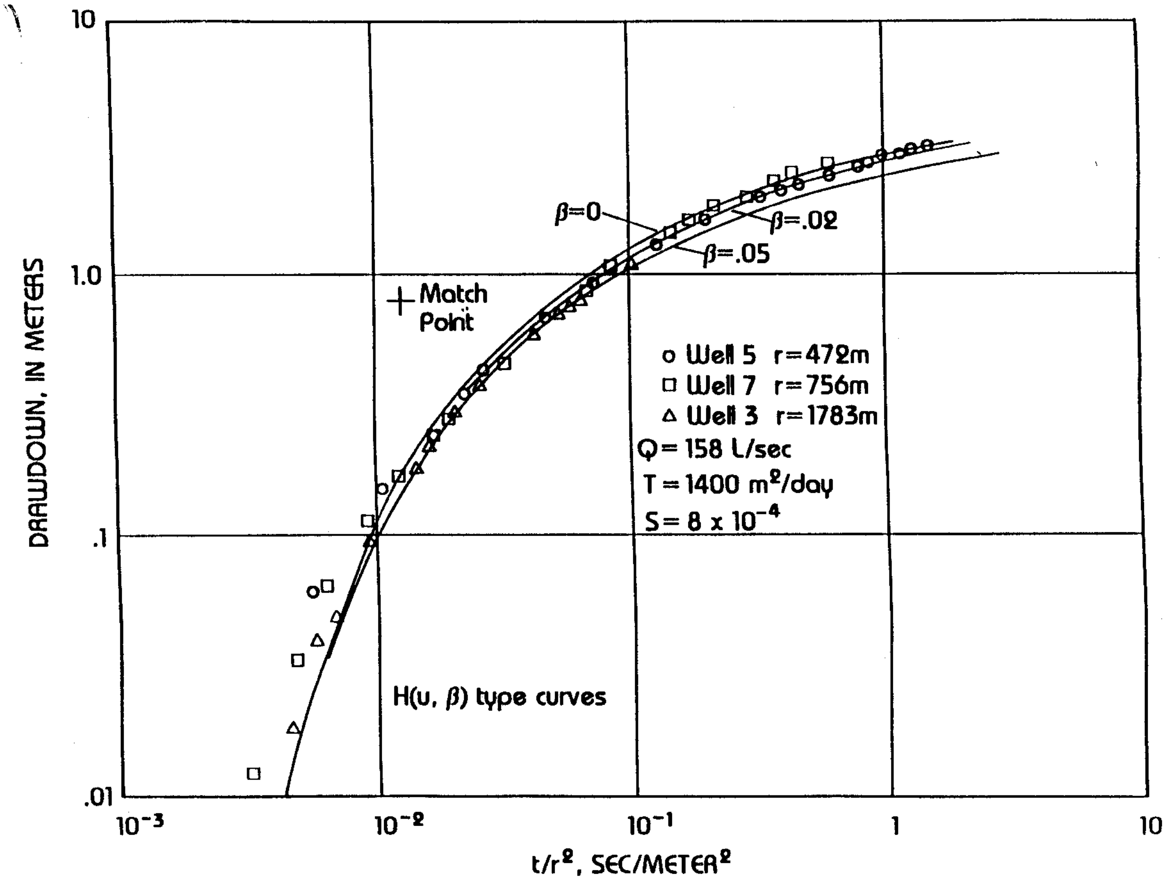

FIG. 3 Graph Showing Match of Drawdown Data for Three Observation Wells Showing Three Selected β Curves (from Stallman and Weeks (16))

9.6.3 *Calculation*—Show calculations of transmissivity, storage coefficient, and any parameters characterizing the leaky confining beds.

9.7 Evaluate qualitatively the overall accuracy of the test on the basis of the adequacy of instrumentation and observations of stress and response, and the conformance of site assumptions to test results.

10. Precision and Bias

10.1 It is not practical to specify the precision of the procedure in this test method because the response of aquifer systems during aquifer tests is dependent upon ambient system stresses. No statement can be made about bias because no true reference values exist.

11. Keywords

11.1 aquifers; aquifer tests; confined aquifers; confining beds; control wells; ground water; hydraulic properties; leakance; leaky aquifers; observation wells; storage coefficient; transmissivity

REFERENCES

(**1**) Hantush, M. S., “Modification of the Theory of Leaky Aquifers,” *Journal of Geophysical Research*, Vol 65, No. 11, 1960, pp. 3713–3725.

(**2**) Reed, J. E., “Type Curves for Selected Problems of Flow to Wells in Confined Aquifers,” *U.S. Geological Survey Techniques of Water-Resources Investigation Book 3*, Chapter B3, 1980.

(**3**) Hantush, M. S., and Jacob, C. E., “Non-Steady Radial Flow in an Infinite Leaky Aquifer,” *Transactions, American Geophysical Union*, Vol 36, No. 1, 1955, pp. 95–100.

(**4**) Theis, C. V., “The Relation Between the Lowering of the Piezometric Surface and the Rate and Duration of Discharge of a Well Using Ground-Water Storage,” *American Geophysical Union Transactions*, Vol 16, Part 2, 1935, pp. 519–524.

(**5**) Neuman, S. P., and Witherspoon, P.A., “Transient Flow of Ground Water to Wells in Multiple-aquifer Systems,” “Sea-Water Intrusion: Aquitards in the Coastal Ground Water Basin of Oxnard Plain, Ventura County,” *California Department of Water Resources Bulletin 63-4*, 1971, pp. 159–359.

(**6**) Moench, A. F., “Transient Flow to a Large-Diameter Well in an Aquifer With Storative Semiconfining Layers,” *Water Resources Research*, Vol 21, No. 8, 1985, pp. 1121–1131.

(**7**) Neuman, S. P., Witherspoon, P. A., “Theory of Flow in a Confined Two Aquifer System,” *Water Resources Research*, Vol 5, No. 4, 1969, pp. 803–816.

(**8**) Hantush, M. S., “Flow of Ground Water in Relatively Thick Leaky Aquifers,” *Water Resources Research*, Vol 3, No. 2, 1967, pp. 583–590.

(**9**) Hantush, M. S., “Hydraulics of Wells,” Chow, Ven Te, ed., *Advances in Hydroscience*, Vol 1, New York, Academic Press, 1964, pp. 281–432.

(**10**) Spane, F. A., Jr., and Wurstner, S. K., “DERIV: A Computer Program for Calculating Pressure Derivatives for Use in Hydraulic Test Analysis,” *Ground Water*, Vol 31, No. 5, 1993, pp. 814–822.

(**11**) Kruseman, G. P., and deRidder, N. A., “Analysis and Evaluation of Pumping Test Data,” *Publication 47*, International Institute for Land Reclamation and Improvement, Wageningen, The Netherlands, 1990 (Second edition, completely revised).

(**12**) Hantush, M. S., “Tables of the function $H(u,\beta) = \int_u^\infty \frac{e^{-y}}{y} \, erfc \left[\frac{\beta\sqrt{u}}{\sqrt{y(y-u)}} \right] dy$,” *New Mexico Institute of Mining and Technology Professional Paper* 103, 1961.

(**13**) Lohman, S. W., “Ground-Water Hydraulics,” *U.S. Geological Survey Professional Paper 708*, 1972.

(**14**) Dawson, K. J., and Istok, J. D., *Aquifer Testing, Design and Analysis of Pumping and Slug Tests*, Lewis Publishers, Inc., 1991.

(**15**) Weeks, E. P., “Aquifer Tests—The State of the Art in Hydrology,” *Proceedings of the Invitational Well-Testing Symposium*, Berkeley, California, Oct. 19–21, 1977, LBL7027, Lawrence Berkeley Laboratory, pp. 14–26.

(**16**) Stallman, R. W., and Weeks, E. P., Section 2.H. Aquifer tests, in Chapter 2-GROUND WATER, *National Handbook of Recommended Methods for Water-Data Acquisition*, 1980, pp. 2-115 to 2-149.

(**17**) Jorgensen, D. G., “Analog-Model Studies of Ground-Water Hydrology in the Houston District, Texas,” *Texas Water-Development Board Report 190*, 1975.

(**18**) Neuman, S. P., and Witherspoon, P. A., “Field Determination of the Hydraulic Properties of Leaky Multiple Aquifer Systems,” *Water Resources Research*, Vol 8, No. 5, 1972, pp. 1284–1298.